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Estimation of Variance of General Regression Estimator: Higher Level Calibration Approach

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ABSTRACT

In the present investigation, the problem of estimation of variance of the general linear regression estimator has been considered. It has been shown that the efficiency of the low level calibration approach adopted by Särndal (1996) is less than or equal to that of a class of estimators proposed by Deng and Wu (1987). A higher level calibration approach has also been suggested. The efficiency of higher level calibration approach is shown to improve on the original approach. Several estimators are shown to be the special cases of this proposed higher level calibration approach. An idea to find a non – negative estimate of variance of the GREG has been suggested. Results have been extended to a stratified random sampling design. An empirical study has also been carried out to study the performance of the proposed strategies. The well known statistical package, GES, developed at Statistics Canada can further be improved to obtain better estimates of variance of GREG using the proposed higher level calibration approach under certain circumstances discussed in this paper.

KEY WORDS: Calibration; Estimation of variance; Auxiliary information; Ratio and regression type estimators; Model assisted approach.

1. INTRODUCTION

The statisticians are often interested in the precision of survey estimates. The most commonly used estimator of population total/mean is the generalized linear regression (GREG) estimator. Let us consider the simplest case of the GREG where information on only one auxiliary variable is available. Consider a population $\Omega = \{1, 2, ..., N\}$, from which a probability sample $s\,(s \subset \Omega)$ is drawn with a given sampling design, $p(.)$. The inclusion probabilities $\pi_i = Pr(i \in s)$ and $\pi_{ij} \in Pr(i \text{ and } j \in s)$ are assumed to be strictly positive and known. Let y_i be the value of the variable of interest, y, for the i-th population element, with which also is associated an auxiliary variable x_i. For the elements, $i \in s$, we observe (y_i, x_i). The population total of the auxiliary variable x, $X = \sum_{i=1}^{N} x_i$, is assumed to be accurately known. The objective is to estimate the population total $Y = \sum_{i=1}^{N} y_i$. Deville and Särndal (1992) used calibration on known population x-total to modify the basic sampling design weights, $d_i = 1/\pi_i$, that appear in the Horvitz-Thompson (1952) estimator

$$\hat{Y}_{\text{HT}} = \sum_{i=1}^{n} \frac{y_i}{\pi_i} = \sum_{i=1}^{n} d_i y_i. \tag{1.1}$$

A new estimator,

$$\hat{Y}_{\text{DS}} = \sum_{i=1}^{n} w_i y_i \tag{1.2}$$

was proposed by Deville and Särndal (1992), with weights w_i as close as possible in an average sense for a given metric to the d_i, while respecting the calibration equation

$$\sum_{i=1}^{n} w_i x_i = X. \tag{1.3}$$

A simple case considered by Deville and Särndal (1992) is the minimization of chi-square type distance function given by

$$\sum_{i=1}^{n} \frac{(w_i - d_i)^2}{d_i q_i} \tag{1.4}$$

where q_i are suitably chosen weights. In most of the situations, the value of $q_i = 1$. The form of the estimator depends upon the choice of q_i. By minimizing (1.4) subject to calibration equation (1.3) we obtain weights

$$w_i = d_i + \frac{d_i q_i x_i}{\sum_{i=1}^{n} d_i q_i x_i^2}\left(X - \sum_{i=1}^{n} d_i x_i\right). \tag{1.5}$$

Substitution of the value of w_i from (1.5) in (1.2) leads to the traditional regression estimator of total given by

$$\hat{Y}_{\text{DS}} = \sum_{i=1}^{n} d_i y_i + \frac{\sum_{i=1}^{n} d_i q_i x_i y_i}{\sum_{i=1}^{n} d_i q_i x_i^2}\left(X - \sum_{i=1}^{n} d_i x_i\right). \tag{1.6}$$

In this paper, the problem of estimation of variance of the regression estimator (1.6) has been considered at two different levels of calibration. The higher level calibration approach covers a greater variety of estimators than the low level calibration approach adopted by Särndal (1996).

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Higher level calibration approach makes use of known total as well as known variance of the auxiliary character, whereas low level calibration utilizes only known total of auxiliary character.

The section 4 has been devoted to study the stratified sampling design. The original stratum weights are calibrated which results in combined regression and combined ratio estimators in stratified sampling. The estimators of variance of combined regression and combined ratio estimators proposed by Wu (1985) are shown to be the special cases of the low level calibration approach. The higher level calibration approach has been shown to apply to a broader variety of estimators.

2. ESTIMATOR OF VARIANCE OF THE GREG: THE LOW LEVEL CALIBRATION APPROACH

Following model assisted survey sampling approach of Särndal, Swensson and Wretman (1989, 1992), the Yates-Grundy (1953) form of estimator of variance of the estimator (1.6) is given by

$$\hat{V}_{\mathrm{YG}}\left(\hat{Y}_{\mathrm{DS}}\right) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}\left(w_i e_i - w_j e_j\right)^2 \tag{2.1}$$

where $D_{ij} = (\pi_i \pi_j - \pi_{ij})/\pi_{ij}$, $i \neq j$ and $e_i = y_i - \hat{\beta} x_i$ have their usual meanings. This estimator can easily be written as

$$\hat{V}_{\mathrm{YG}}\left(\hat{Y}_{\mathrm{DS}}\right) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}\left(d_i e_i - d_j e_j\right)^2 + \hat{\psi}_1\left(X - \sum_{i=1}^{n} d_i x_i\right) + \hat{\psi}_2\left(X - \sum_{i-1}^{n} d_i x_i\right)^2 \tag{2.2}$$

where

$$\hat{\psi}_1 = \frac{1}{\sum_{i=1}^{n} d_i q_i x_i^2} \sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}\left(d_i e_i - d_j e_j\right)\left(d_i q_i x_i e_i - d_j q_j x_j e_j\right) \tag{2.3}$$

and

$$\hat{\psi}_2 = \frac{1}{2\left(\sum_{i=1}^{n} d_i q_i x_i^2\right)^2}\sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}\left(d_i q_i x_i e_i - d_j q_j x_j e_j\right)^2 \tag{2.4}$$

