Survey Methodology, December 2000
Vol. 26, No. 2, pp. 163-172
Statistics Canada, Catalogue No. 12-001

The Impact of Different Rotation Patterns on the Sampling Variance of Seasonally Adjusted and Trend Estimates

C.H. McLaren and D.G. Steel [1]

Abstract

Many economic and social time series are based on sample surveys which have complex sample designs. The sample design affects the properties of the time series. In particular, the overlap of the sample from period to period affects the variability of the time series of survey estimates, and the seasonally adjusted and trend estimates produced from them. The Census X11 and X11ARIMA packages are commonly used to produce seasonally adjusted estimates and can also be used to produce estimates of trend. This paper considers the implications of different overlap patterns on the sampling variance of seasonally adjusted and trend estimates obtained from time series based on sample surveys.

Key Words: X11; X11ARIMA; Seasonal adjustment; Trend estimation; Rotation patterns.

1. Introduction

Many important time series are based on repeated sample surveys which have complex patterns of sample overlap from period to period. The use of sampling means that the estimated time series have a component of variability due to sampling errors and for many series this will be a major source of variability. The sample design, in particular the overlap pattern, affects the variability of the time series of survey estimates.

Increasingly, analysis of time series is concentrating on assessing underlying patterns of change or trends based on analysis of the seasonally adjusted series. Most government statistical agencies have calculated seasonally adjusted series for many years. Kenny and Durbin (1982) noted that policy analysts frequently say that they are more interested in underlying trends than following irregular fluctuations in the de-seasonalized monthly values. A similar view is expressed by Smith (1997). For more than 10 years the Australian Bureau of Statistics (ABS) has published series of trend estimates obtained by applying Henderson Moving Averages (HMAs) (Henderson 1916) to the seasonally adjusted series to smooth out the irregular components of the series (ABS 1987). Other government statistical agencies also produce trend estimates using a variety of methods (Knowles 1997). Since seasonally adjusted and trend estimates are obtained by processes applied to the original series, they are also influenced by sampling errors. Bell and Kramer (1999) note that the variance of seasonally adjusted estimates will often be dominated by the contribution from sampling error. Some series are based on independent samples over time, but usually the samples used have a degree of overlap from period to period to reduce costs and the standard errors of estimates of change between two consecutive time periods (Kish 1998).

A key issue in the development of the design of a repeated survey is the rotation pattern, that is, the pattern of a selected unit's inclusion in the survey over time, which will determine the sample overlap. The aim of this paper is to determine the effects of the rotation pattern used on the sampling variance of the estimated seasonally adjusted and trend series obtained using the Census X11 method developed by Shiskin, Young and Musgrave (1967) and X11ARIMA developed by Dagum (1980 and 1988). We will focus on the estimates of the level and one period change in the seasonally adjusted and trend estimates.

2. Rotation Patterns

Consider a univariate time series with values y_t, $t = 1, ..., T$, obtained from a repeated sample survey. The observed value at time t is related to the true value of the series in the finite population, Y_t, by

$$y_t = Y_t + e_t$$

where e_t is the sampling error. The series Y_t is thought to consist of trend-cycle, seasonal and irregular components T_t, S_t and I_t, so that

$$y_t = T_t + S_t + I_t + e_t.$$

In some cases a multiplicative decomposition may be more appropriate. Many statistical agencies produce seasonally adjusted series by attempting to estimate S_t and remove it from the series, usually using some combination of linear filters. Most commonly used is the Census X11 method developed by Shiskin *et al.* (1967) and X11ARIMA developed by Dagum (1980 and 1988). Findley, Monsell, Otto, Bell and Pugh (1998) described further enhancements embodied in X12ARIMA. The ABS also publishes trend

1. C.H. McLaren and D.G. Steel, School of Mathematics and Applied Statistics, University of Wollongong, NSW 2522, Australia. E-mail: craigmc@uow.edu.au, dsteel@uow.edu.au.

estimates obtained by applying HMAs to the seasonally adjusted series and encourages users to base their interpretation of the series on these trend estimates (Linacre and Zarb 1991; ABS 1993). The HMAs were originally derived by Henderson (1916) for use in actuarial work and are used within X11, X11ARIMA and X12ARIMA to de-trend series for seasonal adjustment purposes. Kenny and Durbin (1982) and Gray and Thomson (1996) explain the derivation of the HMAs. Users can also produce trend estimates by applying filters to the published seasonally adjusted estimates. Kenny and Durbin (1982) noted that there is no unique definition of trend and that different filters may be used according to the degree of smoothness and sensitivity required. Knowles and Kenny (1997) investigated methods of trend estimation for official statistical series. For monthly series they recommended the use of HMAs, with the length of the filter being 13 or 23 depending on the volatility of the series in question.

The autocorrelation structure of the observed series is determined by the autocorrelation of the series Y_t and e_t, which will then affect the estimates of the trend, seasonally and irregular components. The covariance structure of the sampling error series, e_t, can be estimated from the unit level survey data. By obtaining such estimates, it is possible to obtain estimates of the sampling variance of the estimated trend, seasonally adjusted and irregular series. Various methods for doing this have been proposed; for example Steel and DeMel (1988) considered the effect of linear filters on the spectrum of the sampling error series and Wolter and Monsour (1981) used an approach based on the effect of linear filters on the autocovariance function. Sutcliffe (1993) adopted a similar approach using a linear approximation to the X11 procedure. Pfeffermann (1994) proposed a method which develops an estimate of sampling error directly from the estimated time series using various simplifying assumptions. These approaches do not explicitly model the time series. Other authors, for example Bell and Wilcox (1993), Tiller (1992), Burridge and Wallis (1985) and Hausman and Watson (1985), considered explicit ARIMA models for both the true series and the sampling error series, and concentrated on the estimation of the parameters of the models. These papers do not consider the effect of different rotation patterns and concentrate on producing estimates of the variances of seasonally adjusted estimates for the particular rotation pattern used.

The rotation pattern used in the survey will affect the autocorrelation structure of the sampling error series and hence the sampling variance of the original, seasonally adjusted and trend estimates. Several considerations are taken into account in deciding upon a rotation pattern. High sample overlap between consecutive periods reduces the sampling variance of estimates of change between the periods and high sample overlap between periods 12 months apart reduces the sampling variance of estimates of annual change. The first occasion that a selected unit is included in the survey is usually the most expensive. By keeping selected units in the survey for longer the cost of the survey is reduced. This leads to rotation patterns in which a selected unit is included every period for as long as possible. However, a selected unit must eventually be rotated out of the survey. Besides the ethical consideration of spreading respondent load, there is the possible deterioration in response rate and quality of data reported if the same unit is included for a large number of occasions (see Kalton and Citro 1993, for a discussion of these issues).

