



Catalogue no. 13-604-MIE — No. 42

ISSN: 1707-1739

ISBN: 0-662-35296-3

Research Paper

Income and Expenditure Accounts technical series

Chain Fisher Volume Index Methodology

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Chain Fisher Volume Index Methodology

On May 31, 2001 the quarterly income and expenditure accounts adopted the Fisher index formula, chained quarterly, as the official measure of real gross domestic product in terms of expenditures. This formula was also adopted for the Provincial Accounts on October 31, 2002.

There were two reasons for adopting this formula: to provide users with a more accurate measure of real GDP growth between two consecutive periods and to make the Canadian measure comparable with the Income and Product Accounts of the United States, which has used the chain Fisher index formula since 1996 to measure real GDP.

Ottawa
November 2003

Catalogue no. 13-604-**MIE** no. 42
ISBN: 0-662-35296-3
ISSN: 1707-1739

Catalogue no. 13-604-**MPE** no. 42
ISBN: 0-662-35295-5
ISSN: 1707-1720

Published by authority of the Minister responsible for Statistics Canada

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Table of contents

1.0 Introduction	1
2.0 Building an index and chaining	1
Example 1 : A wine and cheese economy	2
Example 2 : Wine and cheese production without the price effect	3
Example 3 : Another way to remove the price effect	4
3.0 Choice of index.....	5
4.0 Application in the Income and Expenditure Accounts.....	6
Example 4 : In the real world : the deflation method.....	7
4.1 Annual and quarterly National Economic and Financial Accounts.....	9
Level of detail at the national level.....	9
4.2 The problem of inventory.....	9
4.3 The provincial accounts.....	10
Level of detail at the provincial level.....	10
Sources of bias between the national and the provincial systems.....	11
5.0 The problem of non-additivity.....	11
Contribution to a GDP change series.....	12
Appendix I: Transformation of Laspeyres and Paasche indexes.....	13
Appendix II: Contribution to change formula	14
Bibliography	17
Technical series	19

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1.0 Introduction

Growth in the current gross domestic product (GDP) or any other nominal value aggregate can be decomposed into two elements: a “price effect”, or the part of the growth linked to inflation, and a “volume effect”, which covers the change in quantities, quality and composition of the aggregate. The volume effect is presented in the National Accounts by what is referred to as the “real” series (such as the real GDP).

In the Canadian National Accounts, the volume effect is determined using the deflation method, which eliminates the price effect from each component of the aggregate and then aggregates the components thus deflated to obtain the “total” volume effect.

There are several ways to aggregate the components of an aggregate in order to calculate the volume effect. Index number theory offers a wide range of tools to this end. Since spring 2001, Statistics Canada has preferred the chain Fisher index. This measure is theoretically superior to the former fixed-base Laspeyres measure and also makes the Canadian data comparable with the United States’ official measures of economic activity. Furthermore, it offers compliance with the recommendations of the System of National Accounts 1993 (SNA).¹

The following paragraphs provide a simplified explanation of the methods that will henceforth be used by the National Accounts to measure the country’s real economic activity.

2.0 Building an index and chaining

A given nominal aggregate (GDP or other) represents a summation of quantities evaluated in the same monetary unit, at the prices of the current period. To use GDP as an example, this summation can be expressed as $GDP = \sum pq$, which is the sum of all quantities of goods and services transacted in the economy, multiplied by their respective prices. The change or variation in nominal GDP, between a period o and a period t , can therefore be expressed in index² form by:

$$(1) \quad \Delta GDP_{t/o} = \frac{\sum p_t q_t}{\sum p_o q_o}$$

where: $\Delta GDP_{t/o}$ is the GDP variation index
 p_t is the price at time t
 p_o is the price at time o
 q_t is the quantity at time t
 q_o is the quantity at time o

¹ System of National Accounts 1993. Prepared under the auspices of the Inter-Secretariat Working Group on National Accounts: Commission of the European Communities – Eurostat; International Monetary Fund; Organisation for Economic Co-operation and Development; United Nations; World Bank. Brussels/Luxembourg, New York, Paris, Washington, D.C., 1993.

² That is, by establishing a ratio between the size of the current period and the size of a preceding period.

A calculation of a GDP value index is shown in *Example 1: A wine and cheese economy*.

The change obtained by this formula may theoretically be divided into a change in prices and a change in volume. If there were an “average” GDP price then it would be quite simple to divide the change in GDP (given by Equation (1)) by this average price to obtain the average change in quantities. Most of the time in the National Accounts, there is no such average price. Thus, the total change in quantities can only be calculated by adding the changes in quantities in the economy. However, creating such a summation is problematic in that it is not possible to add quantities with physically different units, such as cars and telephones, even two different models of cars. This means that the quantities have to be re-evaluated using a common unit. In a currency-based economy, the simplest solution is to express quantities in monetary terms: once evaluated, that is, multiplied by their prices, quantities can be easily aggregated.

Example 1: A wine and cheese economy...

In an insular economy where there would be only wine and cheese, we find the following portrait of production for the last four quarters :

		Q1	Q2	Q3	Q4
Cheese (kilos)	q	100	105	108	112
	p	15	16	18	20
	v	1,500	1,680	1,944	2,240
Wine (liters)	q	25	30	38	50
	p	22	20	16	12
	v	550	600	608	600
Total GDP	v	2,050	2,280	2,552	2,840

In this economy, the quantity of cheese produced rises regularly, as well as its price. However, the quantity of wine produced raises very quickly as its price drops significantly.

What would the growth of the nominal GDP be between, say, Q1 and Q3? The calculation of the index defined by the equation (1) gives the answer:

$$\Delta GDP_{Q3/Q1} = \frac{\sum p_{Q3}q_{Q3}}{\sum p_{Q1}q_{Q1}} = \frac{(108 \times 18) + (38 \times 16)}{(100 \times 15) + (25 \times 22)} = \frac{2,552}{2,050} = 1.245$$

The growth of the economy, in nominal terms, will therefore be 24.5% between Q1 and Q3 (we get the percentage growth by subtracting 1 from the ratio given by the index and by multiplying the result by 100).