The estimator at (2.1) has been discussed by Särndal *et al.* (1989, 1992, 1996) on different occasions and covers a variety of estimators as discussed below:

For simplicity, let us consider simple random sampling and without replacement (SRSWOR) design *i.e.*, $\pi_i = \pi_j = n/N$ and $\pi_{ij} = n(n-1)/N(N-1)$. Then we have following cases:

Case 2.1: If $q_i = 1$, then (1.6) reduces to the usual regression estimator of total, $\hat{Y}_{\mathrm{GREG}}$ (say). Now if $w_i = d_i$ in (2.1), it reduces to

$$\hat{V}_{\mathrm{YG}}\left(\hat{Y}_{\mathrm{GREG}}\right) = \frac{N^2(1-f)}{n(n-1)}\sum_{i=1}^{n} e_i^2 \tag{2.5}$$

where $f = n/N$ and $e_i = y_i - \hat{\beta} x_i$. Thus (2.5) denotes the usual estimator of variance of the regression estimator (1.6).

Case 2.2: If $q_i = 1/x_i$ then the estimator (1.6) reduces to the ratio estimator of total, $\hat{Y}_{\mathrm{RATIO}}$ (say). The estimator (2.1) reduces to an estimator of variance of the estimator $\hat{Y}_{\mathrm{RATIO}}$, given by

$$\hat{V}_{\mathrm{YG}}\left(\hat{Y}_{\mathrm{RATIO}}\right) = \frac{N^2(1-f)}{n(n-1)}\sum_{i=1}^{n} e_i^2\left\{\frac{X}{\hat{X}}\right\}^2 \tag{2.6}$$

where

$$e_i = y_i - \left(\frac{\bar{y}}{\bar{x}}\right) x_i \text{ and } \hat{X} = \frac{N}{n}\sum_{i=1}^{n} x_i .$$

The estimator at (2.6) is a special case of a class of estimators of variance of the ratio estimator proposed by Wu (1982) as

$$\hat{V}_{\mathrm{YG}}\left(\hat{Y}_{\mathrm{W}}\right) = \frac{N^2(1-f)}{n(n-1)}\sum_{i=1}^{n} e_i^2\left\{\frac{X}{\hat{X}}\right\}^g \tag{2.7}$$

for $g = 2$.

Case 2.3: If $q_i = 1$ and w_i is given by (1.5) then (2.2) and hence (2.1) becomes

$$\hat{V}_{\mathrm{YG}}\left(\hat{Y}_{\mathrm{GREG}}\right) = \frac{N^2(1-f)}{n(n-1)}\sum_{i=1}^{n} e_i^2 + \hat{\psi}_1\left(X - \hat{X}\right) + \hat{\psi}_2\left(X - \hat{X}\right)^2 \tag{2.8}$$

where

$$\hat{\psi}_1 = \frac{(N-n)}{\left(\sum_{i=1}^{n} x_i^2\right) n(n-1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(e_i - e_j\right)\left(x_i e_i - x_j e_j\right) \tag{2.9}$$

and

$$\hat{\psi}_2 = \frac{(N-n)}{2N(n-1)\left(\sum_{i=1}^{n} x_1^2\right)^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(x_i e_i - x_j e_j\right)^2 . \tag{2.10}$$

Deng and Wu (1987) have defined a general class of estimators of the variance of the regression estimator as

$$\hat{V}_{\mathrm{YG}}\left(\hat{Y}_{\mathrm{DW}}\right) = \frac{N^2(1-f)}{n(n-1)} \sum_{i=1}^{n} e_i^2 \left\{\frac{X}{\hat{X}}\right\}^g \tag{2.11}$$

where $e_i = y_i - \hat{\beta} x_i$. The linear form of the class of estimators (2.11) takes the form as

$$\hat{V}_{\mathrm{YG}}\left(\hat{Y}_{\mathrm{DW}}\right) = \frac{N^2(1-f)}{n(n-1)} \sum_{i=1}^{n} e_i^2$$

$$\left[1 + g\left(\frac{X}{\hat{X}} - 1\right) + \frac{g(g-1)}{2}\left(\frac{X}{\hat{X}} - 1\right)^2 + ...\right] \tag{2.12}$$

which is again similar to (2.8). Thus the low level calibration approach considers estimators of variance of estimators of total *i.e.*, both ratio and regression methods of estimation. It is remarkable that there is no choice of q_i which reduces (1.6) to the product method of estimation considered by Cochran (1963). Hence the estimation of variance of product estimator has not been considered. To look at the efficiency of such estimators, we consider an analogue of the general class of estimators for estimating variance of GREG by following Srivastava (1971) as

$$\hat{V}_{\mathrm{S}}\left(\hat{Y}_{\mathrm{GREG}}\right) = \left(\frac{N^2(1-f)}{n(n-1)} \sum_{i=1}^{n} e_i^2\right) H\left(\frac{X}{\hat{X}}\right) \tag{2.13}$$

where $H(.)$ is a parametric function such that $H(1) = 1$ and satisfies certain regularity conditions. Following Srivastava (1971), it is easy to see that analogues of the general class of estimators (2.13) attain the minimum variance of the class of estimators proposed by Deng and Wu (1987) for regression estimator and Wu(1982) ratio estimator. We want to say here that if we will attach any function of the ratio $X/\hat{X}$ to the usual estimator of variance given by

$$\frac{N^2(1-f)}{n(n-1)} \sum_{i=1}^{n} e_i^2,$$

the asymptotic variance of the resultant estimator remains the same. In other words, the efficiency of the estimators of variance of regression estimator (GREG) of total obtained through low level calibration remains less than or equal to the class of estimators proposed by Wu (1982) and Deng and Wu (1987). The weights w_i used to construct estimator of variance of GREG at (2.1) were obtained while estimating the population total and hence named as low level calibration weights for variance estimation. The next section is devoted to the higher level calibration approach where variance of auxiliary character is known. Several new estimators are shown as special cases of the proposed higher level calibration approach.