Rotation patterns vary in terms of the number of times a unit is included in the survey and the time interval between inclusions. We concentrate on monthly labour force surveys (MLFSs). The rotation patterns used in practice are special cases of the a-b-$a(m)$ rotation patterns where selected units are included for a consecutive months, removed from the survey for b months then re-included for a further a months. The pattern is repeated so that selected units are included for a total of m occasions. Rao and Graham (1964) considered the estimation of the finite population means and totals for this class of rotation patterns. The United States Current Population Survey (CPS) uses a 4-8-4(8) pattern (Fuller, Adam and Yansaneh 1992). Putting $b = 0$ gives an *in-for-* m rotation pattern in which selected dwellings are included for m months after which they are removed from the sample. The case $m = 6$ corresponds to the Canadian rotation pattern (Singh, Drew, Gambino and Mayda 1990) and $m = 8$ corresponds to the Australian pattern (ABS 1992). Steel (1997) noted that the British quarterly labour force survey approximately corresponds to a monthly survey with a 1-2-1(5) rotation pattern.

We consider the sampling variance of the seasonally adjusted and trend estimates associated with the rotation patterns currently used in MLFSs and a number of rotation patterns that, while not currently used, may have some desirable properties. This will give an indication of which rotation patterns are better in terms of the component of the variability of the estimated series that is affected by the sample design.

3. Sampling Variance of Seasonally Adjusted and Trend Estimates

Let $\boldsymbol{y}_T$ be the vector containing the values of the time series of survey estimates up to time T and $\boldsymbol{Y}_T$ be the vector containing the true population values. The sampling variance of the original series is denoted by $V(\boldsymbol{y}_T \mid \boldsymbol{Y}_T)$. Consider a linear filter which is used to obtain values from $\boldsymbol{y}_T$ by applying a vector of filter weights $\boldsymbol{w}_t$. The filter weights are non-random and have no connection with the survey weights used in calculating the survey estimates y_t. The filter weight vectors $\boldsymbol{w}_t$ depends on the time period for which the filtered value refers. The weights are constant within the body of the series but may be modified at the beginning and end. The filtered value at time t is

$$\tilde{y}_t = \boldsymbol{w}_t' \boldsymbol{y}_T. \quad (1)$$

Then

$$V(\tilde{y}_t \mid \boldsymbol{Y}_T) = \boldsymbol{w}_t' V(\boldsymbol{y}_T \mid \boldsymbol{Y}_T)\, \boldsymbol{w}_t \qquad (2)$$

is the sampling variance of the filtered value at time t. The sampling error of the filtered value is the difference between $\boldsymbol{w}_t' \boldsymbol{y}_T$ and $\boldsymbol{w}_t' \boldsymbol{Y}_T$, which is conditional on the values of the true series, $\boldsymbol{Y}_T$. This is the difference between the filtered value obtained from the series of estimates ending at time T and the value that would be obtained if that series was observed without sampling error. We focus on this component as it is the sampling variance that can be altered by changing the sample design. The variance associated with $\boldsymbol{Y}_T$ has not been taken into account. Wolter and Monsour (1981) discussed the issue of total variance versus sampling error variance. There may be advantages in considering the total variance in interpreting the resulting series but when we are considering sample design issues, such as the choice of rotation pattern, we focus on the component that is directly affected by decisions made about the sample design. If the sampling error does not contribute significantly to the variability of the series then decisions about the sample design are not as important as they are when the sampling error is a major contributor, although it still seems sensible to use as effective a sample design as possible.

To determine the effect of different rotation patterns on the sampling variance of a particular filtered series, we need an estimate of $V(\boldsymbol{y}_T \mid \boldsymbol{Y}_T)$ for different rotation patterns. Previous work on estimating variances of seasonally adjusted series has either ignored the rotation pattern and assumed independent samples at each time point, or taken it as fixed and used an estimation method that takes it into account. We need a model for $V(\boldsymbol{y}_T \mid \boldsymbol{Y}_T)$ that reflects the effect of the different rotation patterns that could be used.

The analysis of the effect of different rotation patterns is simplified if the series of sampling errors has a stable autocorrelation structure. The precise form of the autocorrelation function will depend on the series and should reflect the complexities of the design. For example Steel and DeMel (1988) suggested a model for the Australian Monthly Labour Force data and Bell and Wilcox (1993) suggested a model for the United States Retail Trade series. Bell and Hillmer (1990) and Miazaki and Dorea (1993) also considered modelling of survey errors by time series models. Dempster and Hwang (1993) and Lee (1990) considered approaches to estimating and modelling sampling error correlations for the US CPS.

Our approach is to assume that the series of sampling errors, e_t has constant variance. A model is needed for the correlation between the sampling errors of y_t and y_{t+s}. All the rotation patterns considered imply that the sample at any particular time will consist of a number of panels. A panel is a set of units that are included and removed from the survey at the same time. When a panel is rotated out of the survey it will be replaced by another panel. The set of panels related in this way is referred to as a rotation group. Most MLFSs use multistage sampling and when a panel is rotated out of the survey it is replaced by another panel of nearby households (see ABS 1992; Singh *et al.* 1990). Hence it is assumed that the sampling correlation between estimates obtained from the same rotation group s periods apart is $r(s)$ if no rotation has occurred and $d(s)$ if rotation has occurred. We will assume that the estimate at time t is, at least approximately, the average of estimates from each rotation group and that estimates from different rotation groups, which will usually be in different PSUs and spatially well separated, are independent.

These assumptions imply that the sampling correlation between y_t and y_{t+s} is

$$R(s) = d(s) + k(s)(r(s) - d(s))$$

where $k(s)$ is proportion of the sample in common between the two time periods. The sample overlap factor $k(s)$ is determined by the rotation pattern. For example, for an *in-for-m* rotation pattern $k(s) = 1 - s/m$, $s = 0, \ldots, m-1$ and zero otherwise, assuming that the same number of dwellings are added and dropped from the sample each month. If different panels in the same rotation group are independent, then $d(s) = 0$, but in general this will not be the case. This model is essentially the same as derived by Scott, Smith and Jones (1977). An example of an *in-for*-4 rotation pattern over an eight month period is illustrated in Table 1. Different panels are denoted by different letters and the subscript indicates the number of times the panel has been included in the survey up to the time period indicated.

Table 1
Structure of *in-for*-4 Rotation Pattern

Rotation	Time Period							
Group	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$
1	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4
2	c_4	d_1	d_2	d_3	d_4	e_1	e_2	e_3
3	f_3	f_4	g_1	g_2	g_3	g_4	h_1	h_2
4	i_2	i_3	i_4	j_1	j_2	j_3	j_4	k_1

In this case $r(2)$ is the correlation arising from say a_2 and a_4, whereas $d(2)$ is the correlation associated with a_3 and b_1. Binder and Hidiroglou (1988) and Fuller *et al.* (1992) provided discussions of the data structure implied by some other rotation patterns.