This calculation can be done for all the periods relative to Q1. At the end, we obtain an index that covers the four periods :

		Q1	Q2	Q3	Q4
ΔGDP	i	1.000	1.090	1.245	1.385

What would the growth be between periods Q3 and Q4? Here, the ratio of the indexes in Q3 and Q4 gives the answer :

$$\Delta GDP_{Q4/Q3} = \frac{1.385}{1.245} = 1.112$$

The growth will therefore be 11.2% between periods Q3 and Q4.

An intuitive way to measure changes in quantity over time is to take the prices available for a given period and to multiply the quantities from the subsequent periods by these same prices. It amounts to re-evaluating current quantities at prices fixed in time, which essentially “removes” the price effect. In mathematical terms, this can be expressed by the formula for the fixed-base Laspeyres index:

$$(2) \quad LQ_{t/o} = \frac{\sum p_o q_t}{\sum p_o q_o}$$

where: $LQ_{t/o}$ is the Laspeyres quantity index
 p_o is the price at time o
 q_t is the quantity at time t
 q_o is the quantity at o

The only difference from Equation (1) is found in the numerator, where the quantities at time t are multiplied this time by the prices at time o . An application of this formula is shown in *Example 2: Wine and cheese production without the price effect*.

It is quite clear with such a formula that the results are highly dependent on the structure of prices at time o . Should this structure change with time, for example as a result of a drop in the price of one component compared with the others, then the index from Equation (2) will eventually be biased by the fact that it is dependent on an outdated price structure.

One way to overcome this type of problem is to periodically update the weighting base to bring it in line with the current period. This technique was used in the past by the System of National Accounts (SNA) when the real series was rebased every five or six years to reflect changes in the price structure.

Example 2 : Wine and cheese production without the price effect...

Still using our wine and cheese economy, this time we want to evaluate the increase in GDP between period Q1 and periods Q2, Q3 and Q4, excluding the price effect. In our table of the economy, we will use the prices in Q1 as « fixed prices » :

		Q1	Q2	Q3	Q4
Cheese (kilograms)	q	100	105	108	112
	p	15	16	18	20
		1,500	1,680	1,944	2,240
Wine (liters)	q	25	30	38	50
	p	22	20	16	12
		550	600	608	600
Total GDP		2,050	2,280	2,552	2,840

Applying equation (2), we obtain, for the growth between Q1 and Q3 :

$$LQ_{Q3/Q1} = \frac{\sum p_{Q1} q_{Q3}}{\sum p_{Q1} q_{Q1}} = \frac{(108 \times 15) + (38 \times 22)}{(100 \times 15) + (25 \times 22)} = \frac{2,456}{2,050} = 1.198$$

The growth in real GDP is 19.8% between Q1 and Q3. The same index can be calculated for all of the periods, still using Q1 as a base :

		Q1	Q2	Q3	Q4
$LQ_{t/Q1}$	i	1.000	1.090	1.198	1.356

This time, the growth between periods Q3 and Q4, excluding the price effect, is 13.2% :

$$\Delta LQ_{Q4/Q3} = \frac{1.356}{1.198} = 1.132$$

It is noteworthy that this growth of real GDP between Q3 and Q4 is greater than the growth in nominal GDP, which is 11.2%. According to this fixed-base Laspeyres index, the implicit price of the GDP, i.e. the nominal GDP divided by the real GDP (we also call it the *general level of prices*), has therefore dropped between Q3 and Q4.

It is possible, however, for the price structure to change more quickly than usual. The weighting base then becomes outdated quickly, perhaps making it necessary to increase the frequency of the rebasing. Ultimately, the weighting base can be systematically moved from period to period so that it is defined as being the period preceding the current period:

$$(3) \quad LQ_{t/t-1} = \frac{\sum p_{t-1}q_t}{\sum p_{t-1}q_{t-1}}$$

where we find, in place of p_o from Equation (2), p_{t-1} . For the current period t , this “mobile-base” index gives the growth in volume weighted according to prices $t-1$. To some extent it incorporates the frequency of the rebasing, thereby eliminating the arbitrariness of a rebasing done only on an “as required” basis.

Example 3: Another way to remove the price effect

In the previous example, the growth in GDP between Q3 and Q4 was evaluated using Q1 prices. However, with time a change in the price structure of our isolated economy is becoming evident: the liter of wine, more expensive than the kilogram of cheese in Q1, becomes less expensive by Q3. Thus, this time, we will measure the growth between Q3 and Q4 using Q3 prices as the base.

		Q1	Q2	Q3	Q4
Cheese (kilograms)	q	100	105	108	112
	p	15	16	18	20
		1,500	1,680	1,944	2,240
Wine (liters)	q	25	30	38	50
	p	22	20	16	12
		550	600	608	600
Total GDP		2,050	2,280	2,552	2,840

Applying equation (3) gives us :

$$LQ_{Q4/Q3} = \frac{\sum p_{Q3}q_{Q4}}{\sum p_{Q3}q_{Q3}} = \frac{(112 \times 18) + (50 \times 16)}{(108 \times 18) + (38 \times 16)} = \frac{2,816}{2,552} = 1.103$$

The growth in real GDP is now evaluated at 10.3% between Q3 and Q4, rather than 13.1% with the fixed-based Laspeyres index. What happened? The change in quantities in the previous box were evaluated using a different price structure: the change in production from 38 to 50 liters of wine was evaluated at 22\$/litre, while now this increase in production is evaluated at 16\$/litre. The increase in « quantity » thus carries less weight in the balance when the aggregation is done.