3. IMPROVED ESTIMATOR OF VARIANCE OF THE GREG: THE HIGHER LEVEL CALIBRATION APPROACH

Here we apply the calibration approach to estimate the variance of GREG estimator at (1.6). The weights D_{ij} of Yates and Grundy (1953) for an estimator of variance given at (2.1) are calibrated such that the estimator of variance for the auxiliary variable has the exact variance. We consider an estimator of variance of GREG

$$\hat{V}_{\mathrm{SS}}\left(\hat{Y}_{\mathrm{GREG}}\right) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \Omega_{ij} (w_i e_i - w_j e_j)^2 \tag{3.1}$$

where Ω_{ij} are the modified weights attached to the quadratic expression by Yates and Grundy (1953) form of estimator and are as close as possible in an average sense for a given measure to the D_{ij} with respect to the calibration equation

$$\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \Omega_{ij} (d_i x_i - d_j x_j)^2 = V_{\mathrm{YG}}\left(\hat{X}_{\mathrm{HT}}\right) \tag{3.2}$$

where

$$V_{\mathrm{YG}}\left(\hat{X}_{\mathrm{HT}}\right) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} (\pi_i \pi_j - \pi_{ij})(d_i x_i - d_j x_j)^2$$

denotes the known variance of the estimator of the auxiliary total $X(=\sum_{i=1}^{N} x_i)$ given by $\hat{X}_{\mathrm{HT}} = \sum_{i=1}^{n} d_i x_i$. To compute the right hand side of (3.2) we need either information on every unit of the auxiliary character in the population, or only $V_{\mathrm{YG}}(\hat{X}_{\mathrm{HT}})$ obtained from a past survey or pilot survey. The examples of a situation where information on every unit of the auxiliary character is known are the establishment turnover recorded from census or administrative records or Business Register (BR) or Australian Taxation Office (ATO). Known variance of the auxiliary character has also been used by Das and Tripathi (1978), Singh and Srivastava (1980), Srivastava and Jhajj (1980, 1981), Isaki (1983), Singh and Singh (1988), Swain and Mishra (1992), Shah and Patel (1996) and Garcia and Cebrian (1996). Singh, Mangat and Mahajan (1995) have reviewed classes of estimators of unknown population parameters making use of the known variance of an auxiliary character. The idea of adjusting D_{ij} weights has also been discussed by Fuller (1970) through a regression type estimation procedure. For simplicity we restrict ourselves to the two dimensional Chi-Square (CS) type distance, D, between two $n \times n$ grids formed by the weights Ω_{ij} and D_{ij} for $i, j = 1, 2, ..., n$, given by

$$D = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{(\Omega_{ij} - D_{ij})^2}{D_{ij}Q_{ij}} \tag{3.3}$$

In most of the situations $Q_{ij} = 1$ but other types of weights can also be used. We will show that the ratio type adjustment using known variance of auxiliary character is a special case for a particular choice of Q_{ij}. Minimization of (3.3) subject to (3.2) leads to modified optimal weights given by

$$\Omega_{ij} = D_{ij} + \frac{D_{ij}Q_{ij}\left(d_i x_i - d_j x_j\right)^2}{\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}Q_{ij}\left(d_i x_i - d_j x_j\right)^4}\left[V_{\mathrm{YG}}\left(\hat{X}_{\mathrm{HT}}\right) - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}\left(d_i x_i - d_j x_j\right)^2\right] \tag{3.4}$$

for the optimal choice of Lagrange Multiplier λ, given by

$$\lambda = \frac{V_{\mathrm{YG}}\left(\hat{X}_{\mathrm{HT}}\right) - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}\left(d_i x_i - d_j x_j\right)^2}{\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}\; Q_{ij}\left(d_i x_i - d_j x_j\right)^4}. \tag{3.5}$$

Its proof is given in the Appendix. Substitution of Ω_{ij} from (3.4) in (3.1) leads to the following regression type estimator,

$$\hat{V}_{\mathrm{SS}}\left(\hat{Y}_{\mathrm{GREG}}\right) = \hat{V}_{\mathrm{YG}}\left(\hat{Y}_{\mathrm{DS}}\right) + \hat{B}_1\left[V_{\mathrm{YG}}\left(\hat{X}_{\mathrm{HT}}\right) - \hat{V}_{\mathrm{YG}}\left(\hat{X}_{\mathrm{HT}}\right)\right] \tag{3.6}$$

where

$$\hat{B}_1 = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}Q_{ij}\left(d_i x_i - d_j x_j\right)^2\left(w_i e_i - w_j e_j\right)^2}{\sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}Q_{ij}\left(d_i x_i - d_j x_j\right)^4} = \frac{\hat{\mu}_{22}}{\hat{\mu}_{04}} \text{ (say)} \tag{3.7}$$

$\hat{V}_{\mathrm{YG}}(\hat{X}_{\mathrm{HT}}) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}\left(d_i x_i - d_j x_j\right)^2$ and $\hat{V}_{\mathrm{YG}}(\hat{Y}_{\mathrm{DS}})$ is given in (2.1). Regression coefficient $\hat{B}_1$ makes use of the known total X of the auxiliary variable and hence can be treated as an improved estimator of regression coefficient by following Singh and Singh (1988). Under the higher level calibration approach, we have the following cases:

Case 3.1: Under SRSWOR sampling design if $q_i = x_i^{-1}$ and $Q_{ij} = (d_i x_i - d_j x_j)^{-2}$ are the weights attached at low level and higher level calibration approach, respectively, then the proposed strategy reduces to

$$\hat{V}_{\mathrm{SS}}\left(\hat{Y}_{\mathrm{Ratio}}\right) = \frac{N^2(1-f)}{n} \times \frac{1}{(n-1)}\sum_{i=1}^{n} e_i^2\left(\frac{X}{\hat{X}}\right)^2\left(\frac{S_x^2}{s_x^2}\right) \tag{3.8}$$

where $s_x^2 = (n-1)^{-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is an unbiased estimator of $S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \bar{X})^2$.