The assumption that the variance of the sampling error series is constant implies that no major changes to the sample design or the population structure occur, at least over the effective length of the filters being considered. The assumption of stable autocorrelations, $r(s)$ and $d(s)$, for the population correlation also implies no major changes to the sample design or population. Estimates for $r(s)$ and $d(s)$ in (3) were obtained from a study by Bell (1998). The values used are from the Australian Labour Force Survey (ALFS) for the proportion of persons employed and also the proportion of persons unemployed and are shown in

Table 2. These were obtained by treating the rotation groups in the ALFS as replicates and measuring the autocorrelation at the rotation group level. A model given in Bell (1998) was used to extrapolate values beyond the given lags.

Table 2
Autocorelations - ALFS

Proportion of employed persons								
lag	1	2	3	4	5	6	7	8
$r(s)$	0.80	0.71	0.64	0.57	0.50	0.45	0.40	0.36
$d(s)$	0.15	0.15	0.14	0.13	0.12	0.11	0.11	0.10
Proportion of unemployed persons								
	1	2	3	4	5	6	7	8
$r(s)$	0.62	0.52	0.44	0.37	0.31	0.26	0.22	0.19
$d(s)$	0.11	0.11	0.10	0.09	0.09	0.08	0.08	0.07

Sutcliffe and Lee (1995) studied the standard errors of seasonally adjusted and trend estimates of level and movement under a small number of different rotation patterns. They assumed a simple geometric decay model for the correlations between survey estimates with a population correlation of $\rho = 0.8$, *i.e.*, $R(s) = \rho^s$, which decreases more rapidly than the values given in Table 2.

4. Linear Approximations for Seasonally Adjusted and Trend Estimates

The X11 method consists of an iterative application of moving averages resulting in a symmetric filter for the central values, and asymmetric filters for the values at the beginning and end of the series. The final seasonally adjusted and trend estimates produced by X11 can be approximated by linear filters. Several authors; for example, Young (1968), Cleveland and Tiao (1976), Wallis (1982), and Sutcliffe (1993), have produced linear approximations to the X11 procedure. The X11ARIMA procedure (Dagum 1980, 1988) is an extension of X11 and extrapolates the original series at both ends by an ARIMA model. The effect of the ARIMA extrapolation can be incorporated into the filter weights and these weights can be applied to the data alone. Dagum, Chhab and Chiu (1996) considered a Cascade method approach, where the Cascade filters are a result of the convolution of the various predetermined linear filters used within both X11 and X11ARIMA. We used this approach to realistically approximate both the X11 and X11ARIMA procedures.

Define the matrix whose rows contain the filter weights of 13 term HMAs for both symmetric and asymmetric filters as $\mathbf{H}_{13}$. The matrix of weights corresponding to a 3×3 moving average (ma) is denoted as $\mathbf{S}_{3\times3}$ and that corresponding to a 3×5 ma is denoted as $\mathbf{S}_{3\times5}$. These are used for estimation of seasonal factors. The matrix $\mathbf{D}$ is defined as a 12 term centered ma and $\mathbf{I}$ is an identity matrix. The notation c indicates the complement of a filter, for example $\mathbf{D}^c = \mathbf{I} - \mathbf{D}$. The Seasonal Adjustment Cascade filters are written as

$$\mathbf{S} = \mathbf{I} - \mathbf{D}^c\,\mathbf{S}_{3\times5}[\mathbf{H}_{13}(\mathbf{D}^c\,\mathbf{S}_{3\times3}\,\mathbf{D}^c)^c]^c.$$

The trend Cascade filters used for the estimation of trend are then found by multiplying the seasonally adjusted filter by a trend filter. At the end of the series the Cascade filters for trend and seasonally adjusted estimates will differ according to whether X11 or X11ARIMA is used.

We consider the following different combinations of the internal filters of X11 and X11ARIMA:

1. Standard X11 Cascade filter: This corresponds to a 13 term HMA for estimation of trend (H13), 3×3 ma for the first estimation of the seasonal factors ($S1_{3\times3}$), 3×5 ma for estimation of seasonal factors ($S2_{3\times5}$), and no modification for outliers.
2. Standard X11 Cascade filter with ARIMA forecasts: This corresponds to use of a H13, $S1_{3\times3}$, $S2_{3\times5}$, and extended forecasts from an ARIMA model of the form $(1-B)(1-B^{12})y_t = (1-0.4B)(1-0.6B^{12})a_t$, where B is the backward shift operator and a_t is a white noise process, and no modification for outliers.
3. Short X11 Cascade filter with ARIMA forecasts: This corresponds to use of a H9, $S1_{3\times3}$, $S2_{3\times5}$, and extended forecasts from a model of the form $(1-B)(1-B^{12})y_t = (1-0.3B)(1-0.3B^{12})a_t$, and no modification for outliers.
4. Long X11 Cascade filter with ARIMA forecasts: This corresponds to use of a H23, $S1_{3\times3}$, $S2_{3\times5}$, and extended forecasts from a model of the form $(1-B)(1-B^{12})y_t = (1-0.8B)(1-0.8B^{12})a_t$, and no modification for outliers.

Combinations 2 and 3 have been observed by Dagum (1983) to be applicable in a number of cases. The linear approximations chosen allow us to examine the effect of different rotation patterns for a range of filters used in practice, which involve HMAs of different lengths.

For each combination of filters the corresponding Cascade filter provides a vector of filter weights for the seasonally adjusted estimates and a different weight vector for the final trend estimates. These can then be substituted into equation (2) to obtain the sampling variances for a particular rotation pattern by using the appropriate values for $V(\boldsymbol{y}_T \mid \boldsymbol{Y}_T)$. When computing change estimates the data vector $\boldsymbol{y}_T$ remains unchanged and the weights that are applied change. For example, $\boldsymbol{w}_{t+1} - \boldsymbol{w}_t$ can be used for a one month difference. This basic approach is the same as that adopted by Wolter and Monsour (1981) who proposed estimating the variance of seasonally adjusted estimates using (2) with weights chosen that reasonably approximate the seasonal adjustment process and using a survey based estimate of $V(\boldsymbol{y}_T \mid \boldsymbol{Y}_T)$. We also consider trend filters and different realisations of X11ARIMA and rotation patterns.

The X11ARIMA models considered in this paper are representative of those commonly used in practice.

