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The Canadian Productivity Review

An Update on Depreciation Rates for the Canadian Productivity Accounts

by John Baldwin, Huju Liu and Marc Tanguay

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.. not available for a specific reference period
... not applicable
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0s value rounded to 0 (zero) where there is a meaningful distinction between true zero and the value that was rounded
p preliminary
r revised
x suppressed to meet the confidentiality requirements of the *Statistics Act*
E use with caution
F too unreliable to be published
* significantly different from reference category ($p < 0.05$)

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An Update on Depreciation Rates for the Canadian Productivity Accounts

by

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The Canadian Productivity Review

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Abstract

This paper generates updated estimates of depreciation rates to be used in the Canadian Productivity Accounts for the calculation of capital stock and the user cost of capital. Estimates are derived from depreciation profiles for a diverse set of assets, based on patterns of resale prices and retirement ages.

A maximum likelihood technique is used to jointly estimate changes in the valuation of assets over the course of their service life, as well as the nature of the disposal process used to discard assets to generate depreciation rates. This method is more efficient than others in producing estimates with less bias and higher efficiency.

The earlier estimates that were derived for the period from 1985 to 2001 are compared with those for the latest period, from 2002 to 2010. On average, the estimates of the depreciation rate for buildings are not found to be significantly different. The aggregate average estimates for machinery and equipment have increased, though this is mainly a result of the compositional effect of those categories with higher depreciation rates (such as computers and communication equipment) becoming increasingly important. The estimates for individual assets for the two periods are rarely different from one another. The data from the two periods are then pooled together, yielding estimates to be used in computing the capital stock. The growth rate of capital stock, estimated with the new depreciations rates, is quite similar to that estimated with the old depreciation rates reported in Statistics Canada (2007).

The *ex post* estimates of length of life that are produced using the aforementioned technique are compared to *ex ante* estimates of expected lives based on surveys, and both types of estimates are found to be much the same.

Executive summary

Estimates of depreciation are required to implement the perpetual inventory method that cumulates estimates of past investment to provide summary measures of the amount of net capital that is being applied to the production process.

Obtaining estimates of the rate of depreciation creates numerous difficulties. While depreciation is a concept that is applied directly to the accounts of companies and is used in the calculation of taxes owed to the government, the commonly used estimates contained in balance sheets are not always perceived as being those required by the productivity program. This can occur for a number of reasons—not the least of which is that depreciation allowances used for taxation purposes may differ from the 'real' rate. This happens either because the tax system lags in terms of changes in the durability and longevity of assets, or because the tax system may deliberately choose a rate that is different from the 'real' rate, because it is attempting to stimulate investment.

Rather than simply taking estimates of depreciation from accounting sources, the statistical community has developed alternate methods of estimating depreciation rates. Both the United States and Canada make use of the prices of used assets to estimate depreciation—the rate at which the value of the asset declines from usage. The difference between the two countries is that estimates in the United States are taken from numerous unconnected databases that provide prices of used equipment, while in Canada, the prices come from a single Capital and Repair Expenditures Survey extending back into the 1980s, which also asks for the prices of assets that are sold.

The Canadian Productivity Accounts also cross-reference estimates of depreciation derived from used-asset prices with estimates derived from *ex ante* estimates of the length of life derived from a question in the Capital and Repair Expenditures Survey. This question asks for estimates of the expected length of life at the time of the initial investment, and makes several assumptions about the profile of the rate of decline of the value of an asset in use (what has been referred to in the literature as the declining-balance rate, or DBR). The latter is estimated here from the actual decline pattern derived from the trajectory of used-asset prices over time.

This paper expands on the earlier work (Statistics Canada 2007). It enlarges the database on used-asset prices, and makes use of additional editing techniques on that database. This enlarges the number of observations to around 52,000. The size of this database is unique.

Several findings are noteworthy. First, the earlier estimates described in Statistics Canada (2007) are broadly confirmed in several aspects. The depreciation profiles generated by the econometric techniques were, on balance, accelerated, producing convex age–price curves. Adding observations to the database for a subsequent period leaves most of the estimates unchanged. Moreover, there is little evidence that depreciation rates have increased in more recent years, although there has been a shift in the composition of assets towards those with higher rates of depreciation, which causes the average depreciation rate to increase.

Second, as was the case in Statistics Canada (2007), the estimates derived from the econometric *ex post* approach, using the trajectory of used-asset prices, compare favourably to the estimates derived from the *ex ante* method, using estimates of the expected length of life of assets derived from the Capital and Repair Expenditures Survey.

Third, the results produced by the *ex ante* and *ex post* approaches are approximately the same for those assets where there are enough observations to provide estimates for both approaches.

Therefore, information from both approaches is combined to generate depreciation rates across the asset classes. These rates are used to estimate capital stock in the Canadian Productivity Accounts. The *ex ante* information that is provided in Statistics Canada's surveys only pertains to the expected length of life of the asset. Derivation of a (geometric) depreciation rate from the expected life of the asset also requires a shape parameter of the rate—what is referred to as the DBR. It is this parameter that determines how much of total lifetime depreciation occurs early in life. The Productivity Accounts make use of information on similar assets where the *ex post* approach has been used to infer what the DBR is likely to be.

After the database has been updated and the estimation techniques, slightly improved, the new growth rates in capital stock and in capital services are not very different than those previously used.

1 Introduction[1]

Studies of asset depreciation are essential to the development of estimates of net capital stock, which make use of the perpetual inventory method for aggregating investment. In the standard perpetual inventory framework, the stock of capital available to economic agents, in any current period, is simply the sum of current investment and cumulative net investment in past periods (i.e., gross accumulated investment minus depreciation). Estimates of depreciation rates are used to turn the cumulative gross investment into net capital stock.

The capital stock, in turn, is an integral part of the Productivity Accounts. The value of net capital stock available for production purposes is the value of gross capital stock minus the value of depreciation, and depreciation estimates require estimates of the depreciation rate of capital.

Disagreements about depreciation profiles give rise to discordant statistical impressions of the amount of capital available to the production process. And, to the extent that there is little evidence that can be used to discriminate among different depreciation profiles that are used to estimate net capital stock, estimates of depreciation are less useful to clients of a statistical agency—because the point estimates provided by these programs must be accompanied by large confidence intervals.

This paper is the third in a series that use Canadian micro-level data on used-asset prices to estimate patterns of economic depreciation. As a first exercise, Gellatly, Tanguay, and Yan (2002) developed depreciation profiles and life estimates for 25 different machinery and equipment assets and 8 structures employing data on used-asset prices, for the period from 1985 to 1996. That paper compared the estimates produced by several alternative estimation frameworks. Then, a particular framework that used a duration model was chosen to provide estimates of depreciation that were incorporated into estimates of the growth in capital stock and capital services for Statistics Canada's productivity program.

The second paper (Statistics Canada 2007) extended the used-asset price database from 1996 to 2001, and applied two additional estimation frameworks to produce perpetual inventory estimates of capital stock. This longer time period and a larger sample of used prices provided over 30,000 observations of used prices on 49 individual assets, which were aggregated into 29 different asset categories—categories that collectively comprise the non-residential portion of the capital stock. This paper developed an estimation technique that, based on extensive Monte Carlo experiments, proved to be superior to alternatives that had previously been used.

Both papers compared the *ex post* estimates of depreciation that are yielded by the used-asset approach to *ex ante* estimates that come from an alternate source of data—survey estimates of the 'expected' life of assets. Statistics Canada's Capital and Repair Expenditures Survey that generates used-asset prices also provides estimates of the expected life of assets. Used-asset prices provide *ex post* information and tell us how assets worked out in practice. 'Predicted' length of life estimates that are provided by businesses when the investment is first made are *ex ante* estimates. Both previous papers find a close similarity between *ex ante* and *ex post* estimates and therefore substantiate the estimates that emerge from the *ex post* framework.

This paper extends the data set from 2002 to 2010, and examines the extent to which depreciation rates have changed in the most recent period. Finding that they are basically unchanged, it then pools the data and obtains new estimates for use in the Productivity Accounts.

1. This paper builds on Statistics Canada (2007) using the concepts and formulae that were developed therein by Marc Tanguay (Tanguay 2005). We refer readers who wish additional information to this earlier paper.

The estimation procedure is, essentially, the same one that Statistics Canada (2007) follows, and was developed by Tanguay (2005). It is an extension of the two-step procedure (made popular by Hulten and Wykoff [1981]) that models the discard function of an asset, and then uses the estimated discard function to correct the selection bias in which only the prices for assets that sell at positive prices are observed, and discards at a zero price are not included in the original estimation procedure. The new procedure that is used here estimates the discard process and the selling price jointly in a simultaneous framework, since joint estimation is more efficient and likely to be less biased (see Statistics Canada 2007).

Section 2 of this paper reviews a range of theoretical and empirical issues that motivate this study. The properties of the data sample are discussed in Section 3. The econometric estimation techniques are outlined in Section 4. Estimates of depreciation rates are presented in Section 5. Estimates of capital stock based on the estimates of depreciation rates are evaluated in Section 6. Section 7 concludes.

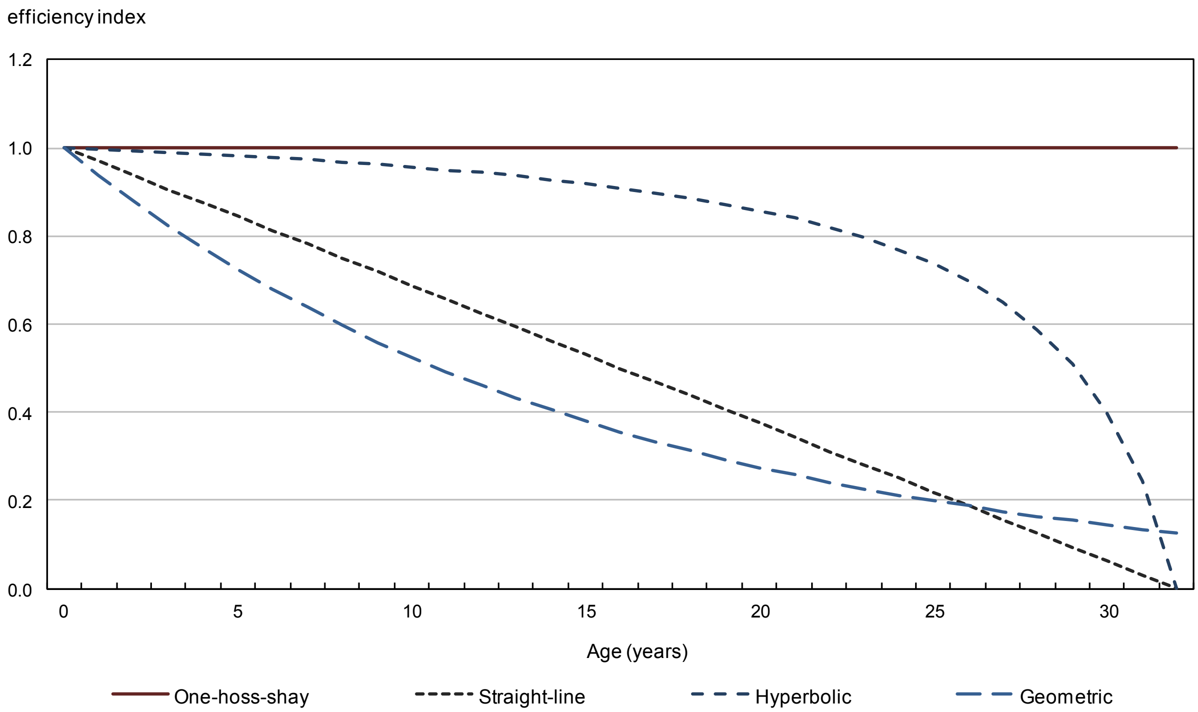
2 Foundations

2.1 Efficiency and depreciation

This study derives measures of the depreciation profile of an asset in use—in other words, the decline in the economic value of an asset that has been used in the production process over time.

To understand how depreciation is estimated, it is useful to start with the concept of an asset's productive efficiency or capacity; that is, its ability to generate an income stream from the production of goods and services over the course of its service life. The productive efficiency is measured with the stream of earnings that the asset is able to produce over time. As the asset experiences wear and tear or obsolescence, the stream of earnings that it produces generally declines. This process is represented graphically in Chart 1 using several different profiles that are assumed to be known with certainty.

Chart 1
Comparative efficiency profiles



Note: Age is for illustration purposes only.
Source: Statistics Canada, authors' calculations based on experimental data.

Four common efficiency profiles, beginning with the one-hoss-shay, are presented.[2] Assets with one-hoss-shay efficiency profiles provide a constant flow of earnings during their productive life T. They retain their full ability to produce goods and services, and generate a constant stream of in-period revenue, until the end of their service life. A second class of assets can be characterized by a concave-to-the-origin efficiency profile. In this case, the decline in efficiency is more pronounced in later periods of service life than in earlier periods. A common representation of this process uses a hyperbolic curve. The third example is provided by assets

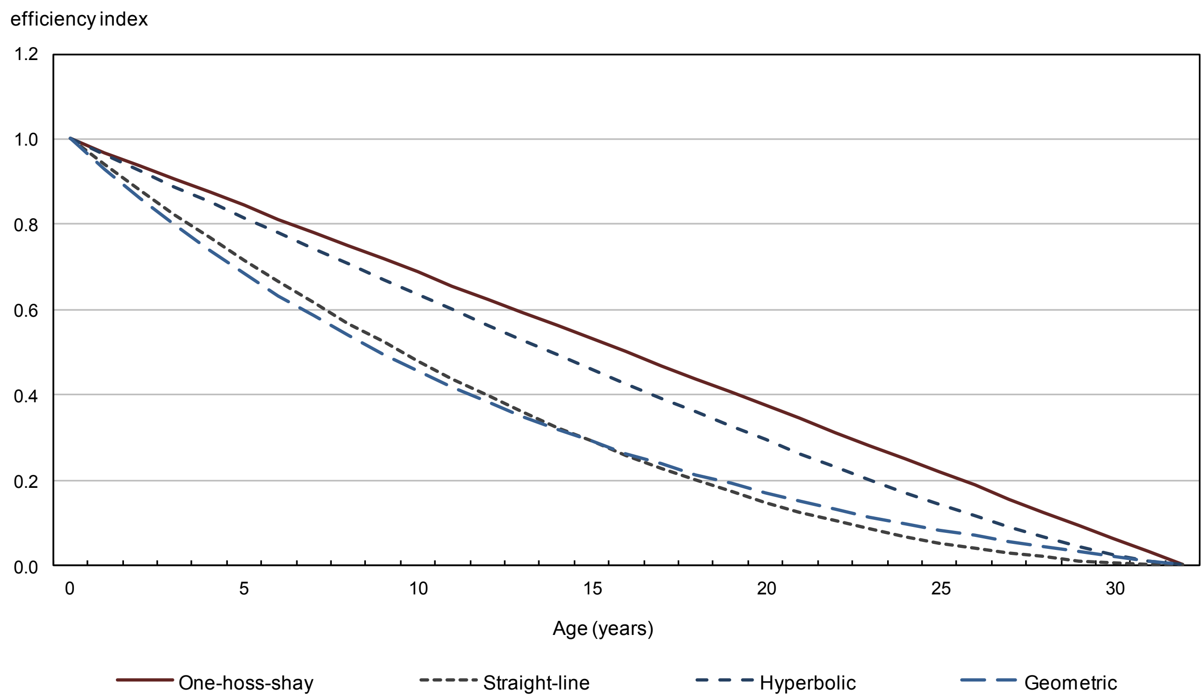
2. Much of this comes directly from Hulten and Wykoff (1981).

that exhibit a straight-line efficiency profile. The productive capacity, and in-period revenues, decline in progressive linear increments over their lifecycle. The fourth example involves assets that exhibit a profile whose earnings stream declines at a constant geometric rate.

Associated with each efficiency profile is an economic depreciation profile, defined as the decline in asset value (or asset price) associated with aging (Fraumeni 1997), under the assumption that the value of an asset, at any point in time, reflects the expected future earnings—the net present value of the future stream of earnings that is expected from owning the asset. Other things being equal, an older asset has less opportunity to generate revenue than a younger asset, which reduces the economic value of the former.

The function $f(y)$ will be used here to refer to the loss in value of an asset per unit of time t. The patterns of economic depreciation that correspond to the efficiency profiles presented in Chart 1 are given in Chart 2. These stylized relationships between asset efficiency and depreciation involve several simplifications: first, that service lives and efficiency patterns are known with certainty; second, that asset prices reflect the actualized value of this future stream of revenues, where these revenues are a linear function of the capacity of the asset; and third, that there is no discounting of future returns.

Chart 2
Corresponding depreciation profiles



Note: Age is for illustation purposes only.
Source: Statistics Canada, authors' calculations based on experimental data.

Under these assumptions, one-hoss-shay efficiency profiles will give rise to linear depreciation patterns, as older assets, while still generating the same in-period revenue as their younger counterparts, decline in value by a constant amount per period.[3] Linear efficiency profiles follow a more accelerated pattern, with higher losses in value earlier in service life. Hyperbolic, straight

3. A linear depreciation profile is depicted here simply to illustrate the incremental decline in present value, as the asset progresses through its service life. Note, however, that the depreciation curve corresponding to a one-hoss-shay efficiency profile will not be linear if (1) the duration of service life is not known with certainty, or (2) the value of the asset's productive capacity is discounted in future periods.

line, and geometric efficiency patterns give rise to an age–price profile that is convex to the origin.[4]

Deriving algebraic representations for the concepts of efficiency and depreciation in a world of certainty is straightforward. Consider for simplicity the one-hoss-shay case, in which there is no reduction in the asset's capacity over the course of its productive life.