Case 3.2: If $q_i = 1$ and $Q_{ij} = 1 \; \forall \, i \; \& \; j$, then we have

$$\hat{V}_{\mathrm{YG}}\left(\hat{Y}_{\mathrm{GREG}}\right) = \frac{N^2(1-f)}{n(n-1)}\sum_{i=1}^{n} e_i^2 + \hat{\psi}_1\left(X - \hat{X}\right) + \hat{\psi}_2\left(X - \hat{X}\right)^2 + \hat{\psi}_3\left(S_x^2 - s_x^2\right) \tag{3.9}$$

where $\hat{\psi}_1$ and $\hat{\psi}_2$ are given by (2.9) and (2.10), respectively, and

$$\hat{\psi}_3 = \frac{N^2(1-f)}{n\sum_{i=1}^{n}\sum_{j=1}^{n}(x_i - x_j)^4}\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\left\{(x_i - x_j)(e_i - e_j) + \frac{\left(X - \hat{X}\right)(x_i - x_j)^2}{\sum_{i=1}^{n} x_i^2}\right\}^2\right] \tag{3.10}$$

Without loss of generality, the estimators of variance of GREG given at (3.8) and (3.9) are neither members of a low level calibration approach nor of the class of estimators by Deng and Wu (1987). These estimators are members of the analogues of classes of estimators for estimating variance of GREG given by Srivastava and Jhajj (1981) as

$$\hat{V}_{\mathrm{SJ}}\left(\hat{Y}_{\mathrm{GREG}}\right) = \left(\frac{N^2(1-f)}{n(n-1)}\sum_{i=1}^{n} e_i^2\right) H\left(\frac{X}{\hat{X}}, \frac{S_x^2}{s_x^2}\right) \tag{3.11}$$

where $H(.,.)$ is a parametric function such that $H(1, 1) = 1$ and which satisfies certain regularity conditions defined by them. Following Srivastava and Jhajj (1981) and Deng and Wu (1987), it is a class room exercise to see that the class of estimators at (3.11) remains better than the class of estimators defined at (2.11) and hence (2.13).

A difficult issue in using (3.1) is how to get non-negative estimates of variance using calibration. The simplest way is to optimize the CS distance function (3.3) subject to calibration constraint (3.2) along with the conditions $\Omega_{ij} \geq 0 \; \forall \, i, j = 1, 2, ..., n$. While it is difficult to develop a solution to this problem theoretically, well known quadratic programming techniques can yield useful numerical results. Straightforward extension to using other distance functions, as discussed by Deville and Särndal (1992) for instance, to the two dimensional problem due to the indeterminate nature of the D_{ij} weights is not possible. It is open to others to propose new distance functions which guarantee the non-negativity of the weights.

4. STRATIFIED SAMPLING DESIGN

Suppose the population consists of L strata with N_h units in the h-th stratum from which a simple random sample of size n_h is taken without replacement. The total population size $N = \sum_{h=1}^{L} N_h$ and sample size $n = \sum_{h=1}^{L} n_h$. Associated with the i-th unit of the h-th stratum there are two values y_{h_i} and x_{h_i} with $x_{h_i} > 0$ being the covariate. For the h-th stratum, let $W_h = N_h/N$ be the stratum weights, $f_h = n_h/N_h$ the sample fraction, $\bar{y}_h$, $\bar{x}_h$, $\bar{Y}_h$, $\bar{X}_h$ the y- and x- sample and population means respectively. Assume $\bar{X} = \sum_{h=1}^{L} W_h \bar{X}_h$ is known. The purpose is to estimate $\bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h$, possibly by incorporating the covariate information x. The usual estimator of population mean $\bar{Y}$ is given by

$$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h. \tag{4.1}$$

We are considering a new estimator, given by

$$\bar{y}^*_{St} = \sum_{h=1}^{L} W_h^* \bar{y}_h \tag{4.2}$$

with new weights W_h^*. The new weights W_h^* are chosen such that chi-square type distance, given by

$$\sum_{h=1}^{L} \frac{\left(W_h^* - W_h\right)^2}{W_h q_h} \tag{4.3}$$

is minimum subject to the condition

$$\sum_{h=1}^{L} W_h^* \bar{x}_h = \bar{X}. \tag{4.4}$$

Minimization of (4.3) subject to calibration equation (4.4) leads to the combined regression type estimator given by

$$\bar{y}^*_{St} = \sum_{h=1}^{L} W_h \bar{y}_h + \frac{\sum_{h=1}^{L} W_h q_h \bar{x}_h \bar{y}_h}{\sum_{h=1}^{L} W_h q_h \bar{x}_h^2}\left[\bar{X} - \sum_{h=1}^{L} W_h \bar{x}_h\right] \tag{4.5}$$

for the optimum choice of weights given by

$$W_h^* = W_h + \frac{W_h q_h \bar{x}_h}{\sum_{h=1}^{L} W_h q_h \bar{x}_h^2}\left(\bar{X} - \sum_{h=1}^{L} W_h \bar{x}_h\right) \tag{4.6}$$