What would be the growth between Q1 and Q3? Since equation (3) measures only the relationship between the current and the previous period, we cannot deduct this directly from this equation. However, we can multiply the successive growth of each period between Q1 and Q3. For example, if the growth is 2.3% between Q1 and Q2 and 4.3% between Q2 and Q3, the growth between Q1 and Q3 will be $1.023 \times 1.043 = 1.067$, so 6.7%. This calculation, the same as that of compound interest, illustrates the principle of chaining. Applied to our economy, equations (3) and (4) give us :

		Q1	Q2	Q3	Q4
Unchained Laspeyres index		1.000	1.090	1.091	1.103
Chained Laspeyres index		1.000	1.090	1.190	1.313

The growth between Q1 and Q3 will therefore be 19.0% :

$$\Delta LQ_{Q3/Q1} = \frac{1.190}{1.000} = 1.190$$

In the short term, this type of index can be adapted to cover several periods. Equation (3) can be chained by successive multiplications, that is, in each period, it can be multiplied by the results obtained from the preceding period. The prices used for weighting in the resulting chain are very recent prices and never become obsolete. Using our example, a chain index would have the following form:

$$(4) \quad LQ_C = \frac{\sum p_o q_1}{\sum p_o q_o} \times \frac{\sum p_1 q_2}{\sum p_1 q_1} \times \dots \times \frac{\sum p_{t-1} q_t}{\sum p_{t-1} q_{t-1}} \times \dots \times \frac{\sum p_{n-1} q_n}{\sum p_{n-1} q_{n-1}}$$

where n is the number of periods over which the chain index extends. *Example 3: Another way to remove the price effect* shows how such a formula can be used.

The *System of National Accounts, 1993* recommends using chain indexes. Statistics Canada has been following this recommendation since the spring of 2001 for quarterly National Accounts and since the fall of 2002 for the Provincial Economic Accounts. Systematic chaining allows for constant renewal of the weighting base, thus avoiding the problem of outdated data associated with a fixed-base index.

3.0 Choice of index

The previous examples refer to a Laspeyres-type index. However, index number theory provides numerous other indices that differ in the way the components are weighted. For example, although the quantities in the Laspeyres index are weighted with the prices of a previous period, in the Paasche index they are weighted with the prices of the current period:

$$(5) \quad PQ_{t/o} = \frac{\sum p_t q_t}{\sum p_t q_o}$$

where $PQ_{t/o}$ is the Paasche quantity index.

This index is in fact the reciprocal of the Laspeyres index. Used in its fixed-base form, it presents the same problem as that described earlier, but the inverse: it does not adequately reflect changes in the structure of the economy for previous periods. However, the Paasche index can be chained in the same way as the Laspeyres index (as in Equation (4)).

It can be shown that, in general, a Laspeyres quantity index will generate a larger increase over time than a Paasche quantity index. This occurs when prices and quantities are negatively correlated, that is, when goods or services that had become relatively more expensive are replaced by goods and services that have become relatively less expensive. This common substitution effect says to economic theory that the Laspeyres and Paasche indexes set upper and lower limits for a theoretically ideal, less biased, index.

This theoretical index can be approached by a Fisher-type index, representing the geometric mean of a Laspeyres and Paasche index:

$$(6) \quad FQ_{t/o} = \sqrt{LQ_{t/o} \times PQ_{t/o}} = \sqrt{\frac{\sum p_o q_t}{\sum p_o q_o} \times \frac{\sum p_t q_t}{\sum p_t q_o}}$$

where $FQ_{t/o}$ is the Fisher quantity index.

This index is not only superior theoretically, but it also includes a number of desirable properties from the standpoint of the National Accounts. For example, it is “reversible over time”, that is, the index showing the change between period o and period t is the reciprocal of the index showing the change between period t and period o . Another interesting feature is the “reversibility of factors” by which the product of the price and quantity indexes is equal to the index of the change in current values:

$$FP_{t/o} \times FQ_{t/o} = \sqrt{\frac{\sum p_t q_o}{\sum p_o q_o} \times \frac{\sum p_t q_t}{\sum p_o q_t}} \times \sqrt{\frac{\sum p_o q_t}{\sum p_o q_o} \times \frac{\sum p_t q_t}{\sum p_t q_o}} = \frac{\sum p_t q_t}{\sum p_o q_o}$$

This brings us back to our index of nominal change in Equation (1) and the decomposition of the “price effect” and “volume effect” discussed at the start of this paper. From there, it is quite easy to find the implicit Fisher price of GDP by dividing GDP in current dollars by real GDP using the Fisher formula. The Laspeyres and Paasche indexes do not have either of these two properties.

The Income and Expenditure Accounts use the chain Fisher index as a measure of real GDP. Following the same sequence that we used with Equation (4), chaining Equation (6) gives us:

$$(7) FQ_C = \sqrt{\frac{\sum P_o q_1}{\sum P_o q_o} \times \frac{\sum P_1 q_1}{\sum P_1 q_o}} \times \dots \times \sqrt{\frac{\sum P_{t-1} q_t}{\sum P_{t-1} q_{t-1}} \times \frac{\sum P_t q_t}{\sum P_t q_{t-1}}} \times \dots \times \sqrt{\frac{\sum P_{n-1} q_n}{\sum P_{n-1} q_{n-1}} \times \frac{\sum P_n q_n}{\sum P_n q_{n-1}}}$$

This is the formula used as the basis of the calculations of real GDP for the National and Provincial Accounts.