Let $Q(y)$ refer to the efficiency index for specific ages y. The variable y expresses the time at which an atom of value embodied in the asset is lost. $f(y)$ refers to the loss of value per unit of time. Use of the asset for one period exhausts the constant value that the asset could potentially produce. Normalizing over T so that $f(y)$ has the characteristic of a density function gives

$$f(y) = \frac{Q(y)}{\int_0^T Q(y)\,dy} \; for \; 0 < y < T, 0 \; elsewhere. \tag{1}$$

If $Q(y)$ is constant as is the case for one-hoss-shay profile, then $f(y)$ is uniformly distributed between 0 and T. The loss of value will be spread equally over the asset's useful life.

Then,

$$f(y) = 1/T \quad for \; 0 < y < T, \, 0, elsewhere, \tag{2}$$

and the expectation will be provided by

$$E(y) = \int_0^T y \, f(y)\, dy = \left| _0^T \frac{y^2}{2T} = T/2. \right. \tag{3}$$

The expected life of a dollar invested in the asset will be the half of the expected life of the asset itself.

Now, the expected life of a dollar invested is just the average time over which a dollar of investment is lost. Its inverse is just the average rate of depreciation.[5] From Equation (3), it is therefore apparent that the average depreciation is just $2/T$.

In some routines for estimating depreciation, depreciation rates have been calculated indirectly from estimates of the length of life (T) of an asset derived from the tax code as

$$\delta = \frac{DBR}{T} \tag{4}$$

4. Once again, this efficiency–price relationship is conditional on several factors; see Footnote 3. More importantly, the geometric efficiency frontier translates precisely into a geometric depreciation curve, only in the case of an infinite lived asset. In the case of the geometric efficiency profile $(1\text{-}\delta)^y$, as depicted by Figure A.1, the depreciation curve will be $\{(1\text{-}\delta)^y\text{-}(1\text{-}\delta)^T\}/\{1\text{-}(1\text{-}\delta)^T\}$, and this expression collapses to the original geometric when T tends to infinity. When the asset has a fixed and finite life, the depreciation curve always reaches zero at the end of its service life, while the efficiency profile will be truncated, and the depreciation curve is still convex, but slightly more so than a geometric curve.
5. This can be seen directly in the case of the geometric depreciation function, where $\delta = 1/E(y)$. It is shown to be more generally true in Tanguay (2005).

where T is service life, and DBR must be chosen and is referred to as the declining-balance rate.

As Equation (3) shows, for the one-hoss-shay case, the DBR should be chosen as 2, in this instance, to provide an average rate of depreciation when T is known. More generally, the average rate of depreciation can always be calculated as the inverse of $E(y)$.[6]

The cumulative density function (c.d.f.) of y, denoted by $F(y)$, expresses the total proportion of initial value lost since the beginning of the asset's service life. Consequently, economic depreciation can be expressed by 1 minus $F(y)$, which provides $S(y)$, the so-called survival function.

Then

$$S(y) = 1 - \int f(y)dy = 1 - F(y). \tag{5}$$

When the efficiency profile is constant, the economic depreciation is a linear decreasing function, as was shown in Chart 2.

The constant capacity profile is often modified to provide for a gradual reduction of capacity produced by an asset, early in life, with a rapid increase in that decline as the asset approaches its useful length of life T. This type of modification produces a concave capacity curve. One functional form that takes on a concave capacity profile and is used by the Bureau of Labor Statistics (BLS) is the hyperbolic function, which is written as

$$Q(y) = (T - y)/(T - \beta y), \tag{6}$$

where β is a shape parameter. β's upper limit is 1, which produces the case of constant capacity to the end of life T. For $0 < \beta < 1$, the capacity curve will be concave (see Chart 3). If $\beta = 0$, it becomes linear decreasing. For negative values of β, the capacity curve becomes convex.

The density of the hyperbolic capacity profile will be

$$f(y) = \frac{(T - y)\beta^2}{(T - \beta y)T\left[(1-\beta)\ln(1-\beta) + \beta\right]} \; for\, 0 < y < T, 0, elsewhere. \tag{7}$$

When $\beta = 1$, $f(y)$ collapses to the density of a uniform distribution.

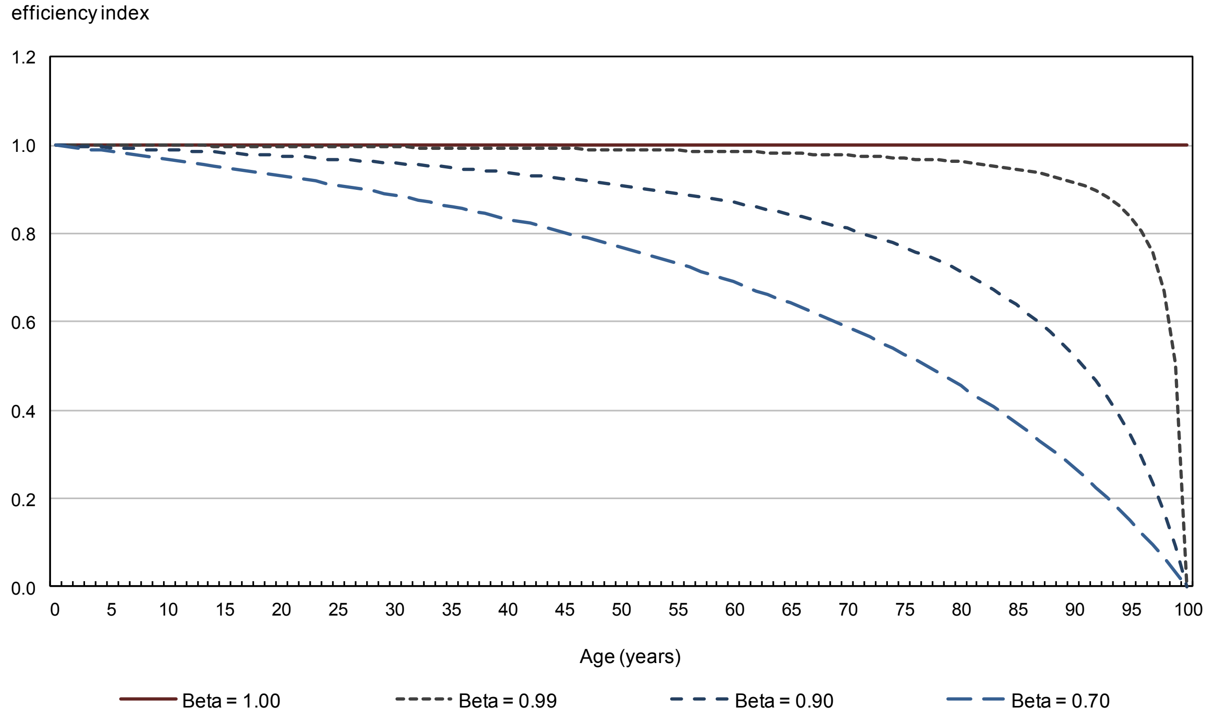
The c.d.f. of y, $F(y)$ will be

$$F(y) = \frac{T(1-\beta)\ln(T - \beta y) + y\beta}{T\left[(1-\beta)\ln(1-\beta) + \beta\right]}. \tag{8}$$

As expected, when $\beta = 1$, the expression collapses to the linear form $F(y) = y/T$. When $\beta = 0$, the above expression is indeterminate, but it converges to a quadratic.

6. See Statistics Canada (2007).

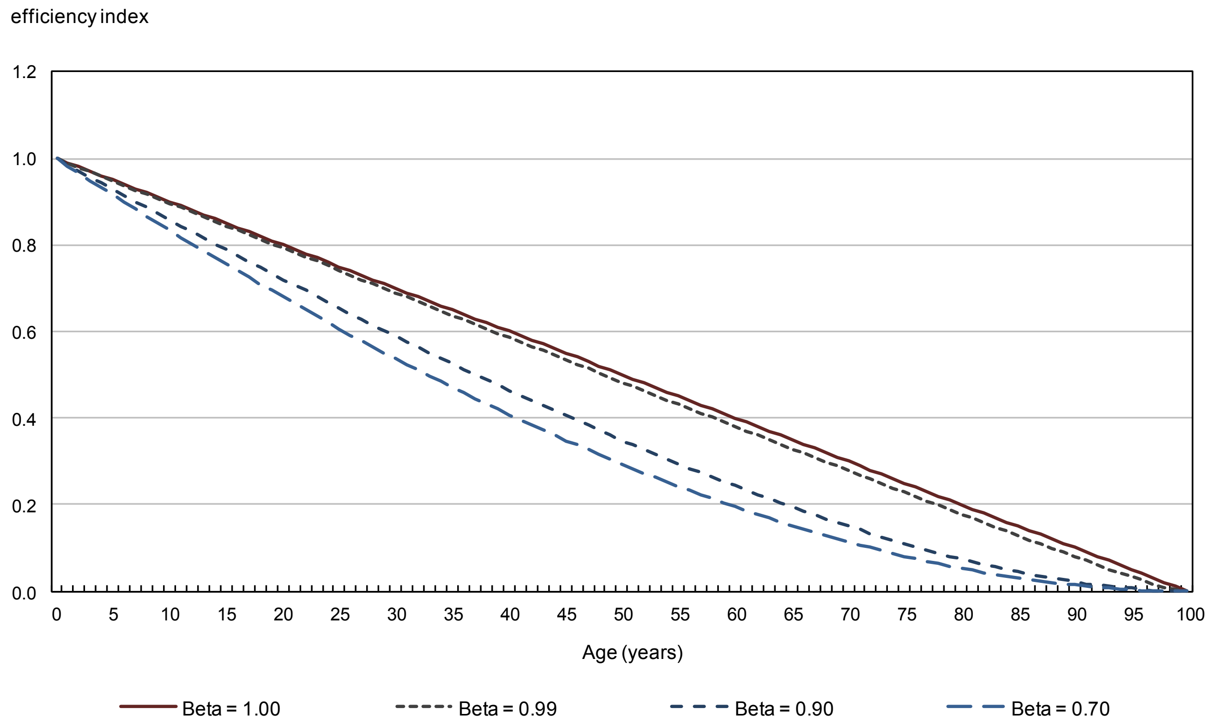
Chart 3
Hyperbolic capacity profiles



Note: Age is for illustration purposes only.
Source: Statistics Canada, authors' calculations based on experimental data.

Depreciation patterns yielded by this survival function depend on the value of β. Chart 4 provides some examples of economic depreciation curves derived from various values of β. When $\beta < 1$, the depreciation curve is always convex.

Chart 4
Economic depreciation curves mapped by hyperbolic capacity profiles



Note: Age is for illustration purposes only.
Source: Statistics Canada, authors' calculations based on experimental data.

In this paper, an alternative and more tractable functional form is used to represent a concave capacity profile, that is

$$f(y) = \frac{k+1}{kT}\left[1-\left(\frac{y}{T}\right)^{k}\right]. \tag{9}$$

The efficiency profile mapped by this function will be concave for any value of k, varying from 1 (linear declining) to infinity (one-hoss-shay). The expectation of y will be

$$E(y) = T\left[\frac{k+1}{2(k+2)}\right]. \tag{10}$$

This means that the DBR associated with Equation (10) is

$$DBR = \left[\frac{2(k+2)}{k+1}\right]. \tag{11}$$

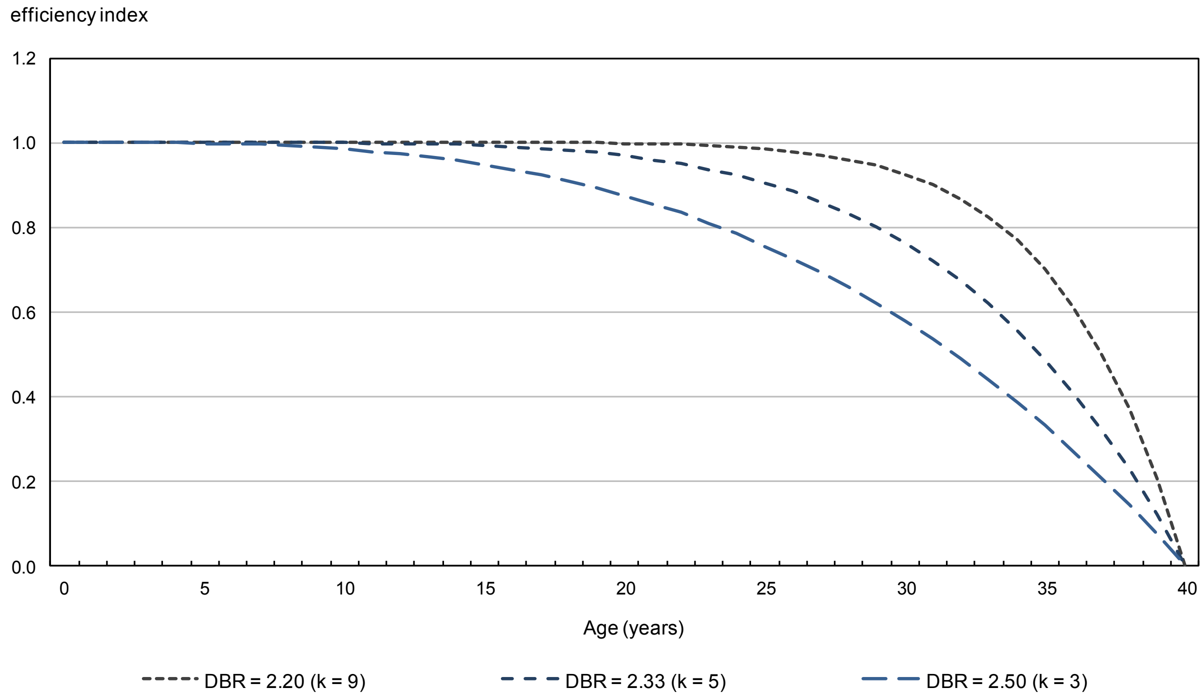
Equation (11) provides a straightforward way to build a mapping between the parameters of the capacity profile and the DBR. Its value will be between 2 and 3.

The c.d.f. related to Equation (9) is

$$F(y) = \frac{k+1}{kT}\left[y - \frac{y^{k+1}}{(k+1)T^k} \right]. \tag{12}$$

Different capacity profiles, using this functional form and the DBR linked to them, are presented in Chart 5.

Chart 5
Concave efficiency profiles and related declining-balance rates (DBRs)



Note: Age is for illustration purposes only. The value of the DBR is determined by the value of k.
Source: Statistics Canada, authors' calculations based on experimental data.

2.2 Efficiency and economic depreciation in a world of uncertainty

In reality, the value of the time of discard (T) is not known with certainty, because some assets will be retired before T, and others will be retired after T. T should, therefore, be treated as a random variable. When this is done, the price profiles will follow a curve that is convex—even when the efficiency profile of an asset is constant.

When a population is composed of assets that each have an efficiency profile coming from a one-hoss-shay and a different time of discard t, the time of discard can be modeled as a random variable having a mean of T but also having the skewed variance of a Weibull function. In this case, it can be demonstrated that the asset price curve is convex, as is the geometric function discussed above.[7]

7. See Statistics Canada (2007) for more detail.

The following example is illustrative.

Let $f(y)$ be a function representing the loss of value per unit of time t, and T be the length of life of the asset. Suppose that $f(y|t)$ corresponds to the constant capacity or efficiency profile and is $(1/t)$, and that the distribution of discard times— $f(t)$ —follows a Weibull distribution with parameters λ and ρ; i.e.,

$$f(t) = \lambda^{\rho} \rho t^{(\rho-1)} e^{-\left((\lambda t)^{\rho}\right)}. \tag{13}$$

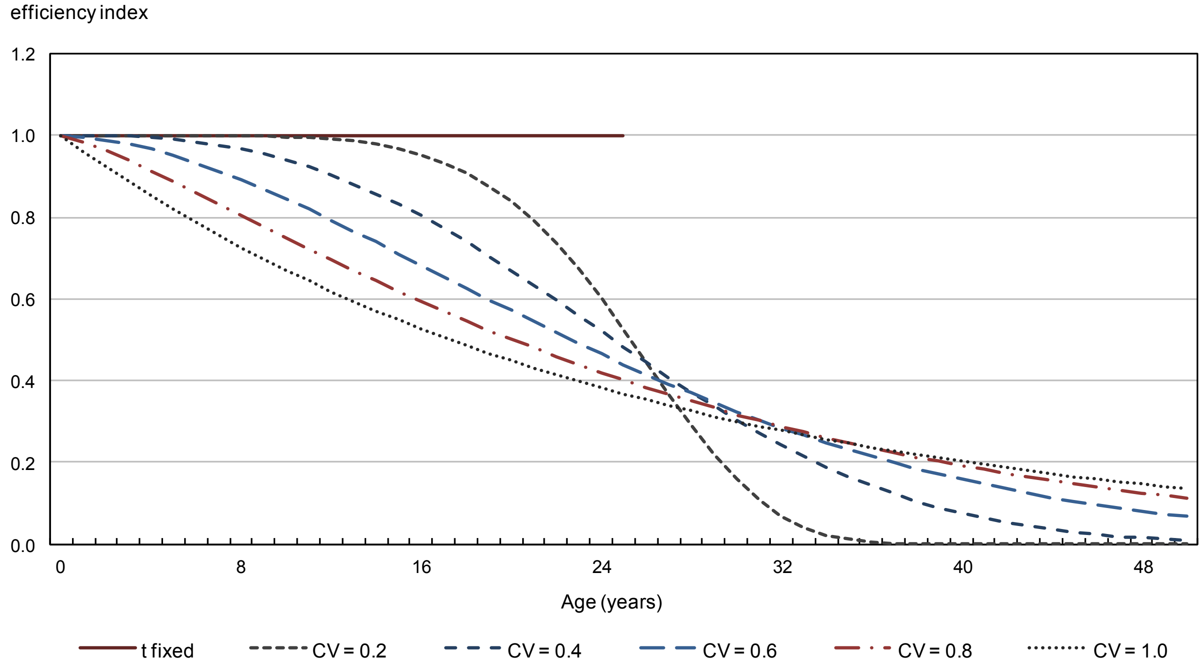
A Weibull function is a commonly used functional form that captures distributions that are skewed, and has the advantage that only two parameters are required for its specification. The discard function of assets is likely to be skewed, with more assets being discarded early in the life of an asset, rather than later in its life. Its first two moments are simple functions of these parameters, and are relatively easy to estimate.