If $q_h = \bar{x}_h^{-1}$ then estimator (4.5) reduces to the well known combined ratio estimator in stratified sampling. The well known estimator of variance of combined regression estimator is given by

$$\hat{V}\left(\bar{y}^*_{St}\right) = \sum_{h=1}^{L} \frac{W_h^2\left(1 - f_h\right)}{n_h} s_{\hat{e}h}^2 \tag{4.7}$$

where

$$s_{\hat{e}h}^2 = (n_h - 1)^{-1} \sum_{i=1}^{n_h} e_{hi}^2$$

is the h-th stratum sample variance and $\hat{e}_{hi} = y_{hi} - \bar{y}_h - b(x_{hi} - \bar{x}_h)$ and $b = \sum_{h=1}^{L} W_h q_h \bar{y}_h \bar{x}_h / \sum_{h=1}^{L} W_h q_h \bar{x}_h^2$ have their usual meaning. The lower level calibration approach yields an estimator of variance of the combined regression estimator as

$$\hat{V}_c\left(\bar{y}^*_{St}\right) = \sum_{h=1}^{L} \frac{D_h W_h^{*2}}{W_h^2} s_{\hat{e}h}^2 \tag{4.8}$$

where

$$D_h = \frac{W_h^2 (1 - f_h)}{n_h}$$

and W_h^* is given by (4.6). If $q_h = \bar{x}_h^{-1}$ then (4.8) reduces to an estimator given by

$$\hat{V}\left(\bar{y}^*_{St}\right)_{\text{RATIO}} = \left(\frac{\bar{X}}{\bar{x}_{St}}\right)^2 \sum_{h=1}^{L} \frac{W_h^2\left(1 - f_h\right)}{n_h} s_{\hat{e}h}^2 \tag{4.9}$$

which is a special case of a class of estimators for estimating the variance of combined ratio estimator given by Wu (1985) as

$$\hat{V}\left(\bar{y}^*_{St}\right)_W = \left(\frac{\bar{X}}{\bar{x}_{St}}\right)^g \sum_{h=1}^{L} \frac{W_h^2\left(1 - f_h\right)}{n_h} s_{\hat{e}h}^2 \tag{4.10}$$

for $g = 2$. The properties of variance estimators of the combined ratio estimator are also studied by Saxena, Nigham and Shukla (1995). In higher level calibration, a new estimator is given by

$$\hat{V}_{St}\left(\hat{Y}_{\text{GREG}}\right) = \sum_{h=1}^{L} \frac{\Omega_h W_h^{*2}}{W_h^2} s_{\hat{e}h}^2 \tag{4.11}$$

where Ω_h are suitably chosen weights such that Chi-Square distance function given by

$$\sum_{h=1}^{L} \frac{(\Omega_h - D_h)^2}{D_h Q_h} \tag{4.12}$$

is minimum subject to higher level calibration equation defined as

$$\sum_{h=1}^{L} \Omega_h s_{hx}^2 = V(\bar{x}_{St}) \tag{4.13}$$

where,

$$V(\bar{x}_{St}) = \sum_{h=1}^{L} W_h^2 \frac{(1-f_h)}{n_h} S_{hx}^2$$

is assumed to be known and $s_{hx}^2 = (n_h - 1)^{-1} \sum_{i=1}^{n_h} (x_{hi} - \bar{x}_h)^2$ is an unbiased estimator of $S_{hx}^2 = (N_h - 1)^{-1} \sum_{i=1}^{N_h} (X_{hi} - \bar{X}_h)^2$. This procedure leads to a new estimator for the variance of the combined regression estimator given by

$$\hat{V}\left(\hat{Y}_{St}\right)_{\text{CLR}} = \hat{V}_{St}\left(\hat{Y}_{\text{GREG}}\right) + \hat{B}_{St}\left[V\left(\bar{x}_{St}\right) - \hat{V}\left(\bar{x}_{St}\right)\right] \tag{4.14}$$

where

$$\hat{B}_{St} = \sum_{h=1}^{L} \frac{W_h^{*2}(1-f_h)}{n_h} Q_h s_{hx}^2 s_{\hat{e}h}^2 \Big/ \sum_{h=1}^{L} \frac{W_h^2 (1-f_h)}{n_h} s_{hx}^4$$

denotes the combined improved estimator of regression coefficient in stratified sampling and

$$\hat{V}(\bar{x}_{St}) = \sum_{h=1}^{L} W_h^2 \frac{(1-f_h)}{n_h} s_{hx}^2$$

is an unbiased estimator of $V(\bar{x}_{St})$. If $q_h = 1/\bar{x}_h$ and $Q_h = 1/s_{hx}^2$, then estimator (4.14) reduces to a new estimator of variance of the combined ratio estimator given by

$$\hat{V}_{St}\left(\hat{Y}_{\text{Ratio}}\right) = \sum_{h=1}^{L} \frac{W_h^2 (1-f_h)}{n_h} s_{\hat{e}h}^2 \left(\frac{\bar{X}}{\bar{x}_{St}}\right)^2 \left\{\frac{V\left(\bar{x}_{St}\right)}{\hat{V}\left(\bar{x}_{St}\right)}\right\} \tag{4.15}$$

which is a ratio type estimator proposed by Wu (1985) for estimating variance of the combined ratio estimator but makes use of extra knowledge of the known variance of the auxiliary variable at the estimation stage. Several more new estimators can be constructed for new choices of weights q_h and Q_h.

5. A WIDER CLASS OF ESTIMATORS

If we define $u = X / \sum_{i=1}^{n} d_i x_i$ and $v = V(\hat{X}_{\text{HT}}) / \hat{V}(\hat{X}_{\text{HT}})$, then a wider class of estimators has been defined as

$$\hat{V}_{\text{SS}}\left(\hat{Y}_{\text{GREG}}\right) = \left\{\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} D_{ij} \left(d_i e_i - d_j e_j\right)^2\right\} H(u,v) \tag{5.1}$$

where $H(u,v)$ is a parametric function of u and v such that $H(1,1) = 1$ and which satisfies certain regularity conditions. Then all estimators obtained from the following functions,

$$H(u,v) = u^{\alpha} v^{\beta}, \; H(u,v) = \frac{1 + \alpha(u-1)}{1 + \beta(v-1)},$$

$$H(u,v) = 1 + \alpha(u-1) + \beta(v-1)$$

and $H(u,v) = \{1 + \alpha(u-1) + \beta(v-1)\}^{-1}$ are special cases of the higher level calibration approach, where α and β are unknown parameters involved in the function $H(u,v)$. Replacing these parameters with their respective consistent estimators in the class of estimators at (5.1) leads to the same asymptotic variance as shown by Srivastava and Jhajj (1983), Singh and Singh (1984) and Mahajan and Singh (1996). The extension of present investigation to two phase sampling following Hidiroglou and Särndal (1995) is in progress.