4.0 Application to the Income and Expenditure Accounts

In practice, the formulae provided above cannot be used as is, given the absence of data on quantities and price levels. The Accounts have only current value (C) series and price indexes (thus, relative prices). Formulas have to be transformed using the fact that the price multiplied by the quantity ($p_t q_t$) equals the series in current dollars (C_t). We then get formulae expressed in terms of nominal series (C_t) and relative prices (p_t/p_{t-1} or the reverse). This then gives us, for Laspeyres (using Equation (3)):

$$(8) \quad LV_{t/t-1} = \frac{\sum \left(\frac{p_{t-1}}{p_t} \right) C_t}{\sum C_{t-1}}$$

... for Paasche (using Equation (5)):

$$(9) \quad PV_{t/t-1} = \frac{\sum C_t}{\sum \left(\frac{p_t}{p_{t-1}} \right) C_{t-1}}$$

... and lastly, for Fisher (geometric mean of Equations (8) and (9)):

$$(10) \quad FV_{t/t-1} = \sqrt{\frac{\sum \left(\frac{p_{t-1}}{p_t} \right) C_t}{\sum C_{t-1}} \times \frac{\sum C_t}{\sum \left(\frac{p_t}{p_{t-1}} \right) C_{t-1}}}$$

It is this formula, chained, that is used in practice. The detail of the transformations can be examined in Appendix I.

Since the series are no longer expressed in terms of quantities, we will now refer to them as volume index. The concept of volume is broader than that of quantity, because it includes variations in quality and ultimately, changes in the composition of the economy.

Example 4: In the real world: the deflation method

In the real world, national accountants don't have prices and quantities as shown in the previous examples (with few exceptions). Rather, they have series in nominal terms (in current dollars) and their corresponding prices, as shown in the table below :

		Q1	Q2	Q3	Q4
Cheese (kilograms)	v	1,500	1,680	1,944	2,240
	p	15	16	18	20
Wine (liters)	v	550	600	608	600
	p	22	20	16	12
Total GDP		2,050	2,280	2,552	2,840

In this example, real GDP will be calculated by the aggregation of the nominal series “deflated” by their prices series. That's what we call the « deflation method ». The aggregation can be done by one or the other indexes presented in the text and expressed in terms of current dollars and prices. For example, the calculation of the Laspeyres index from equation (8) between periods Q2 and Q3 will give us :

$$LV_{Q3/Q2} = \frac{\sum \left(\frac{p_{Q2}}{p_{Q3}} \right) C_{Q3}}{\sum C_{Q2}} = \frac{((16/18) \times 1,944) + ((20/16) \times 608)}{(1,680) + (600)} = \frac{2,488}{2,280} = 1.091$$

The calculation of the Fisher index for the same period will give us :

$$FV_{Q3/Q2} = \sqrt{\frac{\sum \left(\frac{p_{Q2}}{p_{Q3}} \right) C_{Q3}}{\sum C_{Q2}} \times \frac{\sum C_{Q3}}{\sum \left(\frac{p_{Q3}}{p_{Q2}} \right) C_{Q2}}} = \sqrt{\left(\frac{((16/18) \times 1,944) + ((20/16) \times 608)}{(1,680) + (600)} \right) \times \left(\frac{(1,944) + (608)}{((18/16) \times 1,680) + ((16/20) \times 600)} \right)} = 1.084$$

If we make the same calculation for each period, that will give us :

	Q1	Q2	Q3	Q4
Unchained Laspeyres index	1.000	1.090	1.091	1.103
Chained Laspeyres index	1.000	1.090	1.190	1.313
Unchained Fisher index	1.000	1.088	1.084	1.095
Chained Fisher index	1.000	1.088	1.179	1.291

We can see that the chained Fisher index gives us a growth of 17.9% between Q1 and Q3, compared to 19.0% for a chained Laspeyres.

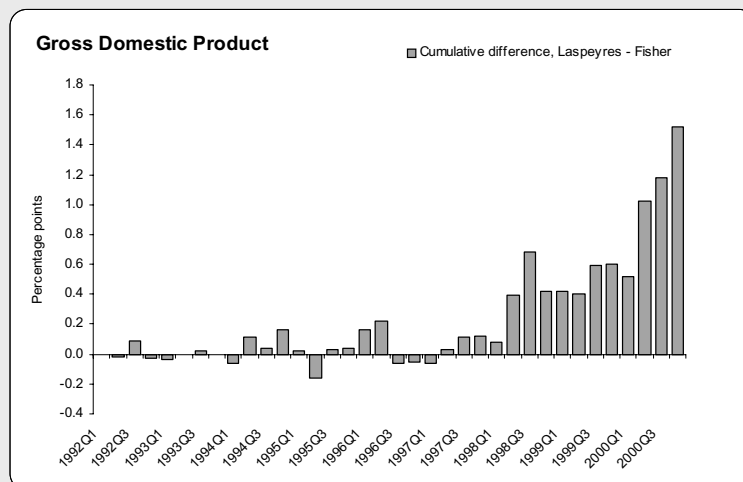
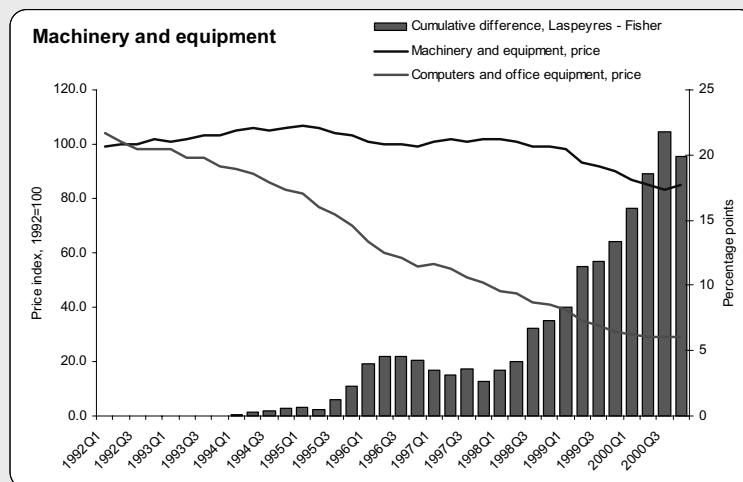
In the 90s, high technology disturbed the price structure ...

