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Understanding Calibration Estimators in Survey Sampling

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ABSTRACT

There exist well known methods due to Deville and Särndal (1992) which adjust sampling weights to meet benchmark constraints and range restrictions. The resulting estimators are known as calibration estimators. There also exists an earlier, but perhaps not as well known, method due to Huang and Fuller (1978). In addition, alternative methods were developed by Singh (1993), who showed that similar to the result of Deville-Särndal, all these methods are asymptotically equivalent to the regression method. The purpose of this paper is threefold: (i) to attempt to provide a simple heuristic justification of all calibration estimators (including both well known and not so well known) by taking a non-traditional approach; to do this, a model (instead of the distance function) for the weight adjustment factor is first chosen and then a suitable method of model fitting is shown to correspond to the distance minimization solution, (ii) to provide to practitioners computational algorithms as a quick reference, and (iii) to illustrate how various methods might compare in terms of distribution of weight adjustment factors, point estimates, estimated precision, and computational burden by giving numerical examples based on a real data set. Some interesting observations can be made by means of a descriptive analysis of numerical results which indicate that while all the calibration methods seem to behave similarly to the regression method for loose bounds, they however seem to behave differently for tight bounds.

KEY WORDS: Benchmark constraints; Distance minimization; Non-negative weights; Range restrictions.

1. INTRODUCTION

In providing estimates from sample surveys, sampling weights are commonly adjusted to obtain calibrated weights in order to match totals or benchmark constraints (BCs) for auxiliary variables. The methods of regression and raking are often used for this purpose. Although these methods have good asymptotic properties (see Deville and Särndal 1992), they may lead to calibrated weights with undesirable (finite sample) properties. The regression method can give negative weights while the raking procedure can produce very high weights. For this reason, range restrictions (RRs) may be imposed on the calibrated weights. It would be desirable to have a calibration method which (i) produces calibrated weights close to the original sampling weights; this can be achieved via minimization of a suitable distance function between the two sets of weights, (ii) meets BCs, and (iii) satisfies RRs. There exist several methods in the literature for weight adjustment under BCs and RRs, see *e.g.*, Deville and Särndal (1992, henceforth referred to as DS) for recent developments, and Huang and Fuller (1978) for earlier developments. For a review, as well as some further work, see Singh (1993, henceforth referred to as Singh). These methods are iterative in nature and can be classified into two families. Family I consists of methods which satisfy BCs after each iteration and continue to iterate until RRs are met. Family II, on the other hand, consists of methods which satisfy RRs after each iteration and continue to iterate until BCs are met. Methods of DS belong to family II while that of Huang-Fuller belongs to family I. Two additional methods, one for each family, were proposed by Singh. Using arguments similar to DS, Singh extended the remarkable result of DS by showing that all of the methods in families I and II are asymptotically equivalent to the regression method.

In Section 2, a non-traditional approach is followed in introducing each method which is expected to help in understanding of calibration estimators. The functional form of the weight adjustment factor is first heuristically motivated and later on a connection between a suitable method of model fitting and minimization of the distance function is made. Alongside, computational algorithms are given as a quick reference for practitioners. A computer program in GAUSS software is available from the second author; see also Singh and Mohl (1997). In Section 3, numerical examples are presented to illustrate various methods using data from Statistics Canada's Family Expenditure (FAMEX) survey. It is of practical interest to see how different calibration methods might compare for a real data set. In particular, we examine by means of a descriptive analysis the impact of RRs on the computational burden, distribution of weight adjustment factors, point estimates and their variance. Related comparative studies on calibration methods based on real data sets are due to Deville, Särndal and Sautory (1993) and Stukel and Boyer (1993). These studies, however, are restricted to family II methods and are primarily concerned with the distribution of weight adjustment factors. Finally, Section 4 contains a discussion.