Additional complications arise from the use of ARIMA forecasts in the X11ARIMA approach. For example, we assume no misspecification of the ARIMA model. The ARIMA model is typically identified and estimated using previous survey data. The sampling error for previous time points could influence the choice of ARIMA model and X11 filters. This could be taken into consideration by modification of the variance in (2).

The initial trend and seasonally adjusted estimates for time t will be made using the time series of estimates ending at time t, that is $\boldsymbol{y}_t$, giving the filtered value $\boldsymbol{w}_t' \boldsymbol{y}_t$. The value that would be obtained if there was no sampling error is $\boldsymbol{w}_t' \boldsymbol{Y}_t$. The sampling error considered in this paper is $\boldsymbol{w}_t' \boldsymbol{y}_t - \boldsymbol{w}_t' \boldsymbol{Y}_t$. As estimates are added to the series the filtered value for time t may change, but there will come a time point, $t+s$, after which there is no appreciable change. The final filtered value for time t based on the survey estimates can be written as $\boldsymbol{w}_t^* \boldsymbol{y}_{t+s}$, for a final symmetric weight vector $\boldsymbol{w}_t^*$. Similarly the final value that would be obtained if there were no sampling error would be $\boldsymbol{w}_t^* \boldsymbol{Y}_{t+s}$. Bell and Kramer (1999) considered the difference $\boldsymbol{w}_t' \boldsymbol{y}_t - \boldsymbol{w}_t^{*\prime} \boldsymbol{Y}_{t+s}$, which includes the forecast error. This difference can be decomposed as

$$\boldsymbol{w}_t' \boldsymbol{y}_t - \boldsymbol{w}_t^{*\prime} \boldsymbol{Y}_{t+s} = (\boldsymbol{w}_t' \boldsymbol{y}_t - \boldsymbol{w}_t' \boldsymbol{Y}_t) + (\boldsymbol{w}_t' \boldsymbol{Y}_t - \boldsymbol{w}_t^{*\prime} \boldsymbol{Y}_{t+s})$$

We have considered how different rotation patterns affect the first term in this decomposition. The second term involves the series observed without sampling error and is unaffected by the sample design, including the rotation pattern. Bell and Kramer (1999) considered the series of US Housing Starts involving five or more units and showed that the total variance of the trend series showed large increases at the end of the series due to forecasting errors. This is due to the revisions in the initial trend estimates that are made as estimates are added to the series. Steel and McLaren (2000) considered the effect of different rotation patterns on the observed revision of the initial trend estimates, which is $\boldsymbol{w}_t' \boldsymbol{y}_t - \boldsymbol{w}_t^{*\prime} \boldsymbol{y}_{t+s}$. They noted that the relative importance of the component due to sampling error will depend on how the true series is evolving around the period being considered.

5. Results

We use filters corresponding to the level and one month difference for both the seasonally adjusted and trend estimates at the very end of the series. Tables 3 to 6 summarise the effect of different rotation patterns for each Cascade filter combination. These tables give, for a selection of rotation patterns, the ratio of the sampling variance of the estimates under consideration divided by the sampling variance that would be obtained when there is complete rotation each month. The ratios obtained in the middle of the series give the same general conclusions (McLaren 1999).

Table 3
Ratio of the Sampling Variance for Chosen Rotation Patterns Divided by the Sampling Variance for an Independent Design (Combination 1)

Rotation Pattern	$\hat{SA}_t$		$\hat{SA}_{t+1} - \hat{SA}_t$		$\hat{T}_t$		$\hat{T}_{t+1} - \hat{T}_t$	
	emp	unemp	emp	unemp	emp	unemp	emp	unemp
complete	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1-2-1(5)	0.99	0.99	0.99	1.00	0.99	1.00	0.68	0.79
1-2-1(8)	0.98	0.99	0.97	0.99	0.98	0.99	0.64	0.77
1-1-1(6)	1.01	1.01	1.00	1.00	1.17	1.14	0.70	0.82
2-2-2(8)	1.02	1.02	0.61	0.71	1.26	1.23	0.83	0.95
2-10-2(4)	1.04	1.04	0.61	0.71	1.35	1.30	1.32	1.26
3-3-3(6)	1.07	1.06	0.48	0.61	1.52	1.44	1.29	1.25
4-8-4(8)	1.10	1.08	0.42	0.57	1.69	1.57	1.42	1.34
6-6-6(12)	1.10	1.08	0.36	0.52	1.76	1.64	1.22	1.22
in-for-6	1.10	1.08	0.36	0.52	1.76	1.64	1.22	1.22
in-for-8	1.09	1.08	0.33	0.50	1.78	1.65	1.06	1.13
no rotation	1.08	1.08	0.24	0.44	1.80	1.69	0.75	0.95

Table 4
Ratio of the Sampling Variance for Chosen Rotation Patterns Divided by the Sampling Variance for an Independent Design (Combination 2)

Rotation Pattern	$\hat{SA}_t$		$\hat{SA}_{t+1} - \hat{SA}_t$		$\hat{T}_t$		$\hat{T}_{t+1} - \hat{T}_t$	
	emp	unemp	emp	unemp	emp	unemp	emp	unemp
complete	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1-2-1(5)	1.01	1.01	0.99	1.00	1.06	1.05	0.69	0.80
1-2-1(8)	1.00	1.00	0.96	0.99	1.07	1.05	0.66	0.78
1-1-1(6)	1.04	1.03	1.00	1.00	1.22	1.17	0.65	0.77
2-2-2(8)	1.05	1.04	0.60	0.71	1.32	1.26	0.81	0.92
2-10-2(4)	1.02	1.03	0.60	0.71	1.26	1.23	1.19	1.17
3-3-3(6)	1.08	1.06	0.49	0.61	1.49	1.40	1.19	1.16
4-8-4(8)	1.06	1.06	0.41	0.56	1.56	1.47	1.13	1.13
6-6-6(12)	1.08	1.07	0.35	0.52	1.67	1.56	0.93	1.01
in-for-6	1.10	1.08	0.36	0.52	1.69	1.56	0.94	1.01
in-for-8	1.11	1.08	0.32	0.49	1.75	1.61	0.82	0.93
no rotation	1.14	1.11	0.24	0.43	1.89	1.73	0.59	0.78

Table 5
Ratio of the Sampling Variance for Chosen Rotation Patterns Divided by the Sampling Variance for an Independent Design (Combination 3)