A change phenomenon in the price structure of the economy was observed in the 1990s with the sharp decline in the price of computer equipment in a context of general price increases in other goods. In reality, this “drop” in prices consisted of increases in “quality”: for example, at the end of the decade, it was possible to acquire a much more powerful computer than ten years earlier, while paying the same price on the market. In other words, a computer of equal quality or capacity cost much less at the end of the decade. While the price of computer hardware fell steadily, there was a substantial increase in investments and in expenditures in this type of equipment.

...and the deflation method was questioned

In such a context, the limitations of the fixed-base Laspeyres-type methodology used until the spring of 2001 quickly became evident. When calculating real GDP in the late nineties, it was quite clear that the impact of the high technologies on the economy was over-valued since the variations revealed by these series were weighted on their much higher 1992 prices (the base year at the time). Given the growing importance of this sector of the economy and the unusual volatility of its prices, the idea of a mobile-base index quickly came to the fore. In the late nineties, several countries around the world had already turned to chain indexes. In Canada, the choice was made to use the chain Fisher index in keeping with the recommendation of the System of National Accounts 1993 (SNA).

The new way of calculating the real series has had a significant impact on quarterly growth rates. For the *machinery and equipment* aggregate for example, the cumulated differences in growth rates calculated by a fixed-based Laspeyres and a chained Fisher reach as much as 20 percentage points between 1992 and 2000 (see graph below). This cumulative difference reach 1.6 percentage points for the GDP over the same period (see second graph).



4.1 Annual and quarterly National Economic and Financial Accounts

Real aggregates published by the Income and Expenditure Division (IED) are calculated using Equation (10) shown above. For each real aggregate, an index is calculated from component series, then chained quarter by quarter as shown in examples 3 and 4. The chained index series thus obtained is then benchmarked to a reference year in order to express it in dollars. Benchmarking consists of putting the level of the chained index series to a level such that, for a given reference year, it is equal to the corresponding aggregate in current dollars, while keeping the quarterly growth rates intact.

Table 1: Level of detail at the national level

Personal expenditure on consumer goods and services	130
Durable goods	22
Semi-durable goods	15
Non-durable goods	14
Services	79
Government current expenditure on goods and services	24
Government gross fixed capital formation	14
Government investment in inventory	1
Business gross fixed capital formation	18
Residential structures	4
Non-residential structures and equipment	14
Non-residential structures	4
Machinery and equipment	10
Business investment in inventory	110
Non-farm	76
Farm	34
Exports of goods and services	69
Goods	64
Services	5
Imports of goods and services	68
Goods	63
Services	5
Statistical discrepancy	1
Gross domestic product at market prices	435
Final domestic demand	186

What is the difference between the base period and the reference period?

The prices used to compile the volume indexes are prices from the **base period**, while the period in which the value of a series in constant dollars is equal to the value of said series in current dollars is the **reference period**. In the former real GDP measure, using the fixed-base Laspeyres method, the reference period and the base period were the same. In a chain volume measure, however, the two periods are not necessarily the same. For example, the chain Fisher series in our publication are referenced in 1997 (current dollars equal constant dollars for 1997) but the base corresponds to a combination of the current period and the period immediately before the current period, because it is a chain Fisher index. The reference period serves only to benchmark the series and a change in the reference period does not change any aspect of the growth rates of the series or the aggregates. The only change occurs with the levels, which are benchmarked on a different value.

For this reason, the chain Fisher series currently published cannot be said to be “at 1997 prices”, because the prices of the reference period are not used in any way in the calculation of the quarters preceding or following the reference year. However, it can be said that these are series expressed in real terms, thereby easing the price effects, at a level at which they are equal to the nominal aggregate level for 1997. In other words, a real series in which the reference year is 1997 is the equivalent of a nominal series in which the price effect has been removed since 1997.

The level of detail - that is, the number of components used in each of the aggregates - is determined by the availability of data and by certain determinants of overall quality (such as the stability of seasonality). At the national level, 435 series in current dollars and the same number of corresponding price series are used to calculate real GDP using the chain Fisher index. The following table shows how these series are distributed between the various aggregates presented in Table 3 of the publication *National Income and Expenditure Accounts, Quarterly Estimates* (13-001).

None of the Fisher index calculations are done on an annual basis. The real annual aggregates are simple averages of the year's four quarters. These are the official measures of real annual national GDP.

4.2 The problem of inventory

For most of the items in publication Table 3 and other tables in real terms, the Fisher calculation does not present any real technical problems. This is not the case for the investment in inventory series, which are first-difference series. Since these series fluctuate around zero, the Laspeyres and Paasche indexes take opposite signs; since Fisher is the geometric mean of these two indexes, it becomes indeterminate.

As published by the IEAD, real investment in inventories is not the result of a direct chained Fisher calculations as shown above, but rather an approximation. The approach used by the IEAD is based on

the fact that an investment in inventory represents the variation of a total stock, which is always positive. In principle, a Fisher index can be calculated on a total stock series. Once this index is benchmarked to the dollar value of a reference year, one can suppose that the first differences of this series, in dollars, represents an estimate of the real series of the investment in inventories.

If such a method is easily applicable to the calculation of the real inventory series, it is however unusable in the context of the calculation of real GDP. Indeed, the calculation of real GDP should be done with series of investment in inventories, and not with series of total stock. To bypass this problem, the IEAD uses two series of total stock rather than one for each series of investment in inventories: a first series of the stock in the current period (with a positive sign); and a second series, of the stock in the previous period (with a negative sign). This last series is in fact a series of total stock with a one-period lag. At any time t , the difference between these two series corresponds to the investment in inventories during the same period. If in the GDP we replace every series of investment in inventories by these two series of stocks, one positive and the other negative and lagged, we can calculate the real GDP with the chained Fisher index formula. The prices used for total stocks are those of investment in inventories.