The next section has been devoted to studying the performance of the higher order calibration approach through simulation.

6. SIMULATION STUDY

Under the simulation study, we have considered comparisons of estimators of variance of ratio estimator as well as that of regression estimator. To avoid any kind of confusion, we have redefined the estimators considered for comparison as follows:

6.1 Ratio Estimator

We have compared the estimators of the variance of the ratio estimator, given by

$$\hat{V}_1\left(\hat{Y}_{\text{RATIO}}\right) = \frac{N^2(1-f)}{n(n-1)} \sum_{i=1}^{n} e_i^2 \left(\frac{X}{\hat{X}}\right)^2 \tag{6.1.1}$$

with the estimator, given by

$$\hat{V}_2\left(\hat{Y}_{\text{RATIO}}\right) = \hat{V}_1\left(\hat{Y}_{\text{RATIO}}\right)\left(\frac{S_x^2}{s_x^2}\right). \tag{6.1.2}$$

6.2 Regression Estimator

We have also compared the estimators of the variance of the regression estimator, given by

$$\hat{V}_1\left(\hat{Y}_{\text{GREG}}\right) =$$

$$\frac{N^2(1-f)}{n(n-1)} \sum_{i=1}^{n} e_i^2 + \hat{\psi}_1\left(X - \hat{X}\right) + \hat{\psi}_2\left(X - \hat{X}\right)^2 \tag{6.2.1}$$

with the estimator, given by

$$\hat{V}_2\left(\hat{Y}_{\text{GREG}}\right) = \hat{V}_1\left(\hat{Y}_{\text{GREG}}\right) + \hat{\psi}_3\,(S_x^2 - s_x^2) \qquad (6.2.2)$$

where $\hat{\psi}_i, i = 1, 2, 3$ have the same meaning as defined earlier.

We have considered two types of populations *viz.* finite populations as well as infinite populations to cover almost all practical situations.

6.3 Finite Populations

In case of finite populations, we have taken a population consisting of $N = 20$ units from Horvitz and Thompson (1952). The study variable, y, is the number of households on i-th block and known auxiliary character, x, is the eye-estimated number of households on the i-th block. All possible samples of size $n = 5$ were selected by SRSWOR, which results in

$$\binom{N}{n} = 15{,}504$$

samples. From the k-th sample, the estimator

$$\hat{Y}_{\text{RATIO}}|_k = \hat{Y}\left(\frac{X}{\hat{X}}\right), \text{ with } \hat{Y} = \frac{N}{n}\sum_{i=1}^{n} y_i$$

was computed. Empirical mean squared error of this estimator was computed as

$$\text{MSE}\left(\hat{Y}_{\text{RATIO}}\right) = \binom{N}{n}^{-1} \sum_{k=1}^{\binom{N}{n}} \left[\hat{Y}_{\text{RATIO}}|_k - Y\right]^2. \qquad (6.3.1)$$

For the k-th sample, the ratio type estimators of variance

$$\hat{V}_h\left(\hat{Y}_{\text{RATIO}}\right)|_k, h = 1, 2,$$

given by (6.1.1) and (6.1.2) respectively, for estimating the variance of the ratio estimator were also obtained. The bias in the h-th ratio type estimator of variance was computed as

$$B\left\{\hat{V}_h\left(\hat{Y}_{\text{RATIO}}\right)\right\} =$$

$$\binom{N}{n}^{-1} \sum_{k=1}^{\binom{N}{n}} \hat{V}_h\left(\hat{Y}_{\text{RATIO}}\right)|_k - \text{MSE}\left(\hat{Y}_{\text{RATIO}}\right) \qquad (6.3.2)$$

and mean squared error was computed as

$$\text{MSE}\left\{\hat{V}_h\left(\hat{Y}_{\text{RATIO}}\right)\right\} =$$

$$\binom{N}{n}^{-1} \sum_{k=1}^{\binom{N}{n}} \left[\hat{V}_h\left(\hat{Y}_{\text{RATIO}}\right)|_k - \text{MSE}\left(\hat{Y}_{\text{RATIO}}\right)\right]^2. \qquad (6.3.3)$$

The percent relative efficiency of the estimator $\hat{V}_2\,(\hat{Y}_{\text{RATIO}})$ with respect to $\hat{V}_1\,(\hat{Y}_{\text{RATIO}})$ was calculated as

$$\text{RE} =$$

$$\text{MSE}\left\{\hat{V}_1\left(\hat{Y}_{\text{RATIO}}\right)\right\} \times 100/\text{MSE}\left\{\hat{V}_2\left(\hat{Y}_{\text{RATIO}}\right)\right\}. \qquad (6.3.4)$$

The coverage by 95% confidence intervals

$$\text{CCI}\left[\hat{V}_h\left(\hat{Y}_{\text{RATIO}}\right)\right]$$

for $h = 1, 2$ were calculated for h-th ratio type estimator of variance by counting the number of times the true population total, Y, falls between the limits defined as

$$\hat{Y}_{\text{RATIO}}|_k \mp t_{n-h-1}(\alpha)\sqrt{\hat{V}_h\left(\hat{V}_{\text{RATIO}}\right)|_k}. \qquad (6.3.5)$$

These results were also obtained from all possible samples of size 6 and 7 and have been presented in Table 1.