Rotation Pattern	$\hat{SA}_t$		$\hat{SA}_{t+1} - \hat{SA}_t$		$\hat{T}_t$		$\hat{T}_{t+1} - \hat{T}_t$	
	emp	unemp	emp	unemp	emp	unemp	emp	unemp
complete	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1-2-1(5)	0.99	0.99	0.96	0.98	0.99	0.99	0.68	0.79
1-2-1(8)	0.97	0.99	0.93	0.97	0.98	0.99	0.64	0.77
1-1-1(6)	1.04	1.02	0.99	0.99	1.11	1.08	0.60	0.72
2-2-2(8)	1.07	1.06	0.60	0.71	1.23	1.19	0.89	0.95
2-10-2(4)	1.05	1.06	0.61	0.72	1.21	1.20	1.07	1.08
3-3-3(6)	1.15	1.12	0.51	0.63	1.41	1.32	1.02	1.02
4-8-4(8)	1.12	1.11	0.44	0.58	1.41	1.35	0.85	0.93
6-6-6(12)	1.14	1.13	0.37	0.53	1.47	1.39	0.69	0.82
in-for-6	1.16	1.13	0.38	0.53	1.49	1.40	0.70	0.81
in-for-8	1.17	1.14	0.34	0.51	1.52	1.42	0.61	0.76
no rotation	1.22	1.17	0.25	0.44	1.62	1.50	0.44	0.64

Table 6
Ratio of the Sampling Variance for Chosen Rotation Patterns Divided by the Sampling Variance for an Independent Design (Combination 4)

Rotation Pattern	$\hat{SA}_t$		$\hat{SA}_{t+1} - \hat{SA}_t$		$\hat{T}_t$		$\hat{T}_{t+1} - \hat{T}_t$	
	emp	unemp	emp	unemp	emp	unemp	emp	unemp
complete	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1-2-1(5)	1.02	1.02	0.99	1.00	1.25	1.19	0.75	0.87
1-2-1(8)	1.02	1.02	0.97	0.99	1.28	1.21	0.70	0.84
1-1-1(6)	1.06	1.04	1.00	1.00	1.49	1.39	0.92	1.01
2-2-2(8)	1.06	1.04	0.60	0.71	1.57	1.47	0.98	1.09
2-10-2(4)	1.00	1.01	0.60	0.70	1.30	1.27	1.49	1.37
3-3-3(6)	1.07	1.05	0.48	0.61	1.64	1.54	1.34	1.36
4-8-4(8)	1.05	1.04	0.41	0.56	1.73	1.63	1.92	1.69
6-6-6(12)	1.08	1.06	0.35	0.51	2.00	1.84	1.87	1.68
in-for-6	1.09	1.07	0.35	0.52	2.00	1.84	1.90	1.70
in-for-8	1.11	1.08	0.32	0.49	2.15	1.96	1.73	1.62
no rotation	1.17	1.12	0.24	0.43	2.56	2.27	1.11	1.33

5.1 X11 – Concurrent Standard Cascade Filters

The results using the standard X11 filters (combination 1) are shown in Table 3. Figures 1(a) to 1(d) show the sampling variance of the level and one month difference for the seasonally adjusted and trend estimates at the end of the series divided by the variance of the original estimate of level plotted against the total number of times a selected unit is included. Results for the variable employment have been plotted for selected a-b-$a(m)$ patterns and the *in-for-m* rotation patterns for m going from 1 to 30. An *in-for*-30 rotation pattern is indicative of having no rotation.

Columns 1 and 2 in Table 3 show that for the variance of the seasonally adjusted level estimates, rotation patterns with no monthly overlap perform well. Using rotation patterns with annual overlap did not help appreciably. However, for the one month change in seasonally adjusted estimates, the benefit of having high monthly overlap becomes evident (see Figure 1(b) and columns 3 and 4 of Table 3). The variances associated with the *in-for-m* rotation patterns are effectively a function of $1/m$, the proportion of the sample that does not overlap. Those rotation patterns used in Canada and Australia perform well. The best option is no rotation but, as discussed in section 2, this is not a practical option. Figures 1(a) and 1(b) show that rotation patterns that have the same degree of monthly sample overlap have similar variances for estimates of the level and one month change in the seasonally adjusted series.