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2. HEURISTIC JUSTIFICATION OF CALIBRATION ESTIMATORS

We will use the following notation. Let n, N denote respectively the sample size and the population size. Let h_k denote the initial or h-weight (used in the expansion or Horvitz-Thompson estimator $\sum_{k=1}^{n} y_k h_k$) for the k-th element where y_k is the value of the study variable. It is assumed that the h-weights include adjustments for any non-response. The parameter of interest is the population total for y, denoted by τ_y. For each k, there are p-auxiliary variables, x_{kj}, $j = 1, ..., p$ for which the population total or benchmark constraint, $\tau_{xj} = \sum_{k=1}^{N} x_{kj}$ for each j is assumed to be known. The transposed p-vector $\boldsymbol{x}_k$ denotes $(x_{k1}, ..., x_{kp})$, the k-th row of the $n \times p$ matrix $\boldsymbol{X}$. Let $c_k^{(\nu)}$ denote the calibrated or c-weight for the k-th element at the ν-th iteration. At $\nu = 0$, $c_k^{(\nu)} = h_k$. The expansion estimators of population totals for variables y and x_j using c-weights at the ν-th iteration are denoted by $\hat{\tau}_y^{(\nu)}$ and $\hat{\tau}_{xj}^{(\nu)}$ respectively.

The RRs are specified by the condition $L \le g_k \le U$ where $g_k = c_k/h_k$ and $L < 1 < U$, where L and U denote suitable lower and upper bounds. The adjustment factors (*i.e.*, g_k's) are also called g-weights. First we consider the unrestricted case (*i.e.*, calibration without RRs) and then the restricted case. All methods in the restricted case require iterations for finding a solution. It is assumed that the iterative process converges in a finite number of iterations.

The criterion for convergence is defined as follows. For the iterative process to meet RRs, a tolerance level ϵ (*e.g.*, .005 or .01) for family I is defined so that the process terminates if the maximum absolute relative error (ARE) for RRs is $\le \epsilon$. Similarly, a tolerance level ($\delta > 0$) for family II is defined for meeting BCs by iterations. The reason for this is that our primary goal is not minimization of the distance function, but to find a solution which satisfies BCs and RRs. In addition to ϵ and δ, a parameter ν_{max} is defined which limits the number of iterations.

There are seven methods considered in this paper, two for the unrestricted case, two for restricted case in family I and the remaining three also for the restricted case but in family II. We have given alternative names to existing methods to facilitate understanding of the relationship between different methods. The naming convention is based on the well known distance measures used in the analysis of count data.

Note that since all the methods are asymptotically equivalent to the regression method, the asymptotic variance of $\hat{\tau}_y$ can be estimated for each method by $\sum_k \sum_l (\pi_{kl} - \pi_k \pi_l)\ \pi_{kl}^{-1}$ $(e_k g_k)(e_l g_l)$, as in DS (equation 3.4) where π_k, π_{kl} are respectively the first and second order inclusion probabilities, e_k are the sample residuals $y_k - \hat{\boldsymbol{B}}' \boldsymbol{x}_k$ with $\hat{\boldsymbol{B}}' = (\boldsymbol{y}' \Gamma_0 \boldsymbol{X})$ $(\boldsymbol{X}' \Gamma_0 \boldsymbol{X})^{-1}$, and Γ_0 is the $n \times n$ matrix diag($\boldsymbol{h}$).