Since y and t are now jointly distributed, the expected efficiency or capacity is no longer constant in this model, despite the fact that each asset is still assumed to follow a constant capacity, and is a function of Weibull parameters. Chart 6 plots expected capacity over time for different Weibull distributions. Alternate distributions are defined in terms of the size of the coefficient of variation, yielded by different values of ρ and λ. The larger the coefficient of variation of the expected duration (a function of ρ and λ), the more convex the expected capacity.[8]

With expected capacity now a convex function of time, the expected value of the asset also follows a convex trajectory, as opposed to the linear trajectory for a fixed capacity investment return function and a fixed retirement date. Chart 7 depicts the economic depreciation profiles that are generated by alternate Weibull functions for the discard process and a constant capacity function.

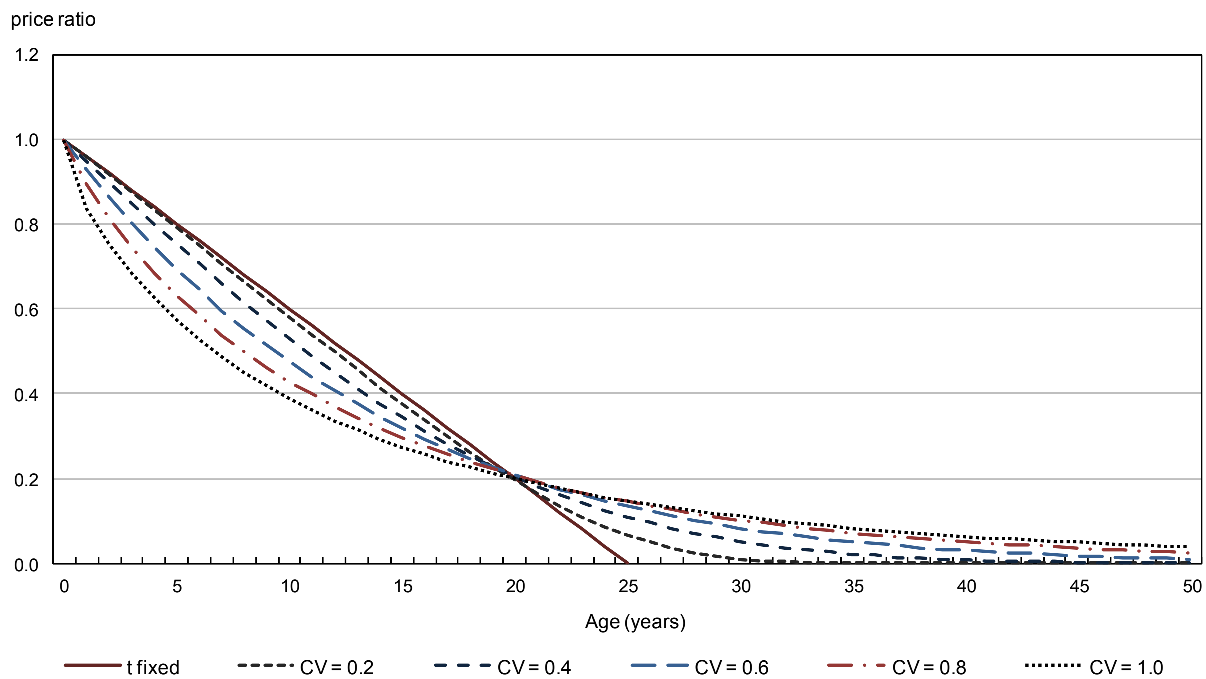
8. The coefficient of variation of a Weibull with scale parameter ρ and shape parameter λ is $\left[\Gamma\left(1+\frac{2}{\lambda}\right)-\left(\Gamma\left(1+\frac{1}{\lambda}\right)\right)^{2}\right]^{1/2} \Big/ \Gamma\left(1+\frac{1}{\lambda}\right)$.

Chart 6
Expected efficiency under Weibull durations



Notes: Age is for illustration purposes only. CV means coefficient of variation. t fixed signifies that the service life of the asset is known with certainty. Conditional capacity is constant.
Source: Statistics Canada, authors' calculations based on experimental data.

Chart 7
Economic depreciation under Weibull durations



Notes: Age is for illustration purposes only. CV means coefficient of variation. t fixed signifies that the service life of the asset is known with certainty. Conditional capacity is constant.
Source: Statistics Canada, authors' calculations based on experimental data.

3 Data source

The data used for this study come from Statistics Canada's annual Capital and Repair Expenditures Survey, an establishment-based survey undertaken by the Investment and Capital Stock Division. In this survey, respondents are asked to report on their sales and discards of fixed assets.

The survey provides detailed information on asset type, gross book value, sale price, and age of each asset that is sold or discarded. The gross book value includes the original investment value plus the capitalized improvements incurred over the life of the asset. Deflators for investment assets were used to express all price information in real dollars.

The basic unit used in this paper is a survival ratio of the value of the original asset, observed at some age t. For an observation i in the sample, the survival ratio is calculated as

$$R_i^t = \frac{SP_i^t}{GBV_i}, \tag{14}$$

where SP_i^t is the selling or discard price of asset i at age t, and GBV is its gross book value. Both numerator and denominator are expressed in constant dollars. R_i^t is, thus, the share of asset value that remains when the asset is sold at some reported age t. If the asset has been retired without a sale, R_i^t is set equal to 0, corresponding to a zero selling price.

Studies that use market prices to estimate depreciation profiles must address issues of data reliability.[9] Traditionally, used-asset samples have not contained information on retirements, which, in turn, will severely bias the estimation of depreciation profiles. The database used for this study contains this information. The previous paper (Statistics Canada 2007) covered a 15-year reporting period (1985 to 2001).

The sample included 30,350 observations on 43 assets, after applying edit routines.[10] The new database that was used for this study added observations from 2002 to 2010. After the filtering process was applied to the new database, 22,129 observations on 32 used assets were added. The breakdown of observations for each asset, and for the two time periods, is provided in Table 1.

Edits were also required to deal with what appeared to be aberrant observations. In some cases, this involved a concentration of non-zero prices near zero. It is likely that many of those observations were, in reality, describing a scrap value, not the value of surviving assets. Therefore, these were classified as discards. A lower bound of 0.06, below which a price ratio was considered to indicate a retirement, was used for this purpose. In addition, aberrant observations for long-lived assets that returned close to their original purchase price were also discarded.

A problem was also encountered with digits preference in the respondents, since there was a concentration of durations on rounding values like 5, 10, 15 and 20 years. This is a typical problem in many surveys, and arises because some respondents tend to round the duration values they report. These patterns of age-rounding can affect the accuracy of estimates. Accordingly, the correction for digit preference, which is described in Gellatly, Tanguay and Yan (2002), is extended to cover all ages, up to 45, using a modified likelihood function for those rounding ages.

9. Once again, for a general discussion of these issues, see Fraumeni (1997).
10. Edit routine is discussed in detail in Appendix A.

While the database provides a unique opportunity to estimate depreciation curves with used-asset price data, it should be recognized that the validity of employing used-asset prices depends on whether these prices reflect the value of representative assets, and that they do not represent 'lemons'.[11] If assets sold in resale markets are inferior to those that owners retain for production, the observed prices are biased downwards. The extent to which the 'lemon' issue limits the utility of used-asset studies is dependent *inter alia* upon one's preconceptions about the extent of the 'lemon' problem and the inability of markets to solve information problems. For instance, the emergence of market intermediaries that provide used-asset information to prospective buyers will reduce the severity of these information asymmetries.

It should be noted that the edit strategy eliminates some of the more apparent 'lemons'—observations with extremely low resale values, relative to like assets early in their service life, and high values later in their later years. Moreover, the estimates of depreciation derived here from used-asset prices are compared to other estimates, so as to cross-reference their accuracy. This and previous papers compare the estimates derived from employing the used-asset prices, which may involve a sample selection problem, to the estimates derived from *ex ante* estimates of length of life derived when the investment is first made, so as to triangulate the results.

In order to take into account potential problems with the use of used prices, the estimates are limited to those assets (mainly machinery and equipment), where the resale market is reasonably active. For example, in engineering construction, less than 40% of the observations had positive prices and, of those, about half had a price ratio lower than 6%. Consequently, engineering construction was removed from the estimation procedure. Only a few classes existed for buildings where there were a reasonable number of transactions—and the econometric framework might, therefore, be expected to do less well here. The observations that provide most of the estimates consist primarily of assets classified as machinery and equipment (about 46,000 observations in total, for 1985 to 2010). The data allow us to estimate depreciation rates directly for 27 major asset categories, out of the 155 assets tracked by Statistics Canada for its investment program.

Finally, concerns over representativeness often come to the forefront when results are based on small samples. Much of empirical work on asset depreciation done by Hulten and Wykoff (1981) has been based on small samples for limited numbers of assets. Here, our database confers the advantage that it consists of a large and diverse set of price information based on the comprehensive Capital and Repair Expenditures Survey, undertaken by Statistics Canada. Over the entire period from 1985 to 2010, the mean number of observations per asset is about 1,200, the minimum is 74, and the maximum is 6,954.

Main statistics for the samples used in the estimation are documented in Tables 2 and 3, including the means and standard deviations of the reselling price ratio, reselling age, and discard age by asset. On average, assets in the buildings class have a higher reselling price ratio than in machinery and equipment. From 1985 to 2001, the mean reselling price ratio for buildings is 0.38, and 0.27 for machinery and equipment (Table 2). From 2002 to 2010, the mean reselling price ratio for buildings is 0.39, and 0.32 for machinery and equipment (Table 3). The reselling and discard ages for buildings, on average, are twice those for machinery and equipment. For example, from 1985 to 2001, the mean reselling age for buildings is 16 years versus 8 years for machinery and equipment.

Across time periods, there is not much change for both buildings and machinery and equipment. For buildings, the average reselling price ratio and average reselling age is 0.38 and 16 years, respectively, in the period from 1985 to 2001, as compared with 0.39 and 15 years, respectively, in the period from 2002 to 2010. For machinery and equipment, the corresponding numbers are

11. See Akerlof (1970).

0.27 and 8 years, in the early period, versus 0.32 and 8 years in the later period. The largest change comes from the discard age for buildings. The mean discard age for buildings is only 14 years in the period from 2002 to 2010, as opposed to 22 years in the early period, a reduction of about one third. The reduction of discard age for machinery and equipment is not significant.

Table 1
Data sample

	1985 to 2001		2002 to 2010	
Asset description and code	**Reselling price**	**Discard age**	**Reselling price**	**Discard age**
	number of observations			
Plants for manufacturing (1001)	352	554	136	260
Warehouses, refrigerated storage, freight terminals (1006)	171	161	130	105
Maintenance garages, workshops, equipment storage facilities (1008)	97	96	55	63
Office buildings (1013)	434	322	262	387
Shopping centres, plazas, malls, stores (1016)	157	175	118	394
Other industrial and commercial (1099)	105	53	164	156
Telephone and cablevision lines, underground and marine cables (3002)	90	106	11	12
Communication towers, antennae, earth stations (3003)	72	85	11	17
Office furniture, furnishing (e.g., desks, chairs) (6001)	1,301	1,837	474	1,816
Computers, associated hardware and word processors (6002)	1,242	1,831	618	2,189
Non-office furniture, furnishings and fixtures (e.g., recreational equipment, etc.) (6003)	458	384	303	545
Scientific, professional and medical devices (6004)	216	336	66	167
Heating, electrical, plumbing, air conditioning and refrigeration equipment (6005)	133	267	26	111
Pollution abatement and control equipment (6006)	22	83	38	111
Motors, generators, transformers, turbines, compressors and pumps of all types (6009)	314	338	155	188
Heavy construction equipment (e.g., loading, hauling, mixing, paving, grating) (6010)	551	76	255	40
Tractors and other field equipment (truck tractors - see 6203) (6011)	360	100	194	69
Capitalized tooling and other tools (hand, power, industrial) (6012)	372	449	128	163
Drilling and blasting equipment (6013)	110	40	33	0
Automobiles and major replacement parts (6201)	1,732	334	825	235
Buses (all types) and major replacement parts (6202)	195	35	103	30
Trucks, vans, truck tractors, truck trailers and major replacement parts (6203)	2,999	893	1,505	421
Other transportation equipment (6299)	137	86	941	259
Computer-assisted processing machinery and equipment (6401, 6402)	560	540	994	1,051
Non-computer-assisted processing machinery and equipment (6601, 6602)	2,959	1,903	1,202	890
Communication and related equipment (6403, 6603)	526	641	337	824
Safety and security equipment and other machinery and equipment (6007, 8999)	634	492	484	514

Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Table 2
Sample statistics, 1985 to 2001

	Resell				Discard age	
	Ratio		Age			
	mean	standard error	mean	standard error	mean	standard error
All buildings	0.380	0.308	16.4	11.7	21.5	12.1
Plants for manufacturing (1001)	0.308	0.291	17.5	12.0	24.3	11.1
Warehouses, refrigerated storage, freight terminals (1006)	0.397	0.292	17.9	9.8	25.9	14.1
Maintenance garages, workshops, equipment storage facilities (1008)	0.287	0.279	16.1	9.4	23.7	11.5
Office buildings (1013)	0.411	0.304	17.2	13.4	21.3	13.2
Shopping centres, plazas, malls, stores (1016)	0.500	0.308	15.0	10.9	9.7	6.0
Other industrial and commercial (1099)	0.342	0.284	18.2	10.9	22.3	9.2
Telephone and cablevision lines, underground and marine cables (3002)	0.449	0.390	9.2	7.3	17.7	8.1
Communication towers, antennae, earth stations (3003)	0.339	0.279	13.5	9.8	22.6	9.4
All machinery and equipment	0.271	0.259	8.5	5.2	10.2	5.3
Office furniture, furnishing (e.g., desks, chairs) (6001)	0.301	0.252	8.5	4.1	10.7	4.3
Computers, associated hardware and word processors (6002)	0.312	0.278	4.8	2.0	5.8	2.1
Non-office furniture, furnishings and fixtures (e.g., recreational equipment, etc.) (6003)	0.264	0.267	8.3	4.2	8.8	3.9
Scientific, professional and medical devices (6004)	0.274	0.281	9.1	4.9	11.5	5.2
Heating, electrical, plumbing, air conditioning and refrigeration equipment (6005)	0.251	0.267	9.9	5.4	14.5	6.7
Pollution abatement and control equipment (6006)	0.232	0.293	14.4	7.5	13.0	6.2
Motors, generators, transformers, turbines, compressors and pumps of all types (6009)	0.264	0.272	11.6	6.5	15.5	7.1
Heavy construction equipment (e.g., loading, hauling, mixing, paving, grating) (6010)	0.267	0.242	9.9	5.0	11.0	5.5
Tractors and other field equipment (truck tractors - see 6203) (6011)	0.253	0.216	9.1	4.3	10.8	4.8
Capitalized tooling and other tools (hand, power, industrial) (6012)	0.248	0.260	9.6	5.3	9.2	4.0
Drilling and blasting equipment (6013)	0.309	0.249	8.2	4.8	11.7	5.1
Automobiles and major replacement parts (6201)	0.343	0.240	4.5	2.1	5.6	2.4
Buses (all types) and major replacement parts (6202)	0.159	0.201	12.9	5.1	11.1	4.9
Trucks, vans, truck tractors, truck trailers and major replacement parts (6203)	0.256	0.247	7.4	3.5	8.5	3.5
Other transportation equipment (6299)	0.233	0.252	10.0	5.7	10.2	5.0
Computer-assisted processing machinery and equipment (6401, 6402)	0.257	0.292	9.9	6.0	12.1	5.7
Non-computer-assisted processing machinery and equipment (6601, 6602)	0.219	0.249	11.7	6.1	13.2	5.5
Communication and related equipment (6403, 6603)	0.285	0.292	7.5	4.5	9.5	4.3
Safety and security equipment and other machinery and equipment (6007, 8999)	0.324	0.288	9.5	5.5	11.7	5.7

Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Table 3
Sample statistics, 2002 to 2010