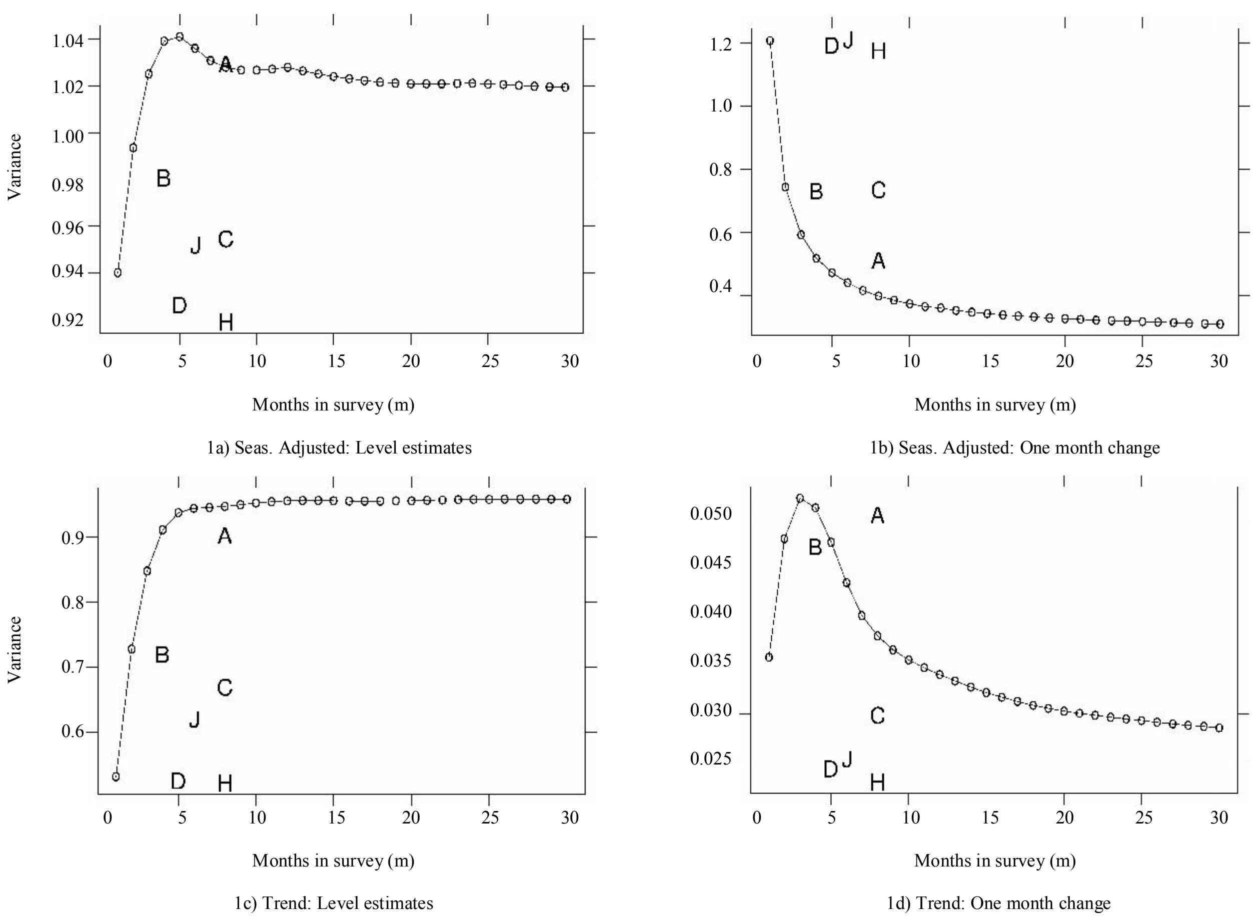


Figure 1. Ratio of the sampling variance to the variance of the original series for chosen rotation patterns for combination 1 (X11) for the variable employment where A = 4-8-4(8), B = 2-10-2(4), C = 2-2-2(8), D = 1-2-1(5), H = 1-2-1(8), J = 1-1-1(6).

For the level of trend estimates the variance increases as the amount of monthly sample overlap increases (see Figure 1(c) and columns 5 and 6 of Table 3). For the *in-for-m* rotation patterns there is a rapid increase in variance as m goes from 1 to 5. The rotation patterns of 1-2-1(5) and 1-2-1(8) perform as well as having an independent sample each month and considerably better than rotation patterns that involve monthly overlap. This is primarily due to the fact that for a moving average, it is better to average over independent observations than positively correlated ones. The larger variance of the 1-1-1(6) pattern compared with that of 1-2-1(5) and 1-2-1(8) suggest that, for those patterns with no monthly overlap, the interval between the re-inclusion of units in the sample has some effect.

Figure 1(d) and columns 7 and 8 of Table 3, show that for one month changes in trend estimates the variance increases very rapidly as m increases from 1 to 3 and decreases rapidly as m increases from 4. The *in-for*-3 rotation pattern seems to be the worst among those considered, and the currently used rotation patterns can be significantly improved upon. For example, using a 1-2-1(8) instead of a 4-8-4(8) rotation pattern would reduce the variance in the one month change in trend estimates for employment by 55 percent and 43 percent for unemployment. While the degree of monthly overlap is still a key factor, the pattern of inclusion also plays a role, for example the 2-2-2(8) pattern has lower variance than the *in-for*-2 or 2-10-2(4) patterns. Moreover, for one month changes in the trend estimates the best performing rotation patterns are 1-2-1(5) and 1-2-1(8) which perform considerably better than using complete rotation each month. This result arises because one month changes in trend estimates effectively look at differences in the seasonally adjusted series a few months apart and the 1-2-1(m) rotation patterns lead to positive correlations between estimates 3 months apart. Similar results were obtained in a study by McLaren and Steel (1997) using Sutcliffe's (1993) approximation to X11.

The results show that for the estimation of the current level of trend and the latest movement in trend, the 1-2-1(m) rotation patterns give considerably lower sampling variances than the rotation patterns currently in use.

5.2 X11ARIMA - Concurrent Cascade Filters with Extrapolations

Results for the filter combinations 2, 3 and 4 are given in Tables 4, 5 and 6 respectively. Figures 2(a) to 2(d) present results for combination 4 for employment.

Columns 1 and 2 of Tables 4, 5 and 6 show that rotation patterns with low monthly overlap perform almost as well as complete rotation for seasonally adjusted level estimates. Rotation patterns with high monthly overlap have higher variances, particularly for combination 3 which corresponds to the use of the 9 term HMA.

There is minimal difference between the ratios of the four different combinations for the one month change in the seasonally adjusted estimates (columns 3 and 4 in all tables). Rotation patterns with high monthly sample overlap still perform better than those with low or no monthly overlap regardless of the X11/X11ARIMA combination used.

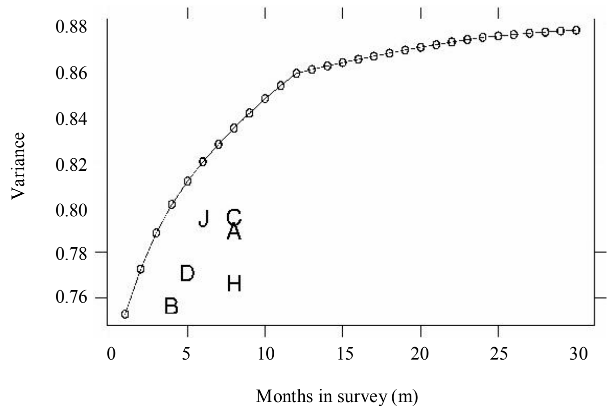
For the level of trend estimates, rotation patterns with a higher degree of sample overlap again have a greater variance ratio. The 1-2-1(5) and 1-2-1(8) rotation patterns still out-perform the other rotation patterns for each combination of filters, although they do not perform as well as an independent sample for combinations 2 and 4.

For one month changes in the trend estimates the better performing rotation patterns are again 1-2-1(5) and 1-2-1(8) which perform better than the independent sample for all four combinations of filters. For combination 3, rotation patterns with high monthly overlap perform equally as well as the 1-2-1(m) rotation patterns. For combinations 2 and 3 the 1-1-1(6) pattern is slightly better than the 1-2-1(m) patterns. Substantial improvements over the currently used rotation patterns can be achieved by using 1-2-1(m) rotation patterns. For example, for the employment variable, changing from an 4-8-4(8) to a 1-2-1(8) would produce gains of 42, 25 and 64 percent using combinations 2, 3 and 4, respectively.

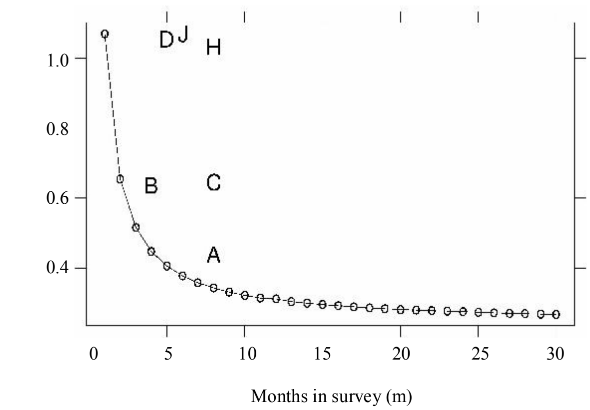
These results are based on the ALFS correlation estimates which, being based on survey estimates, will be subject to sampling error. The trend filters considered are not derived using these estimates. The same general conclusions concerning the impact of different rotation patterns are obtained for the two correlation models which use reasonably different correlations. We believe that the conclusions will apply for the range of correlation models contained between these two models. Similar conclusions are also obtained by McLaren and Steel (1997) using a correlation model derived by Steel (1996) for UK employment and unemployment.