The calculation of the real aggregates of investment in inventories involves the same series of total stock. For each aggregate of investment in inventories, a chain Fisher index is obtained from the series of total stock in the current period and another one from the series of the lagged total stock. Once benchmarked to the reference period, these chained Fisher series can be subtracted from each other to simulate a real series of investment in inventories. This is the way that the real aggregates of the investment in inventories are calculated by the Income and Expenditure Accounts Division.

The methodology for calculating the investment in inventory explains the fact that, in Table 1, there are 110 inventory series used in the Fisher calculation, of which 76 are non-agricultural and 34 agricultural (when, in fact, there are 55 inventory series published in current dollars, of which 38 are non-agricultural and 17 agricultural).

4.3 Provincial Accounts

At the provincial level, real values are calculated the same way as they are at the national level, but on an annual basis. Investment in inventory is calculated according to the methodology described above, on an annual basis, with average prices for the year.

The level of detail of the provincial accounts differs from that of the quarterly national accounts. For each province, 502 series are used in calculating real GDP. Table 2 shows the distribution of these series through the items in Table 3 of the publication *Provincial Economic Accounts* (13-213). This distribution is slightly different than the national structure because of the different availability and quality of provincial data.

Table 2: Level of detail at the provincial level

Personal expenditure on consumer goods and services	130
Durable goods	22
Semi-durable goods	15
Non-durable goods	14
Services	79
Government current expenditure on goods and services	24
Government gross fixed capital formation	3
Government investment in inventory	1
Business gross fixed capital formation	5
Residential structures	3
Non-residential structures and equipment	2
Non-residential structures	1
Machinery and equipment	1
Business investment in inventory	110
Non-farm	76
Farm	34
Exports of goods and services	114
Exports to other countries	57
Exports to other provinces	57
Imports of goods and services	114
Imports from other countries	57
Imports from other provinces	57
Statistical discrepancy	1
Gross domestic product at market prices	502
Final domestic demand	162

Sources of bias between the national and provincial system

The national and provincial systems are additively consistent when expressed in nominal value. However, in real terms, the properties of the chain Fisher index are such that this consistency can no longer be guaranteed.

First, real series based on a chain Fisher calculation are not additive. This means that for each province and for Canada as a whole, the sum of the aggregates will not equal the main aggregate (for example, the sum of the aggregates of Table 3 in our publication does not equal GDP). Another consequence of this non-additivity problem is that for each aggregate, the sum of the provinces will not equal the national level (for example, the sum of expenditure on consumer goods and services in all of the provinces will not equal the national expenditure on consumer goods and services).

Secondly, real series are calculated differently at the national and provincial levels. While the national annual series represent an average of the quarters, the provincial annual series represent a chain Fisher index calculated on the year. These two different methodologies produce different results.

Thirdly, there are theoretically two ways to calculate real series at the national level. Fisher indexes can be calculated with the national series (these being the sum of the provincial series), or calculated directly with the provincial series. Since Fisher is an index sensitive to the number of series involved in the calculation, the two calculations do not produce exactly the same result.

Lastly, the level of detail is different between the national and provincial calculations (435 and 502 series, respectively). Since Fisher is an index sensitive to the number of series involved in the calculation, the difference in level of detail produces an inevitable bias between the national and provincial calculation. For these reasons, it is unlikely that the provincial real GDP series will be additively consistent with the national series.

5.0 The problem of non-additivity

The chain Fisher series published by the Income and Expenditure Accounts Division are not additive, and this problem increases with distance from the reference period. Non-additivity of real series comes both from chaining and from the Fisher formula itself. Chaining destroys the additive consistency of accounting equations and the Fisher formula (as opposed to the Laspeyres formula) doesn't have the additivity property.

The fact that the real series are not additive makes them more difficult to manipulate than in the past, when the calculations were based on a fixed-base Laspeyres index. For example, it becomes difficult to measure the contribution of an individual aggregate or sector to a bigger whole knowing that the sum of the aggregates does not add up to the total. It is also imprudent to create aggregates from other aggregates.

There are a variety of ways to overcome this additivity problem. For some summary analysis, current dollar data may be enough and even desirable, because they reflect the economic structure at current prices. This is especially true if the aggregates being studied do not exhibit large price variations or if these variations are relatively uniform.

For those who want to use real data and create aggregations, one solution is to calculate Fisher indexes using existing Fisher data. Diewert (1978) demonstrated that a Fisher index was approximately consistent, and that therefore it was possible to calculate Fisher indexes from aggregates already in Fisher, what he called a "Fisher of Fishers". This solution provides a valid approximation provided that the aggregates used in the calculation are relatively consistent in terms of prices (this solution should not be used, for example, if the calculation involves inventory series).

A more "structural" solution is to play with the benchmarking frequency. Since additivity decreases with distance from the reference year, rebenchmarking the series to bring the reference year closer may alleviate part of the problem without, however, making the whole strictly additive. It is important to note that, in the case of real data based on chain Fisher index calculations, changing the reference period does not have any impact on the growth rates of real series.

Since it is not possible to make the levels additive, the IEAD, following the lead of the Bureau of Economic Analysis in the United States, suggests a strictly additive decomposition of the variations of the aggregates for tables published from real data. The formula used reweights the contributions to the series in such a way that they become strictly additive at the total variation of the aggregate:

$$(11) \quad \Delta\%_{i,t-1/t} = 100 \times \frac{(p_{t-1}^i + p_t^i / FP_t) \times (q_t^i - q_{t-1}^i)}{\sum_i (p_{t-1}^i + p_t^i / FP_t) \times q_{t-1}^i}$$

Or, in a form that applies to nominal series and to prices,

$$(11a) \quad \Delta\%_{i,t-1/t} = 100 \times \frac{\left(\frac{\sum_i C_t}{\sum_i C_{t-1}} \right) \times \left(C_t \left(\frac{p_{t-1}^i}{p_t^i} \right) - C_{t-1} \right) + FV_t \left(C_t - C_{t-1} \left(\frac{p_t^i}{p_{t-1}^i} \right) \right)}{\sum_i C_t + FV_t \sum_i C_{t-1} \left(\frac{p_t^i}{p_{t-1}^i} \right)}$$

This formula is the basis of the contribution to change series published by the SNA. A detailed mathematical demonstration is available in Appendix II.