2.1 METHOD 1 (Linear Regression or Unrestricted Modified Chi Square, MCS-u)

This method is the simplest and gives rise to the popular generalized regression estimator of Särndal (1980). Here, the model for the adjustment factor is taken to be linear in $\boldsymbol{x}$, *i.e.*, $g_k = 1 + \boldsymbol{x}_k' \lambda$, for some p-vector of model parameters λ which satisfies BCs. That is, $\sum_{k=1}^{n} h_k (1 + \boldsymbol{x}_k' \lambda) x_{kj} = \tau_{xj}$, for all j. This gives rise to $\lambda^{\mathbf{MCS\text{-}u}}$ as $(\boldsymbol{X}' \Gamma_0 \boldsymbol{X})^{-1} (\tau_x - \hat{\tau}_x^{(0)})$. The c-weights remain close to the h-weights in the sense that the above choice of g-weights minimizes the distance function, $\Delta^{\mathrm{MCS\text{-}u}}(\boldsymbol{c}, \boldsymbol{h}) = \sum_{k=1}^{n} (c_k - h_k)^2 / h_k$ subject to BCs. Note that the g-weights could be negative for some k. This is rather undesirable in practice although the simplicity of the method is quite attractive. The computational algorithm for MCS-u is given in Appendix A1.

2.2 METHOD 2 (Raking or Unrestricted Modified Discrimination Information, MDI-u)

This method is also commonly used. Here, the model for the adjustment factor g_k is taken as $\exp(\boldsymbol{x}_k' \lambda)$, thus making it necessarily non-negative. Unlike the case of method 1, the model parameter vector $\lambda^{\mathbf{MDI\text{-}u}}$ is obtained iteratively to meet BCs. The iterations can be started with $\lambda^{\mathbf{MCS\text{-}u}}$ from the GR-estimator, *i.e.*, for iteration 1, set $\lambda^{(1)} = \lambda^{\mathbf{MCS\text{-}u}}$, which implies $c_k^{(1)} = h_k \exp(\boldsymbol{x}_k' \lambda^{(1)})$. These c-weights, in general, do not satisfy BCs. For iteration 2 of this method, the $\lambda^{(1)}$ is adjusted (by a term of smaller order) to define $\lambda^{(2)}$ as $\lambda^{(1)} + (\boldsymbol{X}' \Gamma_1 \boldsymbol{X})^{-1} (\tau_x - \hat{\tau}_x^{(1)})$, where $\Gamma_1 = \mathrm{diag}\,(\boldsymbol{c}^{(1)})$. The λ term is defined similarly for further iterations until convergence, *i.e.*, until BCs are met. The c-weights remain close to h-weights because iterations used in the above method constitute the Newton-Raphson steps for minimizing the distance function, $\Delta^{\mathrm{MDI\text{-}u}}(\boldsymbol{c}, \boldsymbol{h}) = \sum_{k=1}^{n} [c_k \log(c_k/h_k) - c_k + h_k]$ subject to BCs. Note that although the g-weights are non-negative, they could be very high which is clearly not desirable in practice. The computational algorithm for MDI-u is given in Appendix A2.

2.3 METHOD 3 (Modified Huang-Fuller or Scaled Modified Chi Square, SMCS)

This method belongs to family I of the restricted case and is a slight modification of the method due to Huang and Fuller as given in Singh; see also Fuller, Loughin, and Baker (1994). As in regression, the model for the adjustment factor is taken to be linear in $\boldsymbol{x}$. To facilitate the satisfaction of RRs by these adjustments, a scaling factor q_k, $(0 < q_k \le 1)$, is used for each k so that the change in h-weights for those units whose g_k's tend to go outside the bounds $[L, U]$ is reduced. Thus, the g-weight is given by $g_k = 1 + q_k \boldsymbol{x}_k' \lambda$ where the model parameters $\boldsymbol{q}$ and λ are chosen iteratively in the sense that λ is found for a given $\boldsymbol{q}$ and then $\boldsymbol{q}$ is found for a given λ. We start with $q_k^{(0)} = 1$ for all k and set $\lambda^{(1)} = \lambda^{\mathbf{MCS\text{-}u}}$ for iteration 1. Now, clearly $\boldsymbol{c}^{(1)}$ satisfies BCs but RRs need not be satisfied. Depending on the location of g_k's in relation to $[L, U]$, a working rule can be used to define q_k's so that the q_k's discount more for those units which are farther outside of the boundaries than those which are nearer. The scaling factors $q_k^{(1)}$ so determined, define in turn $\lambda^{(2)}$ for iteration 2 as $(\boldsymbol{X}' \Gamma_1 \boldsymbol{X})^{-1} (\tau_x - \hat{\tau}_x^{(1)})$ where $\Gamma_1 = \mathrm{diag}\,(q_k^{[1]} h_k)$, $q_k^{[1]} = q_k^{(0)} q_k^{(1)}$,