6. Discussion

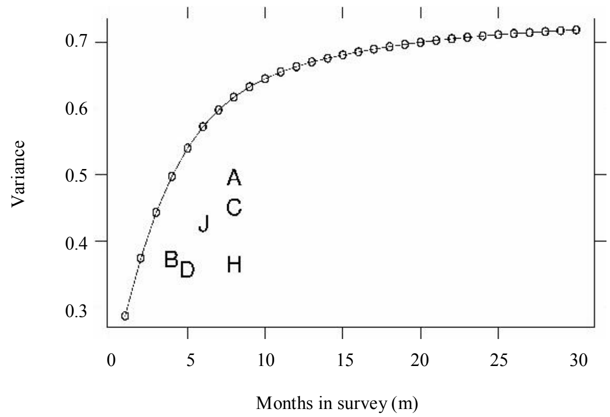
The rotation patterns currently used, such as *in-for*-8, *in-for*-6 and 4-8-4(8), are sensible if the one month change in seasonally adjusted estimates are the key statistics to be analysed. We believe that examination of the one month change in seasonally adjusted estimates is often not a reliable way of assessing current trends. It is necessary to look at the pattern of change over recent months. This can be done using filters to obtain an estimate of the trend. The results here suggest if the main use of the survey is to provide an assessment of trend then quite different rotation patterns should be used. Specifically, the 1-2-1(m) rotation patterns performed well for reducing the variance of the level of trend estimates and the difference between two consecutive trend estimates for a range of different filter combinations. The 1-2-1(m) rotation patterns also performed well for the sampling variance of the seasonally adjusted level estimates. Hence, in designing the rotation pattern for a repeated survey, the relative importance of



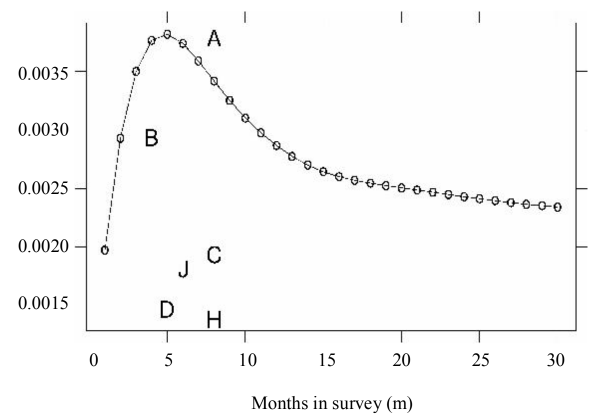
2a) Seas. Adjusted: Level estimates



2b) Seas. Adjusted: One month change



2c) Trend: Level estimates



2d) Trend: One month change

Figure 2. Ratio of the sampling variance to the variance of the original series for chosen rotation patterns for Combination 4 (X11ARIMA) for the variable employment where A = 4-8-4, B = 2-10-2(4), C = 2-2-2(8), D = 1,2-1(5), H = 1-2-1(8), J = 1-1-1(6).

seasonally adjusted and trend estimates needs to be carefully considered. Examining Figures 1 and 2 shows that the rotation pattern 2-2-2(8), is a reasonable compromise if the level and one months change in seasonally adjusted and trend estimates are both considered important. Bell (1999) also considered the effect of four different rotation patterns on the sampling variance of the level and one month change in the original, unadjusted, estimates and also trend estimates obtained using X11 and a 13 point HMA. He also identifies the 2-2-2(8) rotation pattern as a compromise design.

Even if analysts do not formally use trend estimates, the assessment of trend will involve looking at changes in seasonally adjusted estimates a few months apart. McLaren (1999) gives results which show that the 1-2-1 (m) rotation patterns will be suitable if the assessment of trends involve looking at changes in seasonally adjusted estimates over 3 or 6 months. The results also suggest that such rotation patterns perform well for estimates of the change in trend estimates over the most recent 3 and 6 months.

The evaluation criterion used in this paper is the sampling variance of the trend and seasonally adjusted estimates, which is the factor affected by the sample design. Steel and McLaren (2000) considered assessing different rotation patterns in terms of the degree of revisions of these estimates at the end points and reached similar conclusions regarding the rotation patterns.

Acknowledgement

This research was supported by *the Australian Research Council* and *the Australian Bureau of Statistics* (ABS). The views expressed in this paper may not necessarily reflect the views of either organisation. We would like to thank the associate editor and the referees for their comments and Geoff Lee, Andrew Sutcliffe and Phillip Bell from the ABS, and Norma Chhab from Statistics Canada.

References

Australian Bureau of Statistics (1987). *A Guide to Smoothing Time Series - Estimates of "Trend"*. Australian Bureau of Statistics, catalogue no. 1316.0, Canberra.

Australian Bureau of Statistics (1992). *Information Paper: Labour Force Survey Sample Design.* Australian Bureau of Statistics, catalogue no. 6269.0, Canberra.

Australian Bureau of Statistics (1993). *A Guide to Interpreting Time Series - Monitoring "Trends", An Overview.* Australian Bureau of Statistics, catalogue no. 1348.0, Canberra.

Bell, P.A. (1998). *Using State Space Models and Composite Estimation to Measure the Effects of Telephone Interviewing on Labour Force Estimates.* Working Papers in Econometrics and Applied Statistics, No. 98/2, Australian Bureau of Statistics, catalogue no. 1351.0, Canberra.

Bell, P.A. (1999). *The Impact of Sample Rotation Patterns and Composite Estimation on Survey Outcomes.* Working Papers in Econometrics and Applied Statistics, No. 99/1, Australian Bureau of Statistics, catalogue no. 1352.0, Canberra.

Bell, W.R., and Hillmer, S. (1990). Time series methods for survey estimation. *Survey Methodology*, 16, 195-215.

Bell, W.R., and Kramer, M. (1999). Towards variances for X-11 seasonal adjustment. *Survey Methodology*, 25, 13-29.

Bell, W.R., and Wilcox, D.W. (1993). The effect of sampling error on the time series behavior of consumption data. *Journal of Econometrics*, 555, 235-265.

Binder, D.A., and Hidiroglou, M.A. (1988). Sampling in time. *Handbook of Statistics*, (Eds., P.R. Krishaiah and C.R. Rao), Amsterdam: Elsevier Science Publishers, B.V., 6, 187-211.

Burridge, P., and Wallis, K.F. (1985). Calculation of seasonally adjusted series. *Journal of the American Statistical Association*, 80, 541-552.