	Resell				Discard age	
	Ratio		Age			
	mean	standard error	mean	standard error	mean	standard error
All buildings	0.393	0.293	15.4	10.1	13.8	9.3
Plants for manufacturing (1001)	0.334	0.281	19.4	13.4	21.0	10.6
Warehouses, refrigerated storage, freight terminals (1006)	0.293	0.297	18.8	9.9	17.3	10.7
Maintenance garages, workshops, equipment storage facilities (1008)	0.426	0.240	17.5	8.4	20.5	10.5
Office buildings (1013)	0.476	0.288	15.7	9.7	11.3	7.4
Shopping centres, plazas, malls, stores (1016)	0.435	0.288	8.0	6.3	9.3	4.9
Other industrial and commercial (1099)	0.358	0.279	13.9	7.7	13.3	9.2
Telephone and cablevision lines, underground and marine cables (3002)	0.132	0.054	11.1	1.9	16.8	0.6
Communication towers, antennae, earth stations (3003)	0.508	0.435	12.5	2.9	23.6	6.7
All machinery and equipment	0.320	0.293	8.0	4.7	8.5	4.9
Office furniture, furnishing (e.g., desks, chairs) (6001)	0.314	0.287	6.6	3.6	8.4	3.3
Computers, associated hardware and word processors (6002)	0.249	0.270	4.3	1.7	4.8	1.6
Non-office furniture, furnishings and fixtures (e.g., recreational equipment, etc.) (6003)	0.519	0.354	6.9	3.4	8.1	3.4
Scientific, professional and medical devices (6004)	0.373	0.330	9.0	5.6	11.4	5.3
Heating, electrical, plumbing, air conditioning and refrigeration equipment (6005)	0.655	0.303	9.1	4.0	11.0	4.0
Pollution abatement and control equipment (6006)	0.293	0.282	11.3	4.3	11.5	6.0
Motors, generators, transformers, turbines, compressors and pumps of all types (6009)	0.466	0.320	9.3	5.8	14.0	7.6
Heavy construction equipment (e.g., loading, hauling, mixing, paving, grating) (6010)	0.365	0.304	6.9	4.0	10.7	5.9
Tractors and other field equipment (truck tractors - see 6203) (6011)	0.288	0.250	8.8	4.4	13.3	3.8
Capitalized tooling and other tools (hand, power, industrial) (6012)	0.294	0.284	7.1	3.9	8.5	4.2
Drilling and blasting equipment (6013)	0.561	0.254	6.9	4.3	...	...
Automobiles and major replacement parts (6201)	0.314	0.260	5.5	2.5	6.4	2.9
Buses (all types) and major replacement parts (6202)	0.122	0.221	14.8	6.9	22.2	8.0
Trucks, vans, truck tractors, truck trailers and major replacement parts (6203)	0.295	0.264	7.7	3.5	8.8	3.6
Other transportation equipment (6299)	0.314	0.260	9.4	4.3	11.2	4.6
Computer-assisted processing machinery and equipment (6401, 6402)	0.322	0.301	10.3	5.7	12.0	5.8
Non-computer-assisted processing machinery and equipment (6601, 6602)	0.292	0.283	9.5	5.2	12.0	6.1
Communication and related equipment (6403, 6603)	0.283	0.291	7.3	3.0	7.5	3.4
Safety and security equipment and other machinery and equipment (6007, 8999)	0.386	0.350	8.8	5.0	10.6	4.7

... not applicable

Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

4 Estimation framework

The estimation technique that is used here builds on the pioneering work of Hall (1971) and Hulten and Wykoff (1981), and makes use of the econometric methodology developed in Statistics Canada (2007), by Tanguay. The earlier study reported in Statistics Canada (2007) made use of several different convex forms of age–price profiles—a Weibull, an Exponential, and the general form outlined in Equation (9)—and alternate estimation techniques, settling on one that a Monte Carlo experiment identified as having the least bias and the greatest efficiency.

Extensively used in duration analysis, the Weibull distribution is a flexible parametric form, characterized by two parameters, which allows for variable, age-variant rates of depreciation, but can be restricted to produce constant (exponential) rates that are directly comparable to the geometric rates commonly used in depreciation accounting. The third general form was chosen to ask what the form would look like if there was a Weibull discard function and a general concave efficiency profile. The derived equation that characterizes the resulting age–price profile requires the estimation of three parameters.

Previously, it was found that the different functional forms chosen did not yield significant differences in the variable being estimated here—that is, the average depreciation rate. The derived estimates of the average depreciation rate produced depended less on the functional form chosen, and more on making sure that the data used were representative of the entire population of asset transactions, and that edit procedures removed aberrant observations at the two tails of the age distribution—the very young and the very old.

4.1 Survival model

The first step is to consider asset valuation within the standard maximum-likelihood framework.

Let D define a dummy variable describing the two possible life states for a given asset, and let $D=1$ when the asset is dead or retired (its sale value equals zero) and $D=0$ if otherwise. The likelihood of observing an age t is

$$\ell(t) = f(t)^D S(t)^{(1-D)} \tag{15}$$

where $f(t)$ is the density function, and $S(t)$ the survival function[12]—1 minus the cumulative density of $f(t)$.[13]

Equation (15) can be applied to situations in which the event being modeled can be described using binary life states (e.g., ‘alive’ or ‘dead’). If the asset is ‘dead’, the likelihood function reduces to the density function, and gives the probability of death at age t. If the asset is still ‘alive’, the likelihood reduces to the survival function, and gives the probability that it survives until t. The log-likelihood of observing a sample of n observations then takes the form

$$\ln L = \sum_{i=1}^{N} \left[D_i \ln f(t_i) + (1-D_i) \ln S(t_i) \right]. \tag{16}$$

12. $S(t) = 1 - \int f(t)dt = 1 - F(t).$

13. This is consistent with the standard model of survival. See, for example, Cox and Oakes (1984), and Nelson (1982).

Equation (16) can be modified here to characterize the likelihood function of asset's age–price profile, based on the set of observed survival ratios R_i (defined previously by Equation [14]). Each individual atom of value has its own duration, and R_i expresses the proportion of them that survives at some age t, while $1-R_i$ is the proportion lost. Each individual asset is, therefore, considered as a specific cohort of values. The log-likelihood of a sample of n observations (cohorts) becomes

$$\ln L = \sum_{i=1}^{N} \left[(1-R_i) \ln f(y_i) + R_i \ln S(y_i) \right], \tag{17}$$

where y_i is the time at which an atom of value embodied in asset i is lost. The log-likelihood formulation given by Equation (17) has an intuitive interpretation. R_i, the price ratio, represents the amount of asset value that survives to some age y_i multiplied by a corresponding survival probability $S(y_i)$, while $1-R_i$ represents the amount of value lost, multiplied by its failure probability $f(y_i)$.

While well-suited to many survival applications, Equation (17) needs to be modified to produce estimates of economic depreciation. The use of the standard density formulation $f(y_i)$ assumes that asset values remain unchanged in all periods prior to being sold or retired. Embedded, then, in Equation (17), are profiles that are conceptually similar to a "one-hoss-shay"—with asset values remaining at their maximum survival ratio, prior to some age period (the point of transaction y_i) at which some partial or total loss in value is observed. Since this is too restrictive an assumption, Equation (17) is modified to adjust for continuous depreciation by replacing the density term $f(y_i)$ with the cumulative density $F(y_i)$. While the density term $f(y_i)$ assumes that the loss in asset value occurs at y_i, the cumulative density $F(y_i)$ assumes that reductions in value occur before time y_i.

The estimating equation becomes

$$\ln L = \sum_{i=1}^{N} \left[(1-R_i) \ln F(y_i) + R_i \ln S(y_i) \right], \tag{18}$$

where $F(y_i)$ is the probability that asset values will decline at some point prior to y_i.[14]

The survival of an asset is involved with both the survival of an asset's life and its value. That is, y_i and age t are jointly distributed. Assuming a Weibull distribution for t, and a general form of a concave efficiency curve conditional on t, presented in Equation (9),

$$f(y) = \frac{k+1}{kt} \left[1 - \left(\frac{y}{t} \right)^k \right]$$

yields the log of the likelihood function for age–reselling price profiles,

14. This is similar to binary response models where the level of response (time) is observation-specific. Our formulation resembles one of the prototypes listed by Lagakos (1979)—in which observations share a common survival distribution, but different censoring experiences. In our framework, the likelihood function is both left- and right-censored, and the usual indicator variable y is replaced by a survival ratio R_i.

$$\ln L = \sum_{i=1}^{N} W_i (1-c_i) \left| \begin{array}{l} (1-R_i)\log\left[1-e^{-\left((\lambda t_i)^{\rho}\right)} + \lambda t_i \Gamma\left[1-\frac{1}{\rho},(\lambda t_i)^{\rho}\right] + \frac{1}{\rho}\mathrm{E}_{\left(\frac{k+1+\rho}{\rho}\right)}\left[(\lambda t_i)^{\rho}\right]\right] + \\ R_i \log\left[e^{-\left((\lambda t_i)^{\rho}\right)} - \lambda t_i \Gamma\left[1-\frac{1}{\rho},(\lambda t_i)^{\rho}\right] - \frac{1}{\rho}\mathrm{E}_{\left(\frac{k+1+\rho}{\rho}\right)}\left[(\lambda t_i)^{\rho}\right]\right] \end{array} \right|, \quad (19)$$

where $\Gamma(.,.)$ is an incomplete gamma function; $\mathrm{E}(\frac{k+1+\rho}{\rho})$ is an exponential integral of order $\frac{k+1+\rho}{\rho}$ that can be solved using interpolations between integer values of $\mathrm{E}(\frac{k+1+\rho}{\rho})$; c is an indicator for discard (1) and reselling (0); and W_i is a weight for observation i.[15]

4.2 The discard function

The second part of the estimation involves the discard function, so as to correct for the selection bias arising from using only the positive prices that are observed in used-asset markets. Hulten and Wykoff (1981), in their path-breaking estimates, only had price data on used assets and little in the way of information on the discard pattern. That is, they lacked information on the actual discards that were not being observed in used-asset markets that only collected price data for transactions that yielded positive values. In the absence of these data, Hulten and Wykoff made assumptions about the mean length of life and the distribution of discards around this point. In turn, they adjusted downward the positive prices that were observed to average in the missing observations on assets that were discarded at a zero price.

The database used here allows us to estimate the discard process directly. Contrary to most studies that calibrate a retirement distribution around a mean service life, retirement probabilities in this study are directly estimated using information on retirement (that is, transactions characterized by zero prices) and sales of used assets. All the observations (both positive and zeros) are used to estimate the actual discard function, and then this is used to correct the estimators for a proportion of discards at each point of time.[16]

To do so requires an assumption about the discard or retirement pattern. It is assumed that the retirement distributions follow a Weibull specification. The cumulative (D) and density (f) probability functions for retirement are, respectively,

$$D(t;\lambda,\rho) = 1 - Sv(t;\lambda,\rho) = 1 - \exp\left\lfloor -(\lambda t)^{\rho} \right\rfloor \quad (20)$$

$$f(t;\lambda,\rho) = \lambda^{\rho}\rho(t)^{\rho-1}\exp\left\lfloor -(\lambda t)^{\rho} \right\rfloor. \quad (21)$$

The parameters that need to be estimated are the scale parameter, λ, and the shape parameter, ρ, of the Weibull distribution.

To start, let c be a binary variable that takes the value 1 for complete durations, 0 otherwise.

The log-likelihood function becomes

$$l_t = \sum_{i=1}^{N} W_i c_i \log\left[f(t_i;\boldsymbol{\theta})\right] + W_i(1-c_i)\log\left[S(t_i;\boldsymbol{\theta})\right], \quad (22)$$

15. This weight is to correct the potential sample non-randomness. See Statistics Canada (2007) for more details.
16. We also experimented with a version that only used the discard points (the zeros), but discarded this because it involves clear censoring biases that have long been established in the econometrics literature.

where θ represents the parameters to be estimated.

In the case of a Weibull specification, this becomes

$$l^0{}_t = \sum_{i=1}^{N_1} W_i c_i \left[\rho \log(\lambda) + \log(\rho) + (\rho - 1)\log(t_i)\right] - W_i \left(\lambda t_i\right)^{\rho}. \tag{23}$$

Note that Equation (23) is for those ages not affected by the digit-preference problem. The following modifications are made to those rounding ages, since the true ages are not observed. It is assumed that there is an age error parameter to be estimated, e. Therefore, the unobserved true ages lie in the interval $t-e$ and $t+e$. Hence, the probability of observing a rounding age, t, when the duration is complete, is the same as the probability of observing the interval $(t-e, t+e)$, i.e., $F(t+e)-F(t-e)$, where $F(.)$ is a Weibull cumulative distribution function. When the duration is incomplete, i.e., censored, the probability of observing a rounding age t becomes $S(t-e)$, where $S(.)$ is the Weibull survival function. Therefore, the log likelihood function for the rounding age becomes

$$l^1{}_t = \sum_{i=1}^{N_2} W_i c_i \log[F(t_i+e) - F(t_i-e)] + W_i(1-c_i)\log[S(t_i-e)]. \tag{24}$$

4.3 Simultaneous estimation of asset decline and discard function

The asset survival and the discard function are estimated simultaneously; since Statistics Canada (2007) shows that this methodology provides the least bias and the greatest efficiency. The shape of the survival density function will depend on the shape of both the discard function and the efficiency function, and those shapes are likely to be different.[17] A two-step procedure that estimates the discard function first, and then uses its estimates to correct for the selection bias in the decline function, provides biased estimates because it does not use the decline-function information on when assets are still alive to estimate the length of life.[18] A simultaneous framework will force the estimators to respect the consistencies between the two processes generating t and y, given that those processes are related.[19] This consistency can be imposed, even in presence of specification error, when the exact form of the discard model is not known.

17. As was noted in Statistics Canada (2007), approaching the estimation process piecemeal—first estimating a survival curve, then a discard model, and blending the two, risks ignoring links between the two—a Weibull discard function is not necessarily consistent with a Weibull price profile. In fact, the Weibull discard function, along with several common efficiency or capacity profiles, does not yield a Weibull survival curve.
18. It is akin to estimating the length of unemployment by only using observations on those unemployed, who have made the transition back to employment.
19. A reviewer has pointed out that, as the sample data set increases, the bias in the two-step procedure should fall. For more detail on when the simultaneous procedure outperforms the two-step procedure, see Statistics Canada (2007) for a set of Monte Carlo experiments.

For example, if realizations of the random variable, t, are observed for an empirical survival function of y, the system could take the form

$$(i) \quad l_t = f(t;\boldsymbol{\theta}) \tag{25}$$

$$(ii) \quad l_y = S\left(y;\boldsymbol{\theta},\eta\right), \tag{26}$$

where l_t and l_y stand respectively for the likelihood functions of t and y, θ for a vector of parameters common to both functions, and η, the parameter defining the shape of the capacity profile, which is specific to l_y.

The fact that some parameters are shared by the two equations argues for a simultaneous technique that recognizes the consistencies mentioned above. The first equation expresses the physical duration t, while the second corresponds to survival of y, which determines the resale price of used assets. When the price is zero, the information is complete in terms of duration, but left-censored in terms of value. When the price is non zero, the data are right-censored in terms of duration, but provide more information on $S(y)$. A simultaneous estimation framework exploits the complementarities between the information on y and t.

The specification used to estimate the discard function is Weibull, and is provided by Equation (21). The survival curve that is chosen is a general form of a concave efficiency curve, and is provided by Equation (9).

The efficiency profile, mapped by this function, will be concave for any value of k, varying from 1 (linear declining) to infinite (one-hoss-shay).

Estimation of Equation (26), when based only on individual survival ratios, R_i, assumes that depreciation schedules are not correlated with the size—or dollar value—of the asset. To account for dollar value differences across observations, each observation is weighted by its share of total asset value, multiplied by the number of observations in the asset sample.

The observations for the discard function are weighted by the gross book value (GBV) of the asset. The weights serve as proxy for quantities, which are measured by the GBV in constant dollars. These weights are necessary to account for the consolidated reporting of the Capital and Repair Expenditures Survey (several transactions may be reported as a single response), and for the fact that some assets have more capital embedded in them (for example, a two-floor building versus a twenty-floor building).

The discard function is estimated using a maximum likelihood technique that takes into account the digit-preference problem found in the database. The existence of digit preferences means that the independent variable (time) is measured with error. This problem is dealt with by substituting a new variable for age, where a digit-preference problem was identified, as explained in the previous section and by Equation (24).

Equations (19), (23) and (24) are jointly estimated via the maximum likelihood estimation technique, which yields the estimates for the Weilbull distribution and a DBR. Based on the estimates, geometric depreciation rates are then obtained by $DBR / E(t)$.

5 Empirical results

The previous section has outlined the nature of the estimation techniques used. This section presents the estimates of depreciation rates using the simultaneous estimation technique, for a variety of asset classes, and for three time periods, 1985 to 2001, 2002 to 2010, and 1985 to 2010.

While rich in detail, there are potential problems in the data that need to be resolved. A significant problem occurs because of the potential lack of randomness of the sample.

Analysts need to always keep in mind that the data they are using may not have been generated in a random way, and that the sampling technique may have produced a sample that produces a potential bias in its estimates. A classical survey-design process is aimed at reducing these problems. But even here, problems may arise during the survey process. And survey methodologists have designed methods to use post-survey reweighting to address the problem.

The data potentially suffer from non-randomness as a result of the 'purposive' sampling process used to generate the data that is related to the acquisition of assets. Our estimation procedure makes use of asset dispositions. As a result, the data on dispositions may not be ideal for estimation. One manifestation of this problem is the 'lumpiness' at certain ages of the asset that sometimes is seen in the data. In these cases, there is a too narrow range of observations to permit estimation of depreciation rates, ranging from very young ages to very old ages. Or, if the other dimension of the data is considered, the price ratio ranging from 0 to 1, more observations are observed in some groupings than in others.

To address this problem, a reweighting scheme is applied to the estimation procedure (see Statistics Canada 2007, Appendix C).