Contribution to a GDP change series

The contribution of an aggregate to the percentage change in GDP in real terms is presented in Table 4 of the quarterly publication (13-001). Contribution to change tables are also calculated for various grand aggregates (see Tables 18, 21, 24 and 27).

Each of these tables follows the layout of the corresponding real data table. Instead of real data, they show the percentage contribution to the variation of the reference aggregate mentioned in the table's title. For example, Table 4 of our quarterly publication follows the layout of Table 3, and shows the contribution of the aggregates in Table 3 to the percentage growth in real GDP. These contributions are not presented as proportions, but directly as percentage points. For example, a contribution of the aggregate of personal consumption expenditure of 0.453 to real GDP growth of 1.473% means that 0.453 percentage points of the 1.473 are due to personal consumption expenditure.

The formula for percentage contribution to change presented earlier applies only to a single period. To use the same formula over a longer period of time, a Fisher non-chained value is required where the weighting bases correspond to the periods to be analysed. For example, to analyse the growth in durable consumer goods between the fourth quarter of 1996 and the fourth quarter of 2000, it is possible to calculate a Fisher index in which the weighting is explicitly a function of the prices in the fourth quarter of 1996 and of the fourth quarter of 2000. To some degree, it amounts to a fixed-base Fisher index. Once this index has been calculated, the percentage contribution to change formula can be used directly.

Users can do such calculations themselves, if they have all of the series included in the aggregate. Otherwise, they can be prepared by Statistics Canada on request.

Appendix I

Transformation of Laspeyres and Paasche indexes

Laspeyres volume index

The best known volume index is Laspeyres. For this index, weighting is done using prices from a pre-determined base year.

$$(1) \quad LQ_{t/o} = \frac{\sum p_o q_t}{\sum p_o q_o} \quad \text{or, in a version that can be chained:} \quad (2) \quad LQ_{t/t-1} = \frac{\sum p_{t-1} q_t}{\sum p_{t-1} q_{t-1}}$$

where: LQ is the Laspeyres quantity index for period t in relation to period o
 p represents the price series
 q represents the quantity series

By using the identity $C = pq$ (value equals price multiplied by quantity), formula (2) can be expressed in a more usable form:

$$(3) \quad LQ_{t/t-1} = \frac{\sum p_{t-1} \left(\frac{p_t}{p_{t-1}} \right) q_t}{\sum p_{t-1} q_{t-1}} \quad \Rightarrow \quad (4) \quad LQ_{t/t-1} = \frac{\sum \left(\frac{p_{t-1}}{p_t} \right) p_t q_t}{\sum p_{t-1} q_{t-1}}$$

and:

$$(5) \quad LV_{t/t-1} = \frac{\sum \left(\frac{p_{t-1}}{p_t} \right) C_t}{\sum C_{t-1}}$$

Paasche volume index

As well as referring to a previous period, a volume index can be based on prices from the current period. This is known as a Paasche index.

$$(6) \quad PQ_{t/o} = \frac{\sum p_t q_t}{\sum p_t q_o} \quad \text{or, in a version that can be chained:} \quad (7) \quad PQ_{t/t-1} = \frac{\sum p_t q_t}{\sum p_t q_{t-1}}$$

By performing the same substitutions that we did with the Laspeyres index, we get:

$$(8) \quad PQ_{t/t-1} = \frac{\sum p_t q_t}{\sum p_t \left(\frac{p_{t-1}}{p_t} \right) q_{t-1}} \quad \Rightarrow \quad (9) \quad PQ_{t/t-1} = \frac{\sum p_t q_t}{\sum \left(\frac{p_t}{p_{t-1}} \right) p_{t-1} q_{t-1}}$$

and:

$$(10) \quad PV_{t/t-1} = \frac{\sum C_t}{\sum \left(\frac{p_t}{p_{t-1}} \right) C_{t-1}}$$

Appendix II

Contribution to change formula

The axiomatic approach suggested by Yuri Dikhanov (1997) and Christian Ehemann (1997) of the *Bureau of Economic Analysis* is based on the fact that a Fisher volume index is an average of a Laspeyres index, which evaluates the variation in a quantity for a component i at price p_o , and a Paasche index, which evaluates the variation in volume for the same component at price p_t . An additive decomposition of a Fisher index using a weighted average for prices p_o and p_t to evaluate the variation in volume for component i is therefore justified. In general, such a decomposition can take the following form:

$$(1) \quad G(t) = \frac{\sum_i (p_{t-1}^i + \lambda p_t^i) \times q_t^i}{\sum_i (p_{t-1}^i + \lambda p_t^i) \times q_{t-1}^i}$$

where $G(t)$ represents some kind of index. The solution for λ can be deduced as follows:

$$(1.a) \quad G(t) = \frac{\sum_i p_{t-1}^i q_t^i + \lambda \sum_i p_t^i q_t^i}{\sum_i p_{t-1}^i q_{t-1}^i + \lambda \sum_i p_t^i q_{t-1}^i}$$

$$(1.b) \quad \Rightarrow \quad G(t) (\sum_i p_{t-1}^i q_{t-1}^i + \lambda \sum_i p_t^i q_{t-1}^i) = \sum_i p_{t-1}^i q_t^i + \lambda \sum_i p_t^i q_t^i$$

$$(1.c) \quad \Rightarrow \quad G(t) \sum_i p_{t-1}^i q_{t-1}^i + \lambda G(t) \sum_i p_t^i q_{t-1}^i = \sum_i p_{t-1}^i q_t^i + \lambda \sum_i p_t^i q_t^i$$

$$(1.d) \quad \Rightarrow \quad \lambda G(t) \sum_i p_t^i q_{t-1}^i - \lambda \sum_i p_t^i q_t^i = \sum_i p_{t-1}^i q_t^i - G(t) \sum_i p_{t-1}^i q_{t-1}^i$$

thus:

$$(2) \quad \lambda = \frac{\sum_i p_{t-1}^i q_t^i - G(t) \sum_i p_{t-1}^i q_{t-1}^i}{G(t) \sum_i p_t^i q_{t-1}^i - \sum_i p_t^i q_t^i}$$