Cleveland, W.P., and Tiao, G.C. (1976). Decomposition of seasonal time series: a model for the X-11 program. *Journal of the American Statistical Association*, 71, 581-587.

Dagum, E.B. (1980). *The X-11ARIMA Seasonal Adjustment Method.* Catalogue no. 12-564E, Statistics Canada, Ottawa.

Dagum, E.B. (1983). Spectral properties of the concurrent and forecasting seasonal linear filters of the X-11ARIMA method. *The Canadian Journal of Statistics*, 11, 73-90.

Dagum, E.B. (1988). *The X-11ARIMA/88 Seasonal Adjustment Methods - Foundations and User's Manual.* Statistics Canada, Ottawa.

Dagum, E.B., Chhab, N. and Chiu, K. (1996). Derivation and properties of the X-11ARIMA and Census X-11 linear filters. *Journal of Official Statistics*, 12, 329-347.

Dempster, P.A., and Hwang, J.-S. (1993). Component models and Bayesian technology for estimation of state employment and unemployment rates. *Proceedings of the Bureau of the Census Annual Research Conference*, 571-581.

Findley, D.F., Monsell, B.C., Otto, M.C., Bell, W.R. and Pugh, M.G. (1998). New capabilities and methods of the X-12 ARIMA seasonal adjustment program. *Journal of Business and Economic Statistics*, 16, 127-177.

Fuller, W.A., Adam, A. and Yansaneh, I.S. (1992). Estimates for longitudinal surveys with applications to the U.S. Current Population Survey. *Proceedings*: *Symposium 92*, *Design and Analysis of Longitudinal Surveys*, Statistics Canada, 301-324.

Gray, A., and Thomson, P. (1996). Design of moving-average trend filters using fidelity, smoothness and minimum revisions criteria. *Time Series Analysis in Memory of E.J. Hannan*, (Eds., P. Robinson and M. Rosenblatt), 205-219. Springer lecture notes in statistics, 115.

Henderson, R. (1916). Note on graduation by adjusted averages. *Transactions of the Actuarial Society of America*, 17, 43-48.

Hausman, J.A., and Watson, M.W. (1985). Error in variables and seasonal adjustment procedures. *Journal of the American Statistical Association*, 80, 531-540.

Kalton, G., and Citro, C.F. (1993). Panels surveys: Adding the fourth dimension. *Survey Methodology*, 19, 205-215.

Kenny, P.B., and Durbin, J. (1982). Local trend estimation and seasonal adjustment of economic and social time series. *Journal of the Royal Statistical Society A*, 145, 1-41.

Kish, L. (1998). Space/time variations and rolling samples. *Journal of Official Statistics*, 14, 31-46.

Knowles, J. (1997). Trend Estimation Practices of National Statistical Institutes. Office for National Statistics, Methods and Quality Division, UK, MQ 044.

Knowles, J., and Kenny (1997). An Investigation of Trend Estimation Methods. Office for National Statistics, Methods and Quality Division, UK, MQ 044.

Lee, H. (1990). Estimation of panel correlation for the Canadian Labour Force Survey. *Survey Methodology*, 16, 283-292.

Linacre, S., and Zarb, J. (1991). Picking turning points in the economy. *Australian Economic Indicators*, Australian Bureau of Statistics, catalogue no. 1350.0.

McLaren, C.H. (1999). Designing Rotation Patterns and Filters for Trend Estimation in Repeated Surveys. Unpublished PhD Thesis, School of Mathematics and Applied Statistics, University of Wollongong.

McLaren, C.H., and Steel, D.G. (1997). The effect of different rotation patterns on the sampling variance of seasonal and trend filters. *Proceedings of the Section on Survey Research Methods*, American Statistical Association, 1997, 790-795.

Miazaki, E.S., and Dorea, C.C.Y. (1993). Estimation of the parameters of a time series subject to the error of rotation sampling. *Communications in Statistics*, A, 22, 805-825.

Pfeffermann, D. (1994). A general method for estimating the variances of X-11 seasonally adjusted estimators. *Journal of Time Series Analysis*, 15, 85-116.

Rao, J.N.K., and Graham, J.E. (1964). Rotation designs for sampling on repeated occasions. *Journal of the American Statistical Association*, 69, 492-509.

Scott, A.J., Smith T.M.F. and Jones, R.G. (1977). The application of time series methods to the analysis of repeated surveys. *International Statistical Review*, 45, 3-73.

Shiskin, J., Young, A.H., and Musgrave, J.C. (1967). *X-11 Variant of the Census Method II Seasonal Adjustment Program*. Technical Paper 15, Bureau of the Census, U.S. Department of Commerce, Washington, D.C.

Singh, M.P., Drew, J.D., Gambino, J. and Mayda, F. (1990). *Methodology of the Canadian Labour Force Survey*. Catalogue no. 71-526, Statistics Canada.

Smith, T.M.F. (1997). Discussion of paper by Steel. *Journal of the Royal Statistical Society A*, 160, 33-34.

Steel, D.G. (1996). Options for Producing Monthly Estimates of Unemployment According to the ILO Definition. Central Statistical Office, U.K.

Steel, D.G. (1997). Producing monthly estimates of unemployment and employment according to the international labour office definition. *Journal of the Royal Statistical Society A*, 160, 5-46.

STEEL, D.G., and MCLAREN, C.H. (2000). The effect of different rotation patterns on the revisions of trend estimates. *Journal of Official Statistics*, 16, 61-76.

Steel, D.G., and Demel, R. (1988). The Contribution of Sampling Error to the Variability of Statistical Series. Paper Presented at the National Mathematical Sciences Congress, Canberra.

Sutcliffe, A. (1993). *X-11 Time Series Decomposition and Sampling Errors*. Working Papers in Econometrics and Applied Statistics, No 93/2. Australian Bureau of Statistics, catalogue no 1351.

Sutcliffe, A., and Lee, G. (1995). Seasonal Analysis and Sample Design. Paper presented at the Conference of Survey Measurement and Process Quality. Bristol 1995.

Tiller, R.B. (1992). Time series modeling of sample survey data from the U.S. Current Population Survey. *Journal of Official Statistics*, 8, 149-166.

Wallis, K.F. (1982). Seasonal adjustment and revision of current data: linear filters for the X-11-method. *Journal of the Royal Statistical Society A*, 145, 74-85.

Wolter, K.M., and Monsour, N.J. (1981). On the problem of variance estimation for a deseasonalized series. *Current Topics in Survey Sampling*, (Eds., D. Krewski, R. Platek and J.N.K. Rao). New York: Academic Press, 367-407.

Young, A.H. (1968). Linear Approximations to the Census and BLS Seasonal Adjustment Methods. *Journal of the American Statistical Association*, 63, 445-471.