5.1 Estimates of *ex post* rates of depreciation

The estimates of the average depreciation rate, by type of asset in our sample, are reported in Table 4 for machinery and equipment and in Table 5 for buildings. These tables include only those assets for which there were sufficient observations to calculate depreciation rates in Statistics Canada (2007). Differences in the results across the time periods allow a test of the hypothesis that the duration of an asset is shortening and depreciation rates increasing. To this end, the standard errors of the estimates of depreciation rates over different time periods, as well as T-statistics for testing the depreciation rate differences, are reported in Tables 6 and 7.

Comparisons of the set of results across time periods allow us to evaluate the impact of extending the time period. From 1985 to 2001 and from 2002 to 2010, the differences in depreciation rates for most machinery and equipment categories are not statistically significant. The unweighted means are 18.2% and 19.9% for the first and second period, respectively.

In only 4 out of 19 asset classes is the difference statistically significant at a 5% level. These classes are office furniture (6001), computers (6002), communication and related equipment (6403 and 6603), and other machinery and equipment (6007 and 8999) (Table 6). For example, the depreciation rate for computers has increased from less than 0.4 to about 0.48. This reflects the growth of Internet and mobile computation technology in the post-2000 period, which led to faster obsolescence of computer-related equipment and a large revision in the price index series. Similar stories hold for communication and related equipment. The depreciation rates for automobiles, buses and trucks has decreased slightly from the early period to the later period, but the changes are not statistically significant.

Table 4
Depreciation rates for select machinery and equipment categories

	1985 to 2001	2002 to 2010	1985 to 2010
		rate	
Asset description and code			
Office furniture, furnishing (e.g., desks, chairs) (6001)	0.200	0.282	0.227
Computers, associated hardware and word processors (6002)	0.385	0.479	0.431
Non-office furniture, furnishings and fixtures (e.g., recreational equipment, etc.) (6003)	0.224	0.259	0.240
Scientific, professional and medical devices (including measuring, controlling, laboratory equipment) (6004)	0.197	0.198	0.189
Heating, electrical, plumbing, air conditioning and refrigeration equipment (6005)	0.132	0.204	0.130
Pollution abatement and control equipment (6006)	0.115	0.146	0.125
Motors, generators, transformers, turbines, compressors and pumps of all types (6009)	0.117	0.106	0.103
Heavy construction equipment (e.g., loading, hauling mixing, paving, grating) (6010)	0.149	0.155	0.157
Tractors of all types and other field equipment (truck tractors - see 6203) (6011)	0.151	0.175	0.170
Capitalized tooling and other tools (hand, power, industrial) (6012)	0.211	0.221	0.229
Drilling and blasting equipment (6013)	0.168	0.128	0.156
Automobiles and major replacement parts (6201)	0.295	0.287	0.279
Buses (all types) and major replacement parts (6202)	0.125	0.098	0.103
Trucks, vans, truck tractors, truck trailers and major replacement parts (6203)	0.203	0.192	0.201
Other transportation equipment (6299)	0.177	0.171	0.162
Computer-assisted processing machinery and equipment (6401, 6402)	0.142	0.134	0.134
Non-computer-assisted processing machinery and equipment (6601, 6602)	0.136	0.137	0.125
Communication and related equipment (6403, 6603)	0.188	0.222	0.201
Safety and security equipment and other machinery and equipment (6007, 8999)	0.145	0.185	0.167
Mean, all assets	0.195	0.256	0.218
		standard error	
Standard error of mean, all assets	0.004	0.008	0.003

Note: The regrouping of 6401 and 6402, 6601 and 6602, 6403 and 6603, and 6007 and 8999 is due to the concordance of asset codes between the two sub periods.

Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Overall, the mean depreciation rate for these selected machinery and equipment categories as a whole, weighted by their chained dollar investment shares,[20] has increased from 0.20 to 0.26 from 1985 to 2001 and 2002 to 2010 (Table 4). Although the changes in depreciation rates are significant for only 4 types of assets, the increase in the average mean depreciation rate is statistically significant (Table 6). This is mostly due to the increased importance of the 4 types of assets experiencing significant change over the two time periods; their chained dollar investment share increases, from 20% in the early period, to more than 40% in the later period.

The estimates for the entire period, stretching from 1985 to 2010, are obtained by combining the samples for the two periods and estimating the two samples jointly. As a result, the depreciation rates for 1985 to 2010, for most of the assets in machinery and equipment, fall in between the estimates of the two sub-periods (Table 4).[21] Overall, the weighted mean depreciation rate for the selected machinery and equipment for the full period (from 1985 to 2010) is 0.22.

20. The weighted average uses chained dollar investment derived from totals for the total business sector, not just from the survey samples.
21. For some assets, the estimates from the pooled sample do not fall in between the estimates from the two sub-periods. This is because the reweighting applied to the pooled sample could be different to the weights used in the two sub-periods.

Table 5
Depreciation rates for select buildings categories

	1985 to 2001	2002 to 2010	1985 to 2010
		rate	
Asset description and code			
Plants for manufacturing, processing and assembling goods (1001)	0.08	0.08	0.07
Warehouses, refrigerated storage, freight terminals (1006)	0.06	0.09	0.06
Maintenance garages, workshops, equipment storage facilities (1008)	0.07	0.08	0.07
Office buildings (1013)	0.06	0.10	0.07
Shopping centres, plazas, malls, stores (1016)	0.11	0.18	0.16
Other industrial and commercial (1099)	0.07	0.11	0.10
Telephone and cablevision lines, underground and marine cables (3002)	0.14	0.16	0.14
Communication towers, antennae, earth stations (3003)	0.10	0.09	0.10
Mean, all assets	0.09	0.13	0.10
		standard error	
Standard error of mean, all assets	0.01	0.04	0.02

Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Turning to buildings, the depreciation rates of half of the eight selected assets (i.e., warehouses [1006], office buildings [1013], shopping centres [1016], and other industrial and commercial buildings [1099]) have increased, and both their discard age and reselling age have declined (Table 5). Specifically, from the early sub-period sample to the late sub-period sample, their mean discard age decreased from about 20 years to only 11 years, and their mean reselling age, from about 17 years to 15 years. These differences yield much smaller estimates of expected life for the later sub-period, causing the depreciation rates to increase. This increase may also have come from the restructuring of some industries—transportation, retail and wholesale—during the post-2000 period.

These estimates are not very different from recent estimates derived from Japan. Nomura and Momose (2008) and Nomura and Suga (2014) estimate the depreciation rates using the Survey on Capital Expenditures and Disposables (CED) in Japan, between 2005 and 2006, for both machinery and equipments and buildings. Their estimates for buildings range from 0.08 to 0.15.

Overall, the weighted mean depreciation rate for buildings, in the 1985-to-2001 period, is about 0.09, and it increases to 0.13 in the 2002-to-2010 period. However, this increase is not statistically significant at the 5% level (Table 7). Over the entire period from 1985 to 2010, the mean depreciation rate is about 0.1.

Table 6

Standard errors for select machinery and equipment categories and a test for differences between periods

	Estimate of depreciation rate 1985 to 2001	Estimate of depreciation rate 2002 to 2010	Difference in estimates between periods
	standard error		t-test
Asset description and code			
Office furniture, furnishing (e.g., desks, chairs) (6001)	0.005	0.009	7.761 *
Computers, associated hardware and word processors (6002)	0.014	0.016	4.416 *
Non-office furniture, furnishings and fixtures (e.g., recreational equipment, etc.) (6003)	0.010	0.048	0.730
Scientific, professional and medical devices (including measuring, controlling, laboratory equipment) (6004)	0.011	0.020	0.039
Heating, electrical, plumbing, air conditioning and refrigeration equipment (6005)	0.011	0.490	0.147
Pollution abatement and control equipment (6006)	0.010	0.013	1.855
Motors, generators, transformers, turbines, compressors and pumps of all types (6009)	0.006	0.006	1.285
Heavy construction equipment (e.g., loading, hauling mixing, paving, grating) (6010)	0.008	0.013	0.409
Tractors of all types and other field equipment (truck tractors - see 6203) (6011)	0.007	0.014	1.564
Capitalized tooling and other tools (hand, power, industrial) (6012)	0.013	0.238	0.042
Drilling and blasting equipment (6013)	0.013	0.032	1.165
Automobiles and major replacement parts (6201)	0.017	0.046	0.161
Buses (all types) and major replacement parts (6202)	0.023	0.007	1.097
Trucks, vans, truck tractors, truck trailers and major replacement parts (6203)	0.008	0.007	1.029
Other transportation equipment (6299)	0.010	0.008	0.442
Computer-assisted processing machinery and equipment (6401, 6402)	0.018	0.007	0.386
Non-computer-assisted processing machinery and equipment (6601, 6602)	0.007	0.009	0.057
Communication and related equipment (6403, 6603)	0.012	0.010	2.231 *
Safety and security equipment and other machinery and equipment (6007, 8999)	0.007	0.013	2.689
Mean, all assets	0.004	0.008	7.122 *

* significantly different from reference category (p<0.05)

Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Table 7

Standard errors for select categories of buildings and a test for differences

	Estimate of depreciation rate 1985 to 2001	Estimate of depreciation rate 2002 to 2010	Difference of estimates between periods
	standard error		t-test
Asset description and code			
Plants for manufacturing, processing and assembling goods (1001)	0.004	0.012	0.259
Warehouses, refrigerated storage, freight terminals (1006)	0.004	0.011	2.012 *
Maintenance garages, workshops, equipment storage facilities (1008)	0.006	0.585	0.013
Office buildings (1013)	0.004	0.008	4.104 *
Shopping centres, plazas, malls, stores (1016)	0.012	0.011	4.322 *
Other industrial and commercial (1099)	0.005	0.016	2.306 *
Telephone and cablevision lines, underground and marine cables (3002)	0.059	0.026	0.361
Communication towers, antennae, earth stations (3003)	0.009	0.008	0.890
Mean, all assets	0.007	0.037	1.029

* significantly different from reference category (p<0.05)

Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

5.2 *Ex ante* versus *ex post* estimates of depreciation and length of life

Direct estimates of δ can also be derived from information on the length of life of the asset (T). For many years, the latter method was the most common, and T was determined from accounting information—often associated with tax laws.

Straight-line patterns of depreciation assume equal dollar value depreciation at all stages of an asset's lifecycle. Per-period depreciation for a dollar of investment takes the form,

$$D = \frac{1}{T}$$

where T is service life. Although the dollar loss is equal from period to period, the rate of depreciation, that is, the percent change in asset value from period to period, increases progressively over the course of an asset's service life.

Alternately, constant geometric rates can be calculated indirectly from estimates of the length of life (T) of an asset derived from the tax code as

$$\delta = \frac{DBR}{T}$$

where T is service life, and DBR is chosen exogenously to provide a decline profile. The value of the DBR determines, other things being equal, the extent to which asset values erode more rapidly early in the lifecycle (Fraumeni 1997). Higher values of the DBR bring about higher reductions in asset value earlier in service life, giving rise to more convex (i.e., accelerated) depreciation profiles.

Double-declining-balance rates (DDBRs), which set the value of the DBR equal to 2, have been used extensively, in practice. In their estimates of capital stock, Christensen and Jorgenson (1969) employ DDBRs to estimate rates of economic depreciation. One advantage of the DDBR is that it provides a conceptual 'bridge' back to the straight-line case, anchoring the midpoints of the depreciation schedules at an equivalent age point. Indeed, the average depreciation rate in the straight-line case will match the constant rate derived from a DBR of 2.

Statistics Canada's Capital and Repair Expenditures Survey asks not only for the price of assets upon disposition, but also for the anticipated length of service life when investments are first reported to the agency. Use of the anticipated length of service life, along with a declining-balance constant, provides an alternate way to estimate the average depreciation rate (δ=DBR/L—see Statistics Canada, 2007).

Estimates of depreciation using the expected length of life are *ex ante* measures, and they may, therefore, suffer from inaccurate forecasts of the *ex ante* length of life. Differences may also occur if service lives have been changing over time, if the *ex post* rates make use of data that precede data used for the *ex ante* estimates.

There are several other reasons why the *ex ante* rates may differ from the *ex post* rates, which have to do with the concept of an anticipated length of life, all of which stem from the fact that managers may have a different concept in mind than the expected age of discard. For example, managers may have in mind the expected time before disposal, which could be the point at which they sell the asset, rather than the point at which they discard it. For example, buyers of fleet autos may have in mind the point at which they dispose of the car after the first (three-year) lease. Or managers may have in mind the point at which they expect to lose half of their asset value. In both of these cases, the *ex ante* concept may turn out to be less than the *ex post* estimate.

Another reason for possible discrepancies between *ex ante* and *ex post* rates arises from the heterogeneity of some asset classes. In this case, the composition of the sample of discards may be quite different from the population of investments that is used to calculate the *ex ante* length of life. A good example is the class of shopping centres, plazas, malls and stores (1016). Major shopping centres involve large investments with long service lives, and they probably dominate the investment population that supplies the *ex ante* rates. On another hand, strip malls with shorter lives are likely to be more heavily weighted in the observations on discards. This would produce an *ex ante* estimate that is higher than the *ex post* estimate derived from the pattern of actual discards.

The data source that provides an estimate of the expected *ex ante* length of life offers a much larger number of observations per asset than is available for the *ex post* estimate, and this is a distinct advantage. For example, the period from 1985 to 2001 can generate estimates of the *ex ante* expected life for 139 assets and more than 90,000 observations in total. The later period, from 2002 to 2010, contains more than 167,000 observations for almost 200 assets.

As attractive as this alternate *ex ante* technique is, it still requires the estimation (choice) of the declining-balance rate (DBR). The choice of a DBR, in itself, involves uncertainty. The DBR can be chosen as 2, as often happens in the accounting world. But, essentially, this involves an assumption that the associated efficiency or capacity frontier of the asset is constant. If the profile is concave, the DBR will typically be greater than 2 but less than 3.[22]

To compare the *ex post* estimates to the *ex ante* estimates, the DBRs that are yielded by the *ex post* technique are used, and these are substituted into the formula $\delta = \mathrm{DBR/T}$ using an *ex ante* length of life to yield a depreciation rate. Asset-specific estimates of the mean *ex ante* service life (T) are taken from the Capital and Repair Expenditures Survey.[23] The resulting estimates are then compared to the *ex post* rates that are derived from the data sets for the two time periods, jointly, in Tables 8 and 10.

The two sets of estimates are quite similar for buildings (Table 8). The mean *ex post* depreciation rate for buildings is 9.7%. It is 9.2% for the *ex ante* estimate calculated with estimated DBRs and *ex ante* lives.

22. It should be noted that, if the efficiency frontier takes on the profile of a logistic curve (initially concave, but then reversing itself to become convex), the DBR may be greater than 3.
23. Mean *ex ante* service life for each asset is weighted by the corresponding reported value of investment (deflated to 1997 dollars using asset-specific deflator).

The estimates of the expected discard age taken from the simultaneous estimate and the *ex ante* expected discard length of life for buildings are also included in Table 8. The two are quite close—25.6 and 25.1 years, respectively. In conclusion, for long-lived assets in the buildings category, the data cannot distinguish between the *ex post* and *ex ante* estimates, the exception being the class of shopping centres, plazas, malls and stores, which may suffer from the data problem discussed previously. The *ex post* estimated expected discard age for shopping centres is about 15 years and substantially shorter than reported average *ex ante* service life, 26 years. For this paper and the Productivity Accounts, it was decided to use the *ex ante* depreciation rate of about 9.1% instead of the *ex post* 16% for shopping centres.

In Table 9, the expected length of life for a select set of engineering assets, for which adequate data are available, is included in order to estimate the discard function, even if the price survival ratio is likely to be deficient (because most assets are discarded at zero price and not sold for positive value). Again, the *ex post* estimates are close to the *ex ante* estimates for the long-lived assets, 29 years versus 25 years.

These two results suggest that the use of the *ex ante* estimates of length of lives, along with an imputed DBR, for those long-lived assets with infrequent sales, where used-asset prices do not exist, promises a reasonable method of filling in the data set of depreciation rates for those categories where used-asset prices are not available.

As can be seen from Table 10, there are differences between the *ex post* and *ex ante* estimates of depreciation and length of life for machinery and equipment. The average assessed *ex post* estimates of life are higher than the *ex ante* expected service life estimates—11.5 years versus 8.2 years. As a result, the average *ex post* depreciation rates are 21.8% versus 29.7% for the *ex ante* estimate.