$\lambda^{(2)}$ satisfying BCs after the iteration. Note that under usual regularity conditions, $\lambda^{(2)}$ differs from $\lambda^{(1)}$ only by a term of smaller order, since the maximum absolute difference $|q_k^{(1)} - 1|$ is small. Next, if $\boldsymbol{c}^{(2)}$ after iteration 2 does not satisfy RRs, the scaling factors $q_k^{(2)}$ are defined appropriately and compounded with $q_k^{[1]}$ to get $q_k^{[2]}$ for use in iteration 3. The $\lambda^{(3)}$ for iteration 3 is then obtained as before so that BCs are satisfied after the iteration. Iterations continue until convergence, *i.e.*, until RRs are met. The weight vector $\boldsymbol{c}^{\mathbf{SMCS}}$ is close to $\boldsymbol{h}$ because at each iteration ν, $\boldsymbol{c}^{(\nu)}$ minimizes the distance function $\Delta_\nu^{\mathrm{SMCS}}(\boldsymbol{c},\boldsymbol{h}) = \sum_{k=1}^n (c_k - h_k)^2/h_k q_k^{[\nu-1]}$ subject to BCs, where $q_k^{[\nu-1]} = q_k^{(0)} q_k^{(1)} \ldots q_k^{(\nu-1)}$ for $\nu \geq 1$. Note that unlike the previous methods, the distance function varies from iteration to iteration.

The computational algorithm for SMCS is given in Appendix A3. Note that in the algorithm, $[L, U]$ is shrunk to $[L', U']$ by means of a parameter α where $L' = \alpha L + 1 - \alpha$, $U' = \alpha U + 1 - \alpha$, and $0 < \alpha \leq 1$. This implies that some units that are inside $[L, U]$ but close to the boundary are also discounted. This helps to speed up the convergence. Another parameter β, $0 \leq \beta \leq 1$ is also introduced to allow differential discounting of different units.