The same process was repeated for the regression estimator

$$\hat{Y}_{\text{GREG}}|_k = \hat{Y} + \left(\sum_{i=1}^{n} x_i y_i \Big/ \sum_{i=1}^{n} x_i^2\right)\left(X - \hat{X}\right)$$

of total obtained from (1.6) under a SRSWOR design. The biases, relative efficiency and CCI were obtained by using h-th estimator of variance of the regression estimator, $\hat{V}_h\,(\hat{Y}_{\text{GREG}})|_k$ for $h = 1, 2$, given by (6.2.1) and (6.2.2), respectively. The results obtained have been presented in Table 2. In addition, it was observed that for $n = 5$, 0.020% estimates of variance obtained from the estimator $\hat{V}_1\,(\hat{Y}_{\text{GREG}})$ and 0.022% estimates obtained from the estimator $\hat{V}_2\,(\hat{Y}_{\text{GREG}})$ were negative. Similar results were observed for more natural populations given by Cochran (1963) and Sukhatme and Sukhatme (1970). Over all, second order calibration estimators perform better than first order calibration in case of the finite populations.

In real life situations, the study variable and auxiliary variables may follow certain kinds of distributions like normal, beta or gamma *etc*. In order to see the performance of the proposed strategies under such circumstances, we generated artificial populations and considered the problem of estimation of finite population mean through simulation as follows.

Table 1

Comparison of $\hat{V}_2\left(\hat{Y}_{\text{RATIO}}\right)$ with $\hat{V}_1\left(\hat{Y}_{\text{RATIO}}\right)$ for finite populations

n	$\text{B}\left[\hat{V}_1\left(\hat{Y}_{\text{RATIO}}\right)\right]$	$\text{B}\left[\hat{V}_2\left(\hat{Y}_{\text{RATIO}}\right)\right]$	**RE**	$\text{CCI}\left[\hat{V}_1\left(\hat{Y}_{\text{RATIO}}\right)\right]$	$\text{CCI}\left[\hat{V}_2\left(\hat{Y}_{\text{RATIO}}\right)\right]$
5	-211.33	217.01	166.57	0.93	0.95
6	-141.92	102.00	115.06	0.91	0.92
7	-99.34	58.60	109.23	0.90	0.90

Table 2

Comparison of $\hat{V}_2\left(\hat{Y}_{\text{GREG}}\right)$ and $\hat{V}_1\left(\hat{Y}_{\text{GREG}}\right)$ for finite populations

n	$\text{B}\left[\hat{V}_1\left(\hat{Y}_{\text{GREG}}\right)\right]$	$\text{B}\left[\hat{V}_2\left(\hat{Y}_{\text{GREG}}\right)\right]$	**RE**	$\text{CCI}\left[\hat{V}_1\left(\hat{Y}_{\text{GREG}}\right)\right]$	$\text{CCI}\left[\hat{V}_2\left(\hat{Y}_{\text{GREG}}\right)\right]$
5	-328.49	-194.78	112.04	0.92	0.96
6	-223.92	-136.34	103.02	0.90	0.93
7	-157.88	-94.38	101.21	0.91	0.94

6.4 Infinite Populations

The size N of these populations is unknown. We generated n independent pairs of random numbers y_i^* and x_i^* (say), $i = 1, 2, \ldots, n$, from a subroutine VNORM with PHI = 0.7, $\text{seed}(y) = 8987878$ and $\text{seed}(x) = 2348789$ following Bratley, Fox and Schrage (1983). For fixed $S_y^2 = 50$ and $S_x^2 = 50$, we generated transformed variables,

$$y_i = 3.0 + \sqrt{S_y^2\,(1 - \rho^2)}\, y_i^* + \rho\, S_y x_i^* \qquad (6.4.1)$$

and

$$x_i = 4.0 + S_x x_i^* \qquad (6.4.2)$$

for different values of the correlation coefficient ρ. For the k-th sample, the estimator

$$\hat{\bar{y}}_{\text{RATIO}}\,|_k = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right), \text{ with } \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \text{ and } \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

was computed. Empirical mean squared error of this estimator was computed as

$$\text{MSE}\left(\hat{\bar{y}}_{\text{RATIO}}\right) = \frac{1}{15{,}000}\sum_{k=1}^{15{,}000}\left[\hat{\bar{y}}_{\text{RATIO}}\,|_k - \bar{Y}\right]^2. \qquad (6.4.3)$$

For the k-th sample, the ratio type estimators of variance

$$\hat{V}_h\left(\hat{\bar{y}}_{\text{RATIO}}\right)|_k,\, h = 1, 2,$$

obtained from (6.1.1) and (6.1.2) respectively, for estimating the variance of the ratio estimator of population mean were also derived. The bias in the h-th ratio type estimator of variance was computed as

$$B\left\{\hat{V}_h\left(\hat{\bar{y}}_{\text{RATIO}}\right)\right\} = \frac{1}{15{,}000}\sum_{k=1}^{15{,}000}\hat{V}_h\left(\hat{\bar{y}}_{\text{RATIO}}\right)|_k - \text{MSE}\left(\hat{\bar{y}}_{\text{RATIO}}\right) \qquad (6.4.4)$$

and mean squared error was computed as

$$\text{MSE}\left\{\hat{V}_h\left(\hat{\bar{y}}_{\text{RATIO}}\right)\right\} = \frac{1}{15{,}000}\sum_{k=1}^{15{,}000}\left[\hat{V}_h\left(\hat{\bar{y}}_{\text{RATIO}}\right)|_k - \text{MSE}\left(\hat{\bar{y}}_{\text{RATIO}}\right)\right]^2. \qquad (6.4.5)$$

The percent relative efficiency of the estimator $\hat{V}_2\left(\hat{\bar{y}}_{\text{RATIO}}\right)$ with respect to $\hat{V}_1\left(\hat{\bar{y}}_{\text{RATIO}}\right)$ was calculated as

$$\text{RE} = \text{MSE}\left\{\hat{V}_1\left(\hat{\bar{y}}_{\text{RATIO}}\right)\right\} \times 100 / \text{MSE}\left\{\hat{V}_2\left(\hat{\bar{y}}_{\text{RATIO}}\right)\right\} \qquad (6.4.6)$$

The coverage by 95% confidence intervals

$$\text{CCI}\left[\hat{V}_h\left(\hat{\bar{y}}_{\text{RATIO}}\right)\right] \text{ for } h = 1, 2$$

was calculated for h-th ratio type estimator of variance by counting the number of times the true population mean, $\bar{Y}$, falls between the limits defined as

$$\hat{\bar{y}}_{\text{RATIO}}\,|_k \mp 1.96\sqrt{\hat{V}_h\left(\hat{\bar{y}}_{\text{RATIO}}\right)|_k}. \qquad (6.4.7)$$

These results were obtained for samples of size $n = 60$, 80 and 100 for different values of correlation coefficient as presented in Table 3.