Table 8
***Ex post* versus *ex ante* depreciation rates for buildings, 1985 to 2010**

	Depreciation rate		Expected life	
	***Ex post* simultaneous estimation**	***Ex ante* declining-balance rate from simultaneous estimation**	***Ex post* simultaneous estimation**	***Ex ante* survey**
	rate		years	
Asset description and code				
Plants for manufacturing, processing and assembling goods (1001)	0.072	0.082	29.2	25.4
Warehouses, refrigerated storage, freight terminals (1006)	0.064	0.082	33.0	25.4
Maintenance garages, workshops, equipment storage facilities (1008)	0.066	0.077	31.8	27.2
Office buildings (1013)	0.067	0.078	31.4	27.0
Shopping centres, plazas, malls, stores (1016)	0.156	0.091	14.9	25.8
Autres constructions industrielles et commerciales (1099)	0.096	0.095	21.8	22.1
Telephone and cablevision lines, underground and marine cables (3002)	0.137	0.164	19.4	16.2
Communication towers, antennae, earth stations (3003)	0.099	0.133	27.0	20.1
Mean, all assets	0.097	0.092	25.6	25.1

Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Table 9
***Ex post* versus *ex ante* length of life for engineering construction, 1985 to 2010**

	Expected length of life	
	Ex post	*Ex ante*
	years	
Asset description and code		
Highways, roads, streets, including: logging road, signs, guardrail, lighting, etc. (2202)	22.1	28.7
Rail track and roadbeds including: signals and interlockers (2204)	39.9	26.5
Telephone and cablevision lines, underground and marine cables (3002)	19.4	16.2
Communication towers, antennae, earth stations (3003)	27.0	20.1
Gas mains and services (3201)	43.0	42.3
Bulk storage (3204)	24.9	26.4
Mean, all assets	29.4	25.3

Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Table 10
***Ex post* versus *ex ante* depreciation rates for machinery and equipment, 1985 to 2010**

	Depreciation rate		Expected life	
	***Ex post* simultaneous estimation**	***Ex ante* declining-balance rate from simultaneous estimation**	***Ex post* simultaneous estimation**	***Ex ante* survey**
	rate		years	
Asset description and code				
Office furniture, furnishing (e.g., desks, chairs) (6001)	0.227	0.275	9.2	7.6
Computers, associated hardware and word processors (6002)	0.431	0.430	4.9	4.9
Non-office furniture, furnishings and fixtures (e.g., recreational equipment, etc.) (6003)	0.240	0.293	9.7	7.9
Scientific, professional and medical devices (including measuring, controlling, laboratory equipment) (6004)	0.189	0.243	11.1	8.6
Heating, electrical, plumbing, air conditioning and refrigeration equipment (6005)	0.130	0.161	16.8	13.5
Pollution abatement and control equipment (6006)	0.125	0.167	16.8	12.5
Motors, generators, transformers, turbines, compressors and pumps of all types (6009)	0.103	0.112	20.3	18.8
Heavy construction equipment (e.g., loading, hauling mixing, paving, grating) (6010)[1]	0.157	0.285	14.4	7.9
Tractors of all types and other field equipment (truck tractors - see 6203) (6011)[1]	0.170	0.322	13.6	7.2
Capitalized tooling and other tools (hand, power, industrial) (6012)	0.229	0.309	10.2	7.5
Drilling and blasting equipment (6013)	0.156	0.182	14.2	12.1
Automobiles and major replacement parts (6201)[1]	0.279	0.495	7.5	4.2
Buses (all types) and major replacement parts (6202)[1]	0.103	0.170	22.0	13.4
Trucks, vans, truck tractors, truck trailers and major replacement parts (6203)	0.201	0.304	10.5	6.9
Other transportation equipment (6299)	0.162	0.279	13.0	7.5
Computer-assisted processing machinery and equipment (6401, 6402)	0.134	0.177	15.6	11.8
Non-computer-assisted processing machinery and equipment (6601, 6602)	0.125	0.190	16.7	11.0
Communication and related equipment (6403, 6603)	0.201	0.233	10.4	9.0
Safety and security equipment and other machinery and equipment (6007, 8999)	0.167	0.220	13.7	10.4
Mean, all assets	0.218	0.297	11.5	8.2

1. Asset categories where there are large differences between *ex ante* and *ex post* lengths of life.

Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Most of the larger differences occur in five categories—heavy construction (6010), tractors (6011), automobiles (6201), buses (6202) and trucks (6203), where about 70% of the increase in the average results from moving from *ex post* to *ex ante* depreciation rates. These are all categories where heavy motive equipment is found. The differences in these categories are consistent with the explanation that some managers have the concept ‘time to disposal’ rather than ‘time to discard’, when answering the question about the *ex ante* expected length of life. And this may occur if the equipment is purchased for specific construction projects. Using the *ex ante* estimate of the average DBR from machinery and equipment in general, along with the *ex ante* length of life for specific assets where there are not enough observations to estimate the *ex post* depreciation rate, promises to provide robust estimators for capital stock.

Explanations for differences between the *ex ante* and the *ex post* estimates must also account for the fact that the prices of used assets may only imperfectly reflect the future stream of earnings of the assets for several reasons. The used assets that are sold may have a higher proportion of ‘lemons’ than the capital stock in general and, therefore, may not reflect the average value in use. In addition, the price data used in estimating age–price profiles may be subject to more reporting error than the expected length of life data. In the face of all these potential problems, it is perhaps surprising to find as much congruence between the two estimates.

6 Capital stock

6.1 The effect of alternate depreciation rates on capital stock

In the previous section, estimates of depreciation rates for two different periods have been presented, as well as a new set for the two periods taken together. At issue is whether the rates of growth in the capital stock and the levels of capital stock differ when the new observations are added to the database.

In order to evaluate estimates derived from different time periods for the entire set of assets used in the Productivity Accounts, the following approach was adopted.

1. For those assets where used-asset prices for *ex post* estimates of depreciation exist, the depreciation rate, using the simultaneous estimation approach, is used.
2. For these estimates, an implicit DBR is calculated using Equation (4), the *ex post* depreciation rate, and the *ex ante* length of life.
3. For those machinery and equipment and building assets where heterogeneity or data availability prevent us from estimating a relevant *ex post* depreciation rate, an *ex ante* depreciation rate is obtained by dividing an imputed DBR by the *ex ante* service life. The imputed DBR for a given asset is derived from its corresponding average DBR from the 22 group levels when available, otherwise, from the general class of the asset.
4. For the engineering asset estimates, there are few *ex post* estimates as guides. Therefore, the *ex ante* depreciation rates are calculated. But the imputed DBR used is derived from combining available *ex post* estimates for all assets in building and engineering construction.
5. Mining, and oil and gas exploration are treated differently. The *ex ante* life for mining exploration is derived from the average of *ex ante* lives of mining-related engineering construction and for oil and gas exploration from that of oil- and gas-related engineering construction.
6. For research and development services, the estimates used by the Canadian National Accounts are adopted. That is, the *ex ante* service life is assumed to be 7 years and the DBR, 1.65.
7. For software, the estimates used by the Canadian National Accounts are also adopted, and the DBR is assumed to be equal to 1.65.
8. In the interest of simplification, the DBRs are averaged across all machinery and equipment assets, all buildings, and all engineering construction, giving estimates of 2.2, 2.2, and 2.4, respectively, and these are used with the *ex ante* expected length of lives.[24] The average DBRs show that the rate of decline is slightly above the DDBR of 2.

For easy comparison, the resulting depreciation estimates are reproduced in Table 11 for 21 aggregate asset classes and in Table C.1, in Appendix C, for detailed asset classes under the new asset code classification. The depreciation rates for the old and new estimates are very close on average (Table 11). The weighted average depreciation rate for buildings used in Statistics Canada (2007) is 0.074, and 0.077 in this study. The weighted average depreciation rate for engineering constructions used in Statistics Canada (2007) is 0.122, and 0.079 in this study. The weighted average depreciation rate for machinery and equipment used in Statistics Canada (2007) is 0.228, and 0.234 in this study.

With these estimates in hand, the growth rate in the entire capital stock in the business sector is calculated over the period, from 1960 to 2010. Sub-periods from 1960 to 2000 and from 2001 to 2010 are also provided using the old and new depreciation rates.

24. With the exception of the four assets where the *ex ante* length of life was significantly smaller than the *ex post* rate. In these cases, the *ex post* rate was used.

Estimates of capital stock are generated based on the perpetual inventory model,

$$K(t) = I(t) + (1-\delta)K(t-1) \tag{27}$$

where δ represents a (constant) geometric rate of depreciation that was estimated.

Table 11
Depreciation rates by aggregate asset classes

Asset description and code	New (1985 to 2010)	Statistics Canada (2007)[1]
	percent	
Industry building construction (4001)	7.3	8.9
Commercial building construction (4002)	7.8	6.9
Institutional Building Construction (4003)	6.2	6.8
Marine Engineering Construction (5001)	7.9	7.7
Transportation Engineering Construction (5002)	7.1	6.9
Waterworks Engineering Construction (5003)	5.7	7.0
Sewage Engineering Construction (5004)	7.4	9.3
Electric Power Engineering Construction (5005)	5.8	5.6
Communication Engineering Construction (5006)	12.8	12.0
Oil and Gas Engineering Construction (5007)	7.4	15.3
Mining Engineering Construction (5008)	15.7	14.5
Other Engineering Construction (5089)	10.9	9.0
Trucks, trucks chassis, vans, sport utility vehicles and major replacement parts (6001)	20.1	22.7
Automobiles, vans, and sport utility vehicles and major replacement parts (6003)	27.9	28.0
Tractors of all types and other field equipment (6011)	17.0	17.1
Transportation Equipment (7001)	18.5	13.7
Industrial Machinery (7002)	17.2	17.7
Telecommunication Equipment (7003)	24.9	22.0
Furnitures (7004)	24.8	22.2
Other Machinery and Equipment (7089)	20.2	18.6
Computers (8001)	43.1	46.7

1. Statistics Canada. 2007. *Depreciation Rates for the Productivity Accounts.* The Canadian Productivity Review. No. 5. Ottawa.
Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

The rates of growth of capital stock are presented in Table 12. The rate of growth of capital stock in the business sector, under the old depreciation rates over the period from 1961 to 2010, is 3.58% as compared with 3.65% using the new estimates. There are also small differences only for two sub-periods: 3.81% as opposed to 3.85%, for 1961 to 2000, and 2.7% as opposed to 2.9%, for 2001 to 2010. Differences for the rate of growth for different classes of capital are also small. For machinery and equipment, the number is 4.28% for the entire period with the new depreciation rate versus 4.36% with the old depreciation rates and building and engineering constructions, 3.41% using the new depreciation rates versus 3.28% using the old.

In conclusion, extending the database, improving the imputation methods, and experimenting with additional estimation techniques have a minimal impact on the estimates of the growth in capital stock that were developed previously (Statistics Canada 2007).

Table 12
Estimates of old and new annual growth rates in capital stock

	1961 to 2000	2001 to 2010	1961 to 2010
		percent	
Total capital stock			
With old depreciation rates	3.81	2.69	3.58
With new depreciation rates	3.85	2.90	3.65
Machinery and equipment			
With old depreciation rates	4.63	3.32	4.36
With new depreciation rates	4.56	3.21	4.28
Building and engineering construction			
With old depreciation rates	3.50	2.45	3.28
With new depreciation rates	3.58	2.78	3.41

Sources: Statistics Canada, authors' calculations based on data from multiple sources.

7 Conclusion

Estimates of depreciation are required to implement the perpetual inventory method that cumulates estimates of past investment to provide summary measures of the amount of net capital that is being applied to the production process.

Obtaining estimates of the rate of depreciation creates numerous difficulties. While depreciation is a concept that is applied directly to the accounts of companies and is used in the calculation of taxes owed to the government, the commonly used estimates contained in balance sheets are not always perceived as being those required by the productivity program. This can occur for a number of reasons—not the least of which is that depreciation allowances used for taxation purposes may differ from the 'real' rate. This happens either because the tax system lags in terms of changes in the durability and longevity of assets, or because the tax system may deliberately choose a rate that is different from the 'real' rate, because it is attempting to stimulate investment.

Rather than simply taking estimates of depreciation from accounting sources, the statistical community has developed alternate methods of estimating depreciation rates. Both the United States and Canada make use of the prices of used assets to estimate depreciation—the rate at which the value of the asset declines from usage. The difference between the two countries is that estimates in the United States are taken from numerous unconnected databases that provide prices of used equipment, while in Canada, the prices come from a single Capital and Repair Expenditures Survey extending back into the 1980s, which also asks for the prices of assets that are sold.

The Canadian Productivity Accounts also cross-reference estimates of depreciation derived from used-asset prices with estimates derived from *ex ante* estimates of the length of life derived from a question in the Capital and Repair Expenditures Survey. This question asks for estimates of the expected length of life at the time of the initial investment, and makes several assumptions about the profile of the rate of decline of the value of an asset in use (what has been referred to in the literature as the declining-balance rate, or DBR). The latter is estimated here from the actual decline pattern derived from the trajectory of used-asset prices over time.

This paper expands on the earlier work (Statistics Canada 2007). It enlarges the database on used-asset prices, and makes use of additional editing techniques on that database. This enlarges the number of observations to around 52,000. The size of this database is unique.

Several findings are noteworthy. First, the earlier estimates described in Statistics Canada (2007) are broadly confirmed in several aspects. The depreciation profiles generated by the econometric techniques were, on balance, accelerated, producing convex age–price curves. Adding observations to the database for a subsequent period leaves most of the estimates unchanged. Moreover, there is little evidence that depreciation rates have increased in more recent years, although there has been a shift in the composition of assets towards those with higher rates of depreciation, which causes the average depreciation rate to increase.

Second, as was the case in Statistics Canada (2007), the estimates derived from the econometric *ex post* approach, using the trajectory of used-asset prices, compare favourably to the estimates derived from the *ex ante* method, using estimates of the expected length of life of assets derived from the Capital and Repair Expenditures Survey. It is important to know whether the two estimates yield approximately the same results, since it would suggest that managers can accurately predict the length of life of their assets. It is also important to know whether *ex post* and *ex ante* estimates are approximately the same, since this information is used to produce estimates of depreciation of assets for which the number of observations available cannot be used to generate estimates using the *ex post* technique. There are a large number of fixed assets that fall within the building and engineering construction categories, where an *ex ante* prediction of the length of life exists but where there is an insufficiently large number of used-asset transactions to employ the *ex post* technique.

Third, the results produced by the *ex ante* and *ex post* approaches are approximately the same for those assets where there are enough observations to provide estimates for both approaches. This finding is important because the *ex ante* approach suffers a number of potential problems. Managers have to correctly forecast length of life in a changing world. They also need to have in mind an optimal maintenance schedule when they provide expectations on length of life. The *ex post* approach, in turn, suffers from other difficulties. Discarded data can suffer from a number of imperfections—not the least of which is inadequate recall of the original purchase price, all relevant upgrades, and the asset's age. There is also the potential lemon problem for used-asset prices. Despite these problems, the two techniques provide remarkably similar results. It is rare that applied economists have alternate sources that can be used to assess the validity of results.

Therefore, information from both approaches is combined to generate depreciation rates across the asset classes. These rates are used to estimate capital stock in the Canadian Productivity Accounts. The *ex ante* information that is provided in Statistics Canada's surveys only pertains to the expected length of life of the asset. Derivation of a (geometric) depreciation rate from the expected life of the asset also requires a shape parameter of the rate—what is referred to as the DBR. It is this parameter that determines how much of total lifetime depreciation occurs early in life. The Productivity Accounts make use of information on similar assets where the *ex post* approach has been used to infer what the DBR is likely to be.

After the database has been updated and the estimation techniques, slightly improved, the new growth rates in capital stock and in capital services are not very different than those previously used.

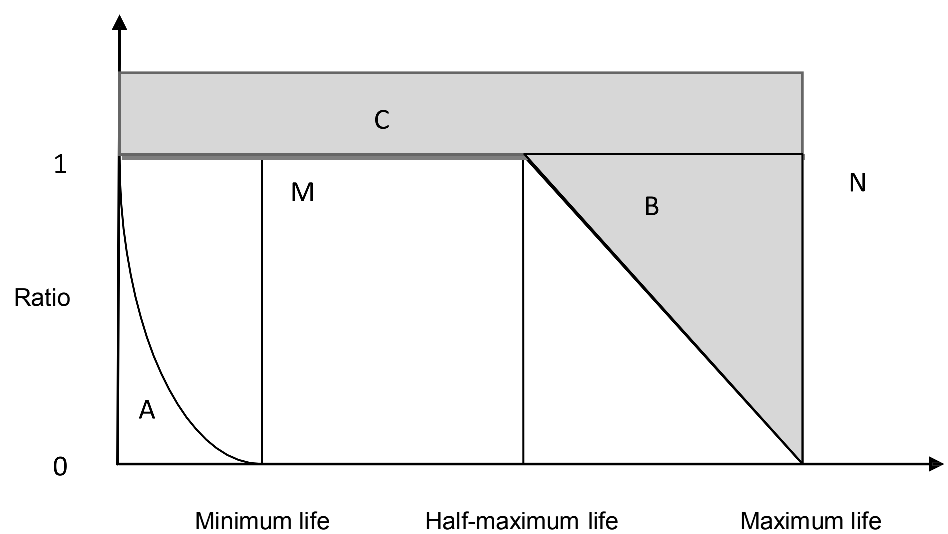
Appendix A Data editing

This paper uses the following editing strategy outlined in the Statistics Canada (2007) study.

First, observations were removed that exhibited either highly undervalued resale prices in early stages of service life, or highly overvalued resale prices at late stages, relative to the majority of observations in their asset categories. In doing so, minimum and maximum survival times for a given asset are calculated using information on both discards. These derive from observations with a selling price of zero, but with information on gross book value (GBV), on age and on *ex ante* surveyed life. The latter information is used, together with the discard information, because the discard ages for building assets in the second time period, 2002 to 2010, are shorter, on average, than those in the first period. Using only discard age would result in observations for building assets with longer lives being trimmed, which would cause depreciation rates to rise.

It is assumed that the retirement age of an asset (expressed in log form) follows a normal distribution. This is represented graphically in Figure A.1. The lower and upper bounds correspond to the youngest and oldest retirement ages, at the 10% significance level. Minimal survival time is defined at the lower bound, and maximum survival time is defined at the upper bound, weighted by an adjustment factor of 1.2.[25]

Figure A.1
Outlier identification



Source: Statistics Canada.

All observations in areas A, B and C are removed from the sample. Area A includes observations which have ‘unreasonably’ low survival rates at an early age. This area is bounded by a quadratic frontier and the minimum age M (i.e., the lower boundary below which zero sale

25. This weighting adjustment was made in order to define roughly symmetrical rejection areas on both sides of the distribution.

prices are rejected). Area B includes observations which have ‘unreasonably’ high survival rates well into their service life. This area is bounded by a linear frontier and point N (maximum life). Area C identifies all observations with survival rates greater than one (i.e., assets that appreciate in constant dollars).