2.4 METHOD 4 (Shrinkage-Minimization, SM)

This method also belongs to family I and is due to Singh. As in regression, the model for the adjustment factor is taken to be linear in $\boldsymbol{x}$, but a new parameter termed the shrinkage factor ψ_k $(0 < \psi_k \leq 1)$ is used for each k so that g_k's meet RRs, *i.e.*, g_k is set at $(1 + \psi_k x_k' \lambda(k))$. Notice that λ is allowed to depend on k through ψ_k and $\boldsymbol{x}_k$. Unlike SMCS, here the g-weights, after discounting, satisfy RRs exactly, *i.e.*, those g-weights which are outside $[L, U]$ are shrunk to lie on or inside the boundary. Therefore, ψ_k's can be defined quite easily in practice. The model parameters ψ and λ are chosen iteratively in a manner analogous to that for SMCS. We start with $\psi_k^{(0)} = 1$ and set $\lambda^{(1)} = \lambda^{\mathbf{MCS\text{-}u}}$ for iteration 1 to obtain $g_k^{(1)}$ as $(1 + \psi_k^{(0)} x_k' \lambda^{(1)})$. Clearly BCs are satisfied after the iteration but RRs need not be. Before iteration 2, $g_k^{(1)}$ is shrunk by $\psi_k^{(1)}$ to obtain $g_k^{(1)*}$ as $(1 + \psi_k^{[1]} x_k' \lambda^{(1)})$ where $\psi_k^{[1]} = \psi_k^{(0)} \psi_k^{(1)}$, which meets RRs. Given $\psi^{[1]}$, $\lambda^{(2)}(k)$ is obtained as $\lambda^{(1)} + (1/\psi_k^{[1]})(\boldsymbol{X}' \Gamma_1 \boldsymbol{X})^{-1} (\tau_x - \hat{\tau}_x^{(1)*}) + \boldsymbol{x}_k'(\boldsymbol{X}' \Gamma_1 \boldsymbol{X})^{-1} (\tau_x - \hat{\tau}_x^{(1)*})\lambda^{(1)}$ where $\Gamma_1 = \mathrm{diag}(\boldsymbol{c}^{(1)*})$, $c_k^{(1)*} = h_k g_k^{(1)*}$, and $\hat{\tau}_x^{(1)*}$ is the expansion estimator using $\boldsymbol{c}^{(1)*}$-weights. Again BCs are satisfied after the iteration but RRs need not be. Note that $\lambda^{(2)}(k)$ differs from $\lambda^{(1)}$ by a term of smaller order uniformly over k. Iterations are continued until convergence, *i.e.*, until RRs are met. The weight vector $\boldsymbol{c}^{\mathbf{SM}}$ is close to $\boldsymbol{h}$ because at each iteration $\nu \geq 1$, $\boldsymbol{c}^{(\nu)}$ minimizes the distance function, $\Delta_\nu^{\mathrm{SM}}(\boldsymbol{c}, \boldsymbol{c}^{(\nu-1)*}) = \sum_{k=1}^n (c_k - c_k^{(\nu-1)*})^2/c_k^{(\nu-1)*}$ subject to BCs. Note that in practice $\boldsymbol{c}^{(\nu)*}$ can be obtained directly from $\boldsymbol{c}^{(\nu)}$ without having to calculate $\psi^{(\nu)}$ separately. As with SMCS, the distance function depends on the iteration.

The computational algorithm is given in Appendix A4. Recall that in the above method, if a g-weight falls outside of the L and U boundaries, an adjustment is made to bring the g-weight back to the L or U boundary. A new parameter α $(0 < \alpha \leq 1)$ is introduced to allow the user to bring the g-weight farther inside the boundary to a point L' or U' $(L' = \alpha L + 1 - \alpha, \quad U' = \alpha U + 1 - \alpha)$. This is somewhat similar to the α parameter of SMCS. Another parameter $\eta(0 < \eta \leq \alpha \leq 1)$ is introduced to adjust the g-weights to the level L' or U' also for those units which are within $[L, U]$, but close to the boundary in that they are outside $[L'', U'']$ where $L'' = \eta L + 1 - \eta$, $U'' = \eta U + 1 - \eta$. All these parameters help speed up the convergence in general.