The same process was repeated for the regression estimator

$$\hat{\bar{y}}_{\text{GREG}}\big|_k = \bar{y} + \hat{\beta}(\bar{X} - \bar{x})$$

of mean obtained from (1.6) under a SRSWR design. The biases, relative efficiency and CCI were obtained by using h-th estimator of variance of the regression estimator,

$$\hat{V}_h(\hat{\bar{y}}_{\text{GREG}})\big|_k \text{ for } h = 1, 2,$$

derived from (6.2.1) and (6.2.2), respectively. The results obtained have been presented in Table 4. We acknowledge that it is worth while studying the proposed strategy through simulation in more detail and its application in actual practice. The empirical study was carried out in FORTRAN-77 using a PENTIUM-120.

7. CONCLUSION

Higher level calibration approach can be used if variance of the auxiliary character is known in addition to the known total of that character. The statistical package GES developed by Statistics Canada can be modified to obtain better estimators of the variance of GREG, useful for surveys where information on variance of auxiliary characters is available or can be calculated.

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Table 3
Comparison of $\hat{V}_2(\hat{\bar{y}}_{\text{RATIO}})$ with $\hat{V}_1(\hat{\bar{y}}_{\text{RATIO}})$ for infinite populations

n	ρ	$B[\hat{V}_1(\hat{\bar{y}}_{\text{RATIO}})]$	$B[\hat{V}_2(\hat{\bar{y}}_{\text{RATIO}})]$	RE	$\text{CCI}[\hat{V}_1(\hat{\bar{y}}_{\text{RATIO}})]$	$\text{CCI}[\hat{V}_2(\hat{\bar{y}}_{\text{RATIO}})]$
	0.1	13.02	10.33	188.7	0.96	0.95
	0.3	8.07	6.35	192.6	0.97	0.95
60	0.5	4.33	3.37	195.9	0.96	0.96
	0.7	1.77	1.37	197.9	0.97	0.97
	0.9	0.33	0.26	197.7	0.99	0.98
	0.1	3.27	2.91	123.2	0.94	0.93
	0.3	2.06	1.84	123.0	0.94	0.94
80	0.5	1.13	1.01	122.7	0.95	0.95
	0.7	0.47	0.42	122.0	0.97	0.96
	0.9	0.08	0.08	119.1	0.98	0.97
	0.1	0.76	0.77	106.1	0.94	0.93
	0.3	0.49	0.49	105.8	0.94	0.94
100	0.5	0.27	0.27	105.3	0.95	0.95
	0.7	0.12	0.12	104.4	0.96	0.95
	0.9	0.02	0.02	102.2	0.97	0.95

Table 4
Comparison of $\hat{V}_2(\hat{\bar{y}}_{\text{GREG}})$ with $\hat{V}_1(\hat{\bar{y}}_{\text{GREG}})$ for infinite populations

n	ρ	$B[\hat{V}_1(\hat{\bar{y}}_{\text{GREG}})]$	$B[\hat{V}_2(\hat{\bar{y}}_{\text{GREG}})]$	RE	$\text{CCI}[\hat{V}_1(\hat{\bar{y}}_{\text{GREG}})]$	$\text{CCI}[\hat{V}_2(\hat{\bar{y}}_{\text{GREG}})]$
	0.1	10.12	8.42	177.6	0.98	0.95
	0.3	5.06	4.33	161.5	0.97	0.95
60	0.5	3.32	2.36	152.5	0.95	0.96
	0.7	0.72	0.38	151.9	0.97	0.95
	0.9	0.13	0.10	147.7	0.99	0.97
	0.1	1.23	1.11	153.9	0.96	0.95
	0.3	1.03	1.01	143.5	0.98	0.94
80	0.5	0.13	0.11	132.8	0.97	0.95
	0.7	0.07	0.06	121.6	0.97	0.95
	0.9	0.02	0.03	117.1	0.96	0.96
	0.1	0.65	0.57	136.1	0.95	0.94
	0.3	0.39	0.32	135.1	0.94	0.94
100	0.5	0.13	0.13	129.6	0.95	0.95
	0.7	0.02	0.02	114.4	0.96	0.95
	0.9	0.01	0.01	112.2	0.97	0.96

APPENDIX

This appendix has been devoted to deriving the optimum value of Ω_{ij} as given in (3.4). The Lagrange's function is given by

$$L = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\left(\Omega_{ij} - D_{ij}\right)^2}{D_{ij}Q_{ij}} -$$

$$2\lambda\left[\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\Omega_{ij}\left(d_i x_i - d_j x_j\right)^2 - V_{\text{YG}}\left(\hat{X}_{\text{HT}}\right)\right]. \quad (\text{A.1})$$

On differentiating (A.1) with respect to Ω_{ij} and equating to zero, we get

$$\Omega_{ij} = D_{ij} + \lambda D_{ij} Q_{ij}\left(d_i x_i - d_j x_j\right)^2. \quad (\text{A.2})$$

On putting (A.2) in (3.2), we get

$$\lambda = \frac{V_{\text{YG}}\left(\hat{X}_{\text{HT}}\right) - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij}\left(d_i x_i - d_j x_j\right)^2}{\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} D_{ij} Q_{ij}\left(d_i x_i - d_j x_j\right)^4}. \quad (\text{A.3})$$

On substituting (A.3) in (A.2), we get the optimum value of Ω_{ij} as given in (3.4).

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