Second, those observations on asset discards that exhibit large GBVs are eliminated; the identification process, in this instance, is carried out on the data as a whole—rather than on an age-cohort-specific basis.

Third, observations that involved ‘abnormally’ low price ratios for relatively young buildings, and observations on buildings whose ages are 3 times longer the average expected surveyed service life, were eliminated.

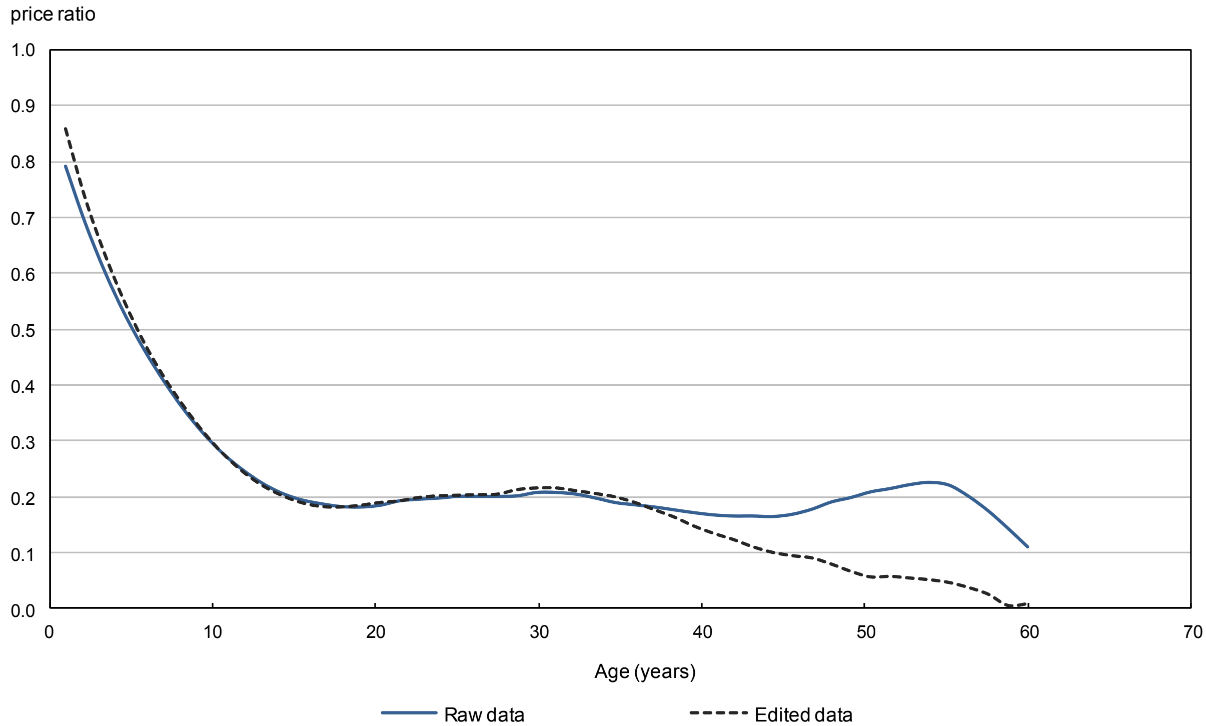
Fourth, additional observations that had an excessively large GBV (which had ‘made it through’ the previous filters) were eliminated.

Fifth, observations in public sectors and automobiles in financial and rental industries for automobiles are removed.

A good editing strategy should still maintain the overall feature of raw data, while getting rid of outliers. Appendix B—Charts B.1 to B.4—plots the smoothed, age–price profiles using a local polynomial method for four types of assets: plants, office furniture, computers and automobiles, for both the raw data and the edited data. The closeness of the two age–price profiles suggests that our edit routine does a good job in this respect.

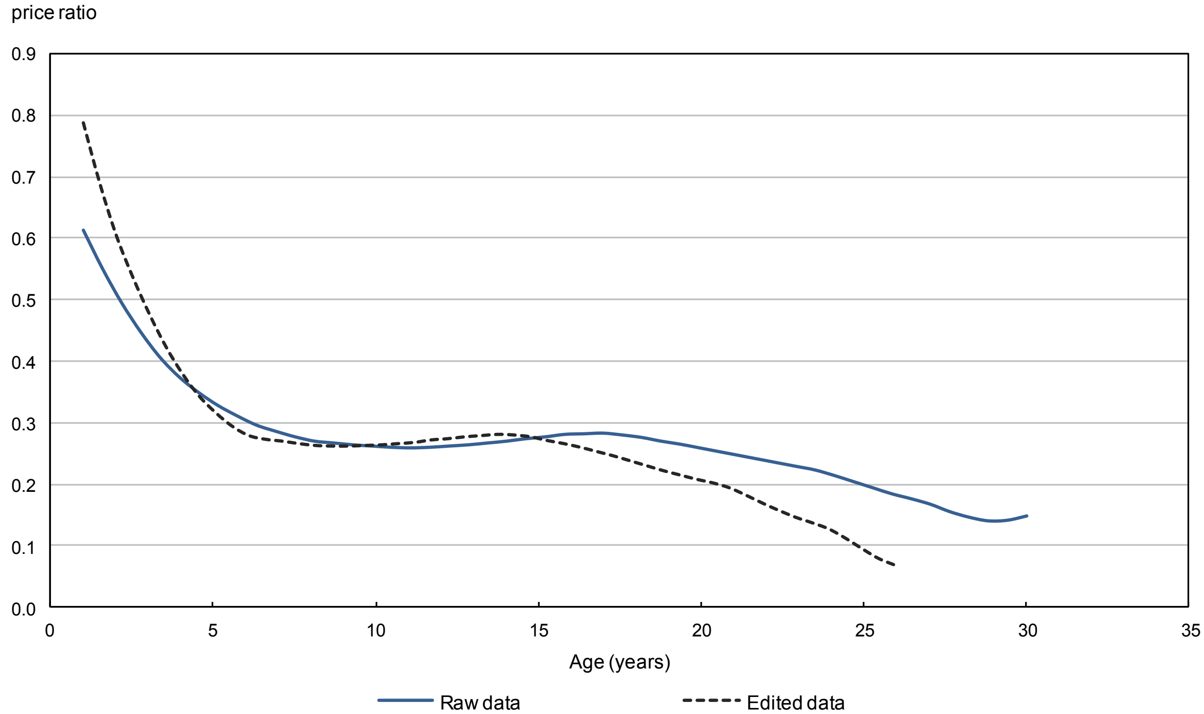
Appendix B Age–price profiles for the raw data and edited data for selected assets

Chart B.1
Manufacturing plants



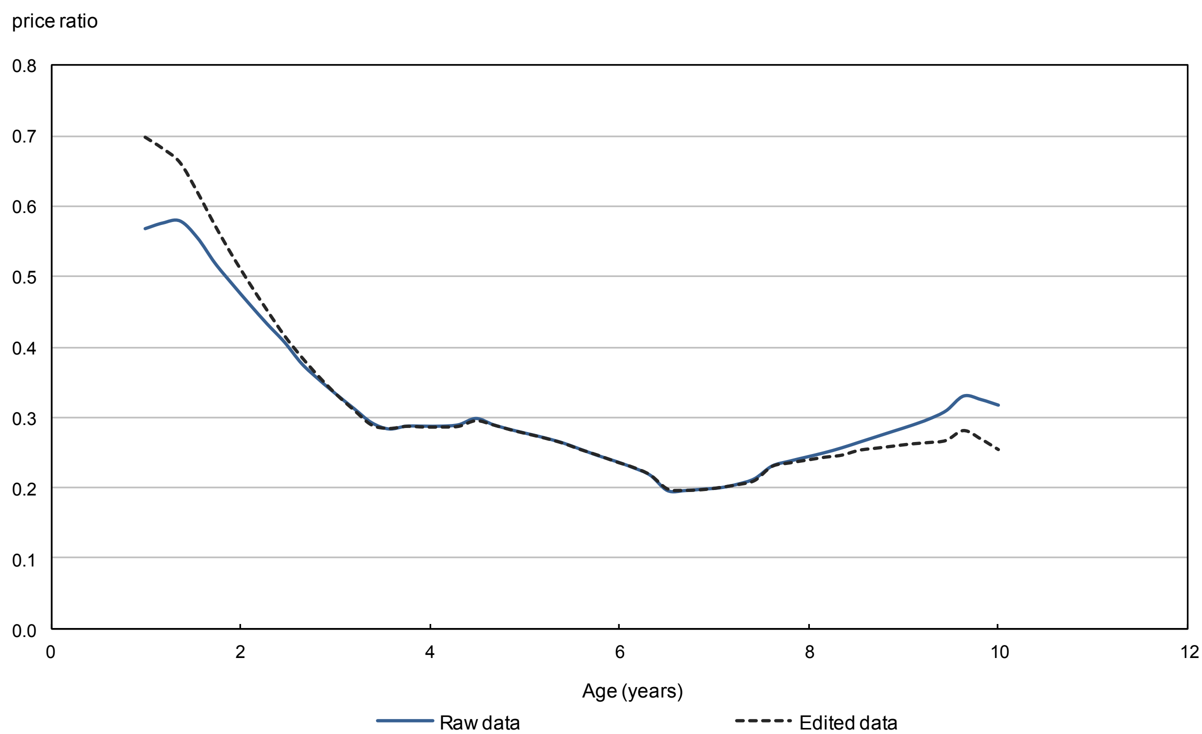
Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Chart B.2
Office furniture



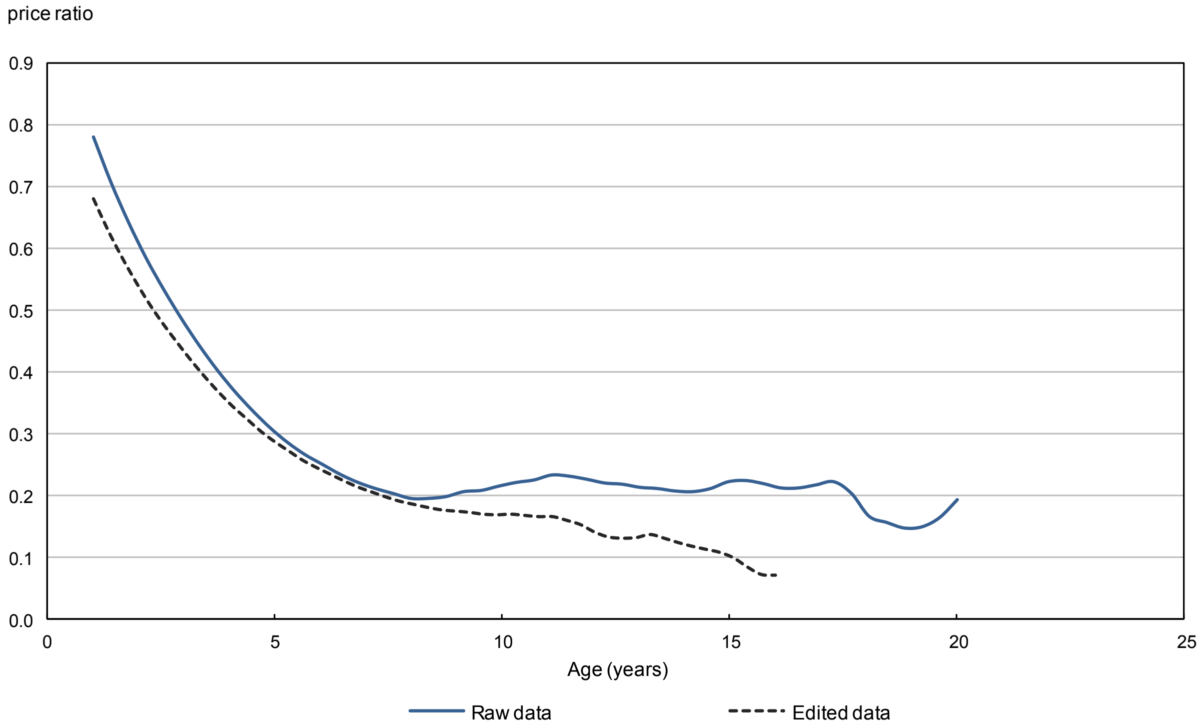
Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Chart B.3
Computers



Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Chart B.4
Automobiles



Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Appendix C Depreciation rates under the new asset code classification

Table C.1-1
List of depreciation rates under the new asset code classification — Building construction (industry, commercial, and institutional)

Major group, asset description and code	Depreciation rates: Updated (1985 to 2010)	Depreciation rates: Statistics Canada (2007)[1]	Surveyed lives (1985 to 2010)
	rate		years
Industry building construction			
Plants for manufacturing (1001)	0.072	0.089	25.4
Maintenance garages, workshops, equipment storage facilities (1008)	0.066	0.084	27.2
Railway shops, engine houses (1009)	0.056	0.080	37.3
Aircraft hangars (1010)	0.078	0.096	26.9
Farm buildings (1021)	0.079	0.095	26.6
Other industrial construction (1097)	0.081	0.085	25.8
Commercial building construction			
Laboratories, research and development centres (1004)	0.069	0.066	31.9
Warehouses, refrigerated storage, freight terminals (1006)	0.064	0.068	25.4
Grain elevators and terminals (1007)	0.080	0.071	27.4
Service stations (include self-serve and car washes) (1011)	0.080	0.123	17.3
Automotive dealerships (1012)	0.100	0.087	22.0
Office buildings (1013)	0.067	0.060	27.0
Hotels, motels, convention centres (1014)	0.081	0.059	27.1
Restaurants, fast food outlets, bars, nightclubs (1015)	0.089	0.087	23.5
Shopping centres, plazas, malls, stores (1016)	0.091	0.070	25.8
Theatres, performing arts and cultural centres (1018)	0.068	0.067	32.1
Indoor recreational buildings (e.g. sport complex, clubhouse, covered stadiums) (1019)	0.074	0.069	29.9
Bunkhouses, dormitories, camp cookeries, camps (1022)	0.163	0.161	13.4
Other commercial construction (1098)	0.101	0.085	20.7
Student residences (exclude residential construction) (1202)	0.056	0.055	39.4
Post offices (1212)	0.067	0.118	32.5
Passenger terminals (e.g. air, boat, bus, rail and other) (2201)	0.075	0.065	29.5
Broadcasting and communication buildings (3001)	0.086	0.086	24.7
Institutional building construction			
Schools (include technical, vocational), colleges, universities and other educational buildings (1201)	0.055	0.062	39.6
Churches and other religious buildings (1203)	0.055	0.047	39.5
Hospitals, health centres, clinics and other health care centres (exclude residential construction) (1204)	0.061	0.061	35.7
Nursing homes, homes for the aged (1205)	0.062	0.060	35.2
Day care centres (1206)	0.068	0.076	31.9
Libraries (1207)	0.055	0.059	39.5
Historical sites (1208)	0.066	0.094	32.9
Penitentiaries, detention centres and courthouses (1209)	0.056	0.060	38.8
Museums, science centres, public archives (1210)	0.066	0.046	33.1
Fire stations, fire halls (1211)	0.059	0.081	37.1
Armouries, barracks, drill halls and other military type structures (1214)	0.069	0.096	31.5
Other institutional and governmental construction (1299)	0.073	0.075	29.6
Other building construction (1999)	0.061	0.071	35.4

1. Statistics Canada. 2007. *Depreciation Rates for the Productivity Accounts.* The Canadian Productivity Review. No. 5. Ottawa.

Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Table C.1-2
List of depreciation rates under the new asset code classification — Engineering construction (marine, transportation, waterworks, sewage, and electric power)

Major group, asset description and code	Depreciation rates: Updated (1985 to 2010)	Depreciation rates: Statistics Canada (2007)[1]	Surveyed lives (1985 to 2010)
	rate		years
Marine engineering construction			
Docks, wharves, piers, terminals (e.g. coal, oil, natural gas, containers, general cargo) (2001)	0.078	0.078	28.5
Dredging and pile driving (2002)	0.091	0.104	24.4
Breakwaters (2003)	0.056	0.211	39.7
Canals and waterways (2004)	0.089	0.046	25.1
Other marine construction (2099)	0.076	0.071	29.3
Transportation engineering construction			
Parking lots and parking garages (1017)	0.098	0.085	22.9
Highways, roads, streets (include logging roads, signs, guardrails, lighting, landscaping, sidewalk, fences) (2202)	0.106	0.089	28.7
Runways (include lighting) (2203)	0.107	0.073	20.8
Rail track and roadbeds (include signals and interlockers) (2204)	0.053	0.060	26.5
Bridges, trestles, overpasses (2205)	0.064	0.062	34.9
Tunnels (2206)	0.042	0.039	52.7
Other transportation construction (2299)	0.080	0.073	28.0
Waterworks engineering construction			
Reservoirs (include dams) (2401)	0.059	0.056	37.9
Trunk and distribution mains (2402)	0.055	0.077	40.7
Water pumping stations and filtration plants (2412)	0.058	0.062	38.8
Water storage tanks (2413)	0.041	0.207	55.1
Other waterworks construction (2499)	0.089	0.092	25.1
Sewage engineering construction			
Sewage treatment and disposal plants (include pumping stations) (2601)	0.078	0.099	28.6
Sanitary and storm sewers, trunk and collection lines, open storm ditches (2602)	0.053	0.076	42.4
Lagoons (2603)	0.096	0.081	23.2
Other sewage system construction (2699)	0.094	0.100	23.8
Electric power engineering construction			
Electric power construction (2801)	0.075	0.096	29.7
Production plant - steam (2811)	0.059	0.055	37.9
Production plant - nuclear (2812)	0.069	0.051	32.3
Production plant - hydraulic (2813)	0.040	0.048	55.3
Overhead cables and lines (include poles, towers and all related parts and costs capitalized to this account) (transmission lines) (2814)	0.051	0.051	43.5
Underground cables and lines (include trenching, tunnels and all related parts and costs capitalized to this account) (transmission lines) (2815)	0.049	0.049	45.2
Overhead cables and lines (include poles, towers and all related parts and costs capitalized to this account) (distribution lines) (2816)	0.075	0.067	32.8
Underground cables and lines (include trenching, tunnels and all related parts and costs capitalized to this account) (distribution lines) (2817)	0.059	0.063	38.1
Other construction (not specified elsewhere) (2899)	0.081	0.063	27.6

1. Statistics Canada. 2007. *Depreciation Rates for the Productivity Accounts.* The Canadian Productivity Review. No. 5. Ottawa.

Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Table C.1-3
List of depreciation rates under the new asset code classification — Engineering construction (communication, oil and gas, mining, and other)

Major group, asset description and code	Depreciation rates: Updated (1985 to 2010)	Depreciation rates: Statistics Canada (2007)[1]	Surveyed lives (1985 to 2010)
	rate		years
Communication engineering construction			
Cables and lines - coaxial, copper, aluminium, etc. (exclude optical fibre) (e.g. aerial, underground and submarine) (3002)	0.137	0.122	16.2
Transmission support structures - towers, poles, conduit (3003)	0.099	0.107	20.1
Optical fibre (e.g. aerial, underground and submarine) (3022)	0.127	0.122	17.5
Other communication construction (3099)	0.142	0.146	15.7
Oil and gas engineering construction			
Oil refineries (1002)	0.126	0.118	17.8
Natural gas processing plants (1003)	0.080	0.106	27.8
Gas mains and services (3201)	0.062	0.070	42.3
Pumping stations, oil (3202)	0.082	0.296	27.2
Pumping stations, gas (3203)	0.075	0.083	29.8
Bulk storage (3204)	0.102	0.113	26.4
Oil pipelines (3205)	0.078	0.116	28.7
Gas pipelines (3206)	0.066	0.081	33.9
Development drilling (3217)	0.072	0.167	31.0
Production facilities (3218)	0.072	0.167	31.0
Enhanced recovery projects (3219)	0.072	0.167	31.0
Drilling expenditures, pre-mining, research and all other costs (3220)	0.072	0.167	31.0
Other oil and gas facilities (3299)	0.074	0.074	30.1
Mining engineering construction			
Mine buildings including headframes, ore bins, ventilation structures, backfill plants and other surface buildings (3401)	0.156	0.180	14.3
Mine buildings for beneficiation treatment of minerals (excluding smelters and refineries) (3402)	0.136	0.168	16.4
Mine shafts, drifts, crosscuts, raises, declines, stoping, etc. (3403)	0.170	0.147	13.1
Tailing disposal system, settling ponds (3404)	0.168	0.157	13.3
Mine site development (3412)	0.157	0.137	14.2
Other engineering construction			
Pollution, abatement and control (1005)	0.106	0.095	21.0
Outdoor recreational facilities (e.g. parks, open stadiums, golf courses, ski resorts) (1020)	0.092	0.099	24.2
Waste disposal facilities (1213)	0.069	0.087	32.2
Irrigation and land reclamation projects (2005)	0.106	0.049	21.0
Other engineering construction (4999)	0.111	0.122	20.1
Other construction (5999)	0.132	0.150	23.8
Trucks, trucks chassis, vans, sport utility vehicles and major replacement parts used for the transport of goods (exclude off-highway trucks) (6001)	0.201	0.227	6.9
Automobiles, vans, sport utility vehicles and major replacement parts used for the transport of persons (exclude hearses and ambulances) (6003)	0.279	0.280	4.2
Tractors of all types and other field equipment (6011)	0.170	0.171	7.2

1. Statistics Canada. 2007. *Depreciation Rates for the Productivity Accounts.* The Canadian Productivity Review. No. 5. Ottawa.
Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Table C.1-4
List of depreciation rates under the new asset code classification — Transportation equipment and industrial machinery

Major group, asset description and code	Depreciation rates: Updated (1985 to 2010)	Depreciation rates: Statistics Canada (2007)[1]	Surveyed lives (1985 to 2010)
	rate		years
Transportation equipment			
Buses, bus chassis and major replacement parts (6002)	0.103	0.149	13.4
Trailers and semi-trailers for the transport of goods (include major replacement parts) (6004)	0.338	0.227	6.2
Locomotives, rolling-stock, street and subway cars, other rapid transit equipment and major replacement parts (6006)	0.096	0.103	18.2
Aircraft, helicopters, aircraft engines and major replacement parts (exclude satellites and flight simulators) (6007)	0.137	0.082	16.1
Ships, boats and floating structures (include drilling rigs) and major replacement parts (6008)	0.109	0.104	20.3
Truck bodies, bus bodies and cargo containers (6009)	0.225	0.188	9.8
Other motor vehicles (e.g. off-highway trucks, all-terrain vehicles, hearses, ambulances, motorcycles) (include major replacement parts) (6010)	0.280	0.201	7.5
Other transportation equipment (6099)	0.167	0.201	13.3
Industrial machinery			
Special purpose motor vehicles and major replacement parts (e.g. mobile cranes, drilling derricks, concrete mixers, snow-blowing vehicles) (6005)	0.197	0.201	10.6
Filtering or purifying equipment for gases (e.g. air separators, electrostatic filters) - computer assisted (7101)	0.143	0.183	14.6
Filtering or purifying equipment for liquids (exclude beverages other than water and the preparation of foodstuffs) (e.g. water, sewage treatment, industrial waste treatment) - computer assisted (7102)	0.158	0.183	13.3
Metal working machinery and equipment (e.g. casting machines, tube and rolling mills, bending, shearing, punching) - non-computer assisted (7205)	0.142	0.168	14.8
Machine tools and tool accessories - non-computer assisted (7206)	0.269	0.168	7.8
Other industry specific processing machinery and equipment (e.g. machinery for the industrial preparation or manufacture of food and beverages, printing machinery, injection-moulding machines) - non-computer assisted (7207)	0.167	0.168	12.5
Packaging and bottling machinery (e.g. cleaning, drying, filling, closing, sealing, capsuling or labelling containers; packing or wrapping) - computer assisted (7103)	0.189	0.183	11.1
Logging machinery and machinery for making pulp, paper or paperboard - computer assisted (7104)	0.129	0.183	16.2
Metal working machinery and equipment (e.g. casting machines, tube and rolling mills, bending, shearing, punching) - computer assisted (7105)	0.150	0.183	14.0
Machine tools and tool accessories - computer assisted (7106)	0.273	0.183	7.7
Other industry specific processing machinery and equipment (e.g. machinery for the industrial preparation or manufacture of food and beverages, printing machinery, injection-moulding machines) - computer assisted (7107)	0.183	0.183	11.5

1. Statistics Canada. 2007. *Depreciation Rates for the Productivity Accounts.* The Canadian Productivity Review. No. 5. Ottawa.
Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Table C.1-5
List of depreciation rates under the new asset code classification — Industrial machinery (continued)

Major group, asset description and code	Depreciation rates: Updated (1985 to 2010)	Depreciation rates: Statistics Canada (2007)[1]	Surveyed lives (1985 to 2010)
	rate		years
Industrial machinery (continued)			
Industrial robots capable of performing a variety of functions by using different tools (exclude material handling equipment, irrigation systems and electric welding machines) - computer assisted (7108)	0.222	0.183	9.4
Machinery for sorting, screening, separating, washing, crushing, grinding, mixing or kneading mineral substances in solid form - computer assisted (7109)	0.177	0.183	11.8
Other processing machinery and equipment - computer assisted (7199)	0.199	0.183	10.5
Filtering or purifying equipment for gases (e.g. air separators, electrostatic filters) - non-computer assisted (7201)	0.146	0.168	14.4
Filtering or purifying equipment for liquids (exclude beverages other than water and the preparation of foodstuffs) (e.g. water, sewage treatment, industrial waste treatment) - non-computer assisted (7202)	0.142	0.168	14.7
Packaging and bottling machinery (e.g. cleaning, drying, filling, closing, sealing, capsuling or labelling containers; packing or wrapping) - non-computer assisted (7203)	0.201	0.168	10.4
Logging machinery and machinery for making pulp, paper or paperboard - non-computer assisted (7204)	0.144	0.168	14.5
Industrial robots capable of performing a variety of functions by using different tools (exclude material handling equipment, irrigation systems and electric welding machines) - non-computer assisted (7208)	0.231	0.168	9.1
Machinery for sorting, screening, separating, washing, crushing, grinding, mixing or kneading mineral substances in solid form - non-computer assisted (7209)	0.190	0.168	11.0
Other processing machinery and equipment - non-computer assisted (7299)	0.174	0.168	12.0
Gas generators, turbines, internal combustion engines and other motors (exclude motors for transportation equipment) and mechanical power transmission equipment (9001)	0.080	0.130	26.0
Non-fuel dispensing pumps, air and gas compressors, fans and blowers (9002)	0.070	0.130	29.8
Air conditioning (exclude portable air conditioners), refrigerating or freezing equipment (9003)	0.182	0.167	12.0
Industrial or laboratory furnaces and ovens, and furnace burners and related equipment (9004)	0.138	0.167	15.8
Well drilling and servicing rigs (other than floating) (9005)	0.156	0.192	12.1
Fork-lift trucks and warehouse trucks (9007)	0.253	0.168	8.3
Construction machinery, mining, oil and gas field machinery (e.g. moving, grading, excavating, compacting, extracting or boring machinery for earth, minerals, ores or snow) (exclude tractors for agricultural work) (9008)	0.157	0.172	7.9
Capitalized tooling (9015)	0.229	0.233	7.5
Gas generators and gas turbines (9050)	0.078	0.130	27.7
Steam and other vapour turbines (9051)	0.078	0.130	27.7
Electric water heaters (9092)	0.185	0.167	11.6
Nuclear reactor parts and fuel elements (9093)	0.107	0.130	20.1
Hydraulic turbines (9094)	0.045	0.130	47.9

1. Statistics Canada. 2007. *Depreciation Rates for the Productivity Accounts.* The Canadian Productivity Review. No. 5. Ottawa.
Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Table C.1-6
List of depreciation rates under the new asset code classification — Industrial machinery (continued), telecommunication equipment, furnitures, and other machinery and equipment

Major group, asset description and code	Depreciation rates: Updated (1985 to 2010)	Depreciation rates: Statistics Canada (2007)[1]	Surveyed lives (1985 to 2010)
	rate		years
Industrial machinery (continued)			
Conveyors, elevators, hoisting and loading and unloading machinery (computer assisted) (9106)	0.204	0.183	10.3
Conveyors, elevators, hoisting and loading and unloading machinery (non-computer assisted) (9206)	0.150	0.168	14.0
Telecommunication equipment			
Broadcasting and radio communication equipment (exclude transmission equipment) (8107)	0.275	0.220	7.6
Radar and navigational instruments (e.g. radar and sonar equipment, radio navigational aid apparatus, GPS receivers) (8109)	0.160	0.220	13.1
Network switching equipment hardware, including IP switches (routers) and PBXs used as public switches (exclude switching software) (8116)	0.232	0.220	9.0
Terminal equipment (e.g. PBXs, telephone, handsets, cellular phones, key systems, modems, palm pilots, fax machines, pagers, satellite terminals / dishes, decoders, set-top boxes) (8117)	0.329	0.220	6.4
Transmission equipment (e.g. transponders, receivers, cross connects, multiplexes, optical electronics, satellite earth stations, cell site equipment, antennas, cable head end equipment and components, cable distribution systems, plant equipment) (8127)	0.204	0.220	10.3
Other communication equipment (8199)	0.200	0.220	10.5
Conventional communication equipment (8299)	0.242	0.220	8.7
Furnitures			
Office furniture (8004)	0.261	0.235	8.0
Other machinery and equipment			
Other furniture, furnishings and fixtures (e.g. hotels, motels, restaurants, hospitals or store furnitures and fixtures) (8005)	0.240	0.214	7.9
Office machinery and equipment (exclude telephone equipment) (8003)	0.306	0.235	6.8
Radios, TVs, stereos, VCRs, DVDs and recorded tapes and discs (8013)	0.227	0.214	5.5
Electric motors and generators (9009)	0.130	0.130	16.1
Electric transformers, converters, inductors, switch gear, and other industrial electric equipment (9010)	0.099	0.130	21.2
Optical instruments and apparatus, laboratory, scientific and material-testing equipment (include flight simulators) (9011)	0.196	0.229	10.7
Instruments and appliances for medical, surgical, dental or veterinary use, or for related purposes (9012)	0.276	0.229	7.6
Measuring, checking or automatically controlling instruments and apparatus (exclude gas, water, and electricity meters) (9013)	0.171	0.233	12.3
Hand tools and power hand tools (9014)	0.250	0.233	9.3
Electric motors and generators (9052)	0.130	0.130	16.1

1. Statistics Canada. 2007. *Depreciation Rates for the Productivity Accounts.* The Canadian Productivity Review. No. 5. Ottawa.
Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

Table C.1-7

List of depreciation rates under the new asset code classification — Other machinery and equipment (continued), oil and gas exploration, mining exploration, research and development, and software

Major group, asset description and code	Depreciation rates: Updated (1985 to 2010)	Depreciation rates: Statistics Canada (2007)[1]	Surveyed lives (1985 to 2010)
	rate		years
Other machinery and equipment (continued)			
Electric transformers, static converters and inductors (9053)	0.099	0.130	21.2
Electric switchgear and switching apparatus (9054)	0.099	0.130	21.2
Electric control and protective equipment (9055)	0.170	0.229	12.3
Measuring, checking or automatically controlling instruments and apparatus (9056)	0.170	0.233	12.3
Electricity meters (9091)	0.091	0.233	24.1
Boilers (9095)	0.180	0.166	12.1
Other machinery and equipment (e.g storage tanks, fire fighting vehicles, portable air conditioners, electric traffic control equipment, gas meters, water meters, electricity meters, central heating boilers, welding equipment) (9099)	0.196	0.166	11.7
Other machinery and equipment (9999)	0.189	0.166	12.1
Computers and related machinery and equipment (exclude software purchased separately) (8001)	0.431	0.467	4.9
Oil and gas exploration			
Exploration drilling (3216)	0.072	0.167	31.0
Geological, geophysical and other exploration and evaluation costs (3221)	0.072	0.167	31.0
Mining exploration			
Mining sites exploration (3411)	0.157	0.137	14.2
Exploration expenses outside the mining site (3413)	0.157	0.137	14.2
Research and development			
Research and development services (6551)	0.236	0.236	7.0
Own account research and development services (except software development) (6552)	0.236	0.236	7.0
Software			
Standard and on the shelf software (8021)	0.550	0.550	3.0
Custom made software prepared by other enterprises (8022)	0.330	0.330	5.0
Own-account software (8023)	0.330	0.330	5.0

1. Statistics Canada. 2007. *Depreciation Rates for the Productivity Accounts.* The Canadian Productivity Review. No. 5. Ottawa.

Source: Statistics Canada, authors' calculations based on data from the Capital and Repair Expenditures Survey.

References

Akerlof, G. 1970. “The market for ‘lemons’: Quality Uncertainty and the Market Mechanism.” *Quarterly Journal of Economics*. 84 (3): 488–500.

Cox, D.R., and D. Oakes. 1984. *Analysis of Survival Data*. New York: Chapman and Hall.

Fraumeni, B.M. 1997. “The Measurement of Depreciation in the U.S. National Income and Product Accounts.” *Survey of Current Business* July: 7–23.

Gellatly, G., M. Tanguay, and B. Yan. 2002. “An Alternative Methodology for Estimating Economic Depreciation: New Results Using a Survival Model.” In *Productivity Growth in Canada – 2002*, ed. J.R. Baldwin and T.M. Harchaoui, Chapter 2, p. 25–65. Statistics Canada Catalogue no. 15-204-X. Ottawa: Statistics Canada.

Hall, R.E. 1971. “The Measurement of Quality Changes from Vintage Price Data.” In *Price Indexes and Quality Change,* ed. Z. Griliches, p. 240–271. Cambridge: Harvard University Press.

Hulten, C.R., and F.C. Wykoff. 1981. “The measurement of economic depreciation.” In *Depreciation, Inflation, and the Taxation of Income from Capital*, ed. C.R. Hulten, p. 81–125. Washington, D.C.: The Urban Institute Press.

Lagakos, S.W. 1979. “General Right Censoring and Its Impact on the Analysis of Survival Data.” *Biometrics* 35 (1): 139–156.

Nelson, W.B. 1982. *Applied Life Data Analysis*. New York: J. Wiley and Sons.

Nomura, K., and F. Momose. 2008. *Measurement of Depreciation Rates based on Disposal Asset Data in Japan.* Economic and Social Research Institute (ESRI) working paper, Government of Japan.

Nomura, K., and Y. Suga. 2014. *Asset Service Lives and Depreciation Rates Based on Disposal Data in Japan.* Economic and Social Research Institute (ESRI) working paper, Government of Japan.

Statistics Canada. 2007. *Depreciation Rates for the Productivity Accounts.* The Canadian Productivity Review, no. 5. Statistics Canada Catalogue no. 15-206-X. Ottawa: Statistics Canada.

Tanguay, M. 2005. “Linking Physical and Economic Depreciation: A Joint Density Approach.” Ottawa: Micro-economic Analysis Division, Statistics Canada. Mimeo.