2.5 METHOD 5 (Linear Truncated or Restricted Modified Chi Square, MCS-r)

This well known method belongs to family II of the restricted case and is due to DS. As in SM, the model for the adjustment factor is taken to be linear in $\boldsymbol{x}$ with a new parameter termed the truncation factor ϕ_k $(0 < \phi_k \leq 1)$ which is used for each k so that g_k's meet RRs, *i.e.*, g_k is set at $(1 + \phi_k x_k' \lambda(k))$. The only difference between the truncation factor ϕ_k used here and the shrinkage factor used in SM is that here those g-weights which are outside $[L, U]$ are always adjusted to lie exactly on the boundary. The model parameters are chosen iteratively. Initially we set $\phi_k^{(0)} = 1$ and at iteration 1, $\lambda^{(1)} = \lambda^{\mathbf{MCS\text{-}u}}$ to obtain $\tilde{g}_k^{(1)} = (1 + \phi_k^{(0)} x_k' \lambda^{(1)})$, which is further adjusted (or truncated) to obtain $g_k^{(1)}$ as $(1 + \phi_k^{[1]} x_k' \lambda^{(1)})$ where $\phi_k^{[1]} = \phi_k^{(0)} \phi_k^{(1)}$, so that RRs are met. However, $\boldsymbol{g}^{(1)}$ may not satisfy BCs. Note that the difference between $\boldsymbol{g}^{(1)}$ and $\boldsymbol{g}^{\mathbf{MCS\text{-}u}}$ is of smaller order. Now, for iteration 2, $\lambda^{(1)}$ is adjusted by a term of smaller order (uniformly over k) to define $\lambda^{(2)}(k)$ as $\lambda^{(1)} + (1/\phi_k^{[1]})(\boldsymbol{X}' \Gamma_1 \boldsymbol{X})^{-1}(\tau_x - \tau_x^{(1)})$, where $\Gamma_1 = \mathrm{diag}(\boldsymbol{h})$ except that the diagonal elements are truncated to zero for all those k for which $\phi_k^{(1)} < 1$, *i.e.*, those units which were truncated at the previous iteration. This discounting of diagonal elements is somewhat similar to using a zero scaling factor in SMCS. In the second iteration, we have $\tilde{g}_k^{(2)} = 1 + \phi_k^{[1]} x_k' \lambda^{(2)}(k)$ and the truncation factors $\phi_k^{(2)}$ are used to obtain $g_k^{(2)}$ which satisfy RRs. The successive iterations are defined in a similar manner. Clearly, unlike SM, here RRs are met at each iteration. Iterations are continued until BCs are met. The weight vector, $\boldsymbol{c}^{\mathbf{MCS\text{-}r}}$ is close to $\boldsymbol{h}$ because the iterations defined above constitute the Newton-Raphson steps for minimizing the distance function $\Delta^{\mathrm{MCS\text{-}r}}(\boldsymbol{c},\boldsymbol{h}) = \sum_k (c_k - h_k)^2/h_k$ if $Lh_k \leq c_k \leq Uh_k$; ∞ otherwise, subject to BCs. The computational algorithm is given in Appendix A5. Note that, in practice, it is more convenient to work with $g_k^{(\nu)}$ directly without having to compute $\phi_k^{(\nu)}$ separately.

2.6 METHOD 6 (Restricted Modified Discrimination Information or MDI-r)

This method also belongs to family II and was proposed by Singh following the lines of DS in developing MCS-r. It is related to MDI-u in the same way as MCS-r is to MCS-u. The basic idea is to adjust parameters ϕ and λ in the adjustment factor $g_k = \phi_k \exp(x_k' \lambda)$ so that RRs and BCs are satisfied. The truncation parameter ϕ is similar to that for MCS-r. This

is done iteratively. Similar to MCS-r, at iteration 1 we set $\tilde{g}_k^{(1)} = \phi_k^{(0)} \exp(\boldsymbol{x}_k' \lambda^{(1)})$ where $\phi_k^{(0)} = 1, \lambda^{(1)} = \lambda^{\text{MCS-u}}$, which is further adjusted by a term of smaller order to obtain $g_k^{(1)}$ as $\phi_k^{[1]} \exp(\boldsymbol{x}_k' \lambda^{(1)})$ so that RRs are met, *i.e.*, it lies in $[L, U]$. Next for iteration 2, $g_k^{(1)}$ is adjusted by a term of smaller order to obtain $g_k^{(2)}$ as $\phi_k^{[2]} \exp(\boldsymbol{x}_k' \lambda^{(2)})$, where $\lambda^{(2)} = \lambda^{(1)} + (\boldsymbol{X}' \Gamma_1 \boldsymbol{X})^{-1} (\tau_x - \hat{\tau}_x^{(1)})$, and $\Gamma_1 = \text{diag}(h_k g_k^{(1)})$ except that the diagonal elements are truncated to 0 for all those k for which $\phi_k^{(1)} < 1$. The truncation factors $\phi_k^{(2)}$ are used to ensure that RRs are met. Iterations are continued until convergence, *i.e.*, until BCs are met. The weight vector $\boldsymbol{c}^{\text{MDI-r}}$ is close to $\boldsymbol{h}$ because the iterations defined above constitute the Newton-Raphson steps for minimizing the distance function $\Delta^{\text{MDI-r}}(\boldsymbol{c}, \boldsymbol{h}) = \sum_{k=1}^{n} [c_k \log(c_k/h_k) - c_k + h_k]$ if $Lh_k \le c_k \le Uh_k$; ∞ otherwise, subject to BCs. Note that in practice, the truncation factors are not needed separately to compute $g_k^{(v)}$. Appendix A6 gives the computational algorithm for MDI-r.