Where $G(t)$ is given by a Laspeyres index, we would have:

$$(2.a) \quad \lambda = \frac{\sum_i p_{t-1}^i q_t^i - \frac{\sum_i p_{t-1}^i q_t^i}{\sum_i p_{t-1}^i q_{t-1}^i} \sum_i p_{t-1}^i q_{t-1}^i}{\frac{\sum_i p_{t-1}^i q_t^i}{\sum_i p_{t-1}^i q_{t-1}^i} \sum_i p_t^i q_{t-1}^i - \sum_i p_t^i q_t^i}$$

which leads immediately to $\lambda = 0$.

Thus, for a Laspeyres index, the contribution $\% \Delta_{i,t-1/t}$ is given by (by subtracting 1 and multiplying formula (1) by 100):

$$(3) \quad \% \Delta_{i,t-1/t} = 100 \times \frac{p_{t-1}^i (q_t^i - q_{t-1}^i)}{\sum_i p_{t-1}^i q_{t-1}^i}$$

which takes us back to formula (3) of the text. This result is understandable in light of the additive consistency of the Laspeyres index.

Where $G(t)$ is given by a Fisher index, we have:

$$(2.b) \quad \lambda = \frac{\sum_i p_{t-1}^i q_t^i - FV_t \sum_i p_{t-1}^i q_{t-1}^i}{FV_t \sum_i p_t^i q_{t-1}^i - \sum_i p_t^i q_t^i}$$

where FV_t is the Fischer volume index at time t . By dividing the numerator and denominator by $\sum_i p_t q_t$, we obtain:

$$(2.c) \quad \lambda = \frac{\frac{\sum_i p_{t-1}^i q_t^i}{\sum_i p_t^i q_t^i} - FV_t \frac{\sum_i p_{t-1}^i q_{t-1}^i}{\sum_i p_t^i q_t^i}}{FV_t \frac{\sum_i p_t^i q_{t-1}^i}{\sum_i p_t^i q_t^i} - 1}$$

or:

$$(2.c') \quad \lambda = \frac{1 - \frac{FV_t}{PP_t}}{\frac{FV_t}{PV_t} - 1}$$

where PP_t is the Paasche price index at time t , and FP_t is the Fisher price index at time t . If we multiply (2.c') by FP_t at the numerator and denominator, we obtain:

$$(2.d) \quad \lambda = \frac{\frac{FP_t}{PP_t} - 1}{LP_t - FP_t}$$

which, multiplied by PP_t at the numerator and denominator, becomes:

$$(2.e) \quad \lambda = \frac{PP_t FP_t - PP_t}{PP_t LP_t - PP_t FP_t}$$

by reducing and using the equivalence $PP_t LP_t = FP_t^2$, we obtain:

$$(2.f) \quad \lambda = \frac{FP_t - PP_t}{FP_t (FP_t - PP_t)} = \frac{1}{FP_t}$$

Formula (1) then becomes:

$$(4) \quad FV_t = \frac{\sum_i (p_{t-1}^i + p_t^i / FP_t) \times q_t^i}{\sum_i (p_{t-1}^i + p_t^i / FP_t) \times q_{t-1}^i}$$

To obtain the contribution of a single component to the percentage growth of the aggregate, we take FV_{t-1} and multiply by 100:

$$(4.a) \quad \Delta\%_{i,t-1/t} = 100 \times \frac{(p_{t-1}^i + p_t^i / FP_t) \times (q_t^i - q_{t-1}^i)}{\sum_i (p_{t-1}^i + p_t^i / FP_t) \times q_{t-1}^i}$$

This formula is sometimes difficult to operationalize given that it is expressed in terms of price and quantity. For a version expressed in terms of current dollar values and prices (as used by the IEAD), we can multiply the numerator and denominator by $FV_t FP_t$:

$$(4.b) \quad \Delta\%_{i,t-1/t} = 100 \times \frac{FV_t FP_t (p_{t-1}^i q_t^i + p_t^i q_t^i / FP_t - p_{t-1}^i q_{t-1}^i - p_t^i q_{t-1}^i / FP_t)}{FV_t FP_t \sum_i p_{t-1}^i q_{t-1}^i + FV_t FP_t \sum_i p_t^i q_{t-1}^i / FP_t}$$

or:

$$(4.c) \quad \Delta\%_{i,t-1/t} = 100 \times \frac{\left(\frac{\sum_i C_t}{\sum_i C_{t-1}} \right) p_{t-1}^i q_t^i + FV_t C_t^i - \left(\frac{\sum_i C_t}{\sum_i C_{t-1}} \right) C_{t-1}^i - FV_t p_t^i q_{t-1}^i}{\sum_i C_t + FV_t \sum_i p_t^i q_{t-1}^i}$$

by consolidating the sums of the values in current dollars:

$$(5) \quad \Delta\%_{i,t-1/t} = 100 \times \frac{\left(\frac{\sum_i C_t}{\sum_i C_{t-1}} \right) \times \left(C_t^i \left(\frac{p_{t-1}^i}{p_t^i} \right) - C_{t-1}^i \right) + FV_t \left(C_t^i - C_{t-1}^i \left(\frac{p_t^i}{p_{t-1}^i} \right) \right)}{\sum_i C_t + FV_t \sum_i C_{t-1} \left(\frac{p_t^i}{p_{t-1}^i} \right)}$$

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