2.7 METHOD 7 (Logit or Generalized Modified Discrimination Information, GMDI)

This is the last method considered. This well known method of family II is due to DS. As in the raking method, we start with $\exp(\boldsymbol{x}_k' \lambda)$ and an inverse logit-type transformation is used to ensure that the adjustment factor satisfies RRs. The model for the adjustment factor is given by $g_k = [(U - 1) + (1 - L) \exp(A\boldsymbol{x}_k' \lambda)]^{-1} [L(U - 1) + U(1 - L) \exp(A\boldsymbol{x}_k' \lambda)]$, where $A = (1 - L)^{-1} (U - 1)^{-1} (U - L)$. This adjustment factor, unlike other methods, lies necessarily inside the interval $[L, U]$, *i.e.*, does not take boundary values. As $L \to 0$ and $U \to \infty$, the factor reduces to the familiar inverse logit form, $\exp(\boldsymbol{x}_k' \lambda)/[1 + \exp(\boldsymbol{x}_k' \lambda)]$. The model parameter λ is obtained iteratively to meet BCs. Starting with $\lambda^{\text{MCS-u}}$ as $\lambda^{(1)}$ for iteration 1, we adjust by a smaller order term to obtain $\lambda^{(2)}$ as $\lambda^{(1)} + (\boldsymbol{X}' \Gamma_1 \boldsymbol{X})^{-1} (\tau_x - \hat{\tau}_x^{(1)})$ where $\Gamma_1 = \text{diag}(h_k d_k^{(1)})$, $d_k^{(1)} = (U - 1)^{-1} (1 - L)^{-1} (U - g_k^{(1)})(g_k^{(1)} - L)$. Further iterations are done in a similar manner until BCs are met. The weight-vector $\boldsymbol{c}^{\text{GMDI}}$ is close to $\boldsymbol{h}$ in the sense that subject to BCs, the above iterative process corresponds to the Newton-Raphson algorithm for minimizing the distance function $\Delta^{\text{GMDI}}(\boldsymbol{c}, \boldsymbol{h})$ given by $A^{-1} \sum_{k=1}^{n} h_k [(g_k - L) \log\{(1 - L)^{-1} (g_k - L)\} + (U - g_k) \log\{(U - 1)^{-1} (U - g_k)\}]$. Appendix A7 gives the computational algorithm for GMDI.

3. NUMERICAL EXAMPLES

3.1 Data Description

We consider application of the seven adjustment methods described above to data from the 1990 Statistics Canada's Family Expenditure (FAMEX) Survey for the two cities (or domains) of Regina and Saskatoon in the province of Saskatchewan. Four study variables are considered: annual expenditures on owned dwelling for repair and renovation, furniture and equipment, ladies' clothing , and men's clothing. The FAMEX survey is a supplementary survey to the Canadian Labour Force Survey (LFS) and, therefore, is based on the LFS design – a multistage stratified cluster sample of households, see Singh *et al.* (1990). Samples are drawn independently from the two cities of Regina and Saskatoon. Respectively for the two cities, the numbers of strata are 30 and 34, and the numbers of primary sampling units (PSUs) selected in the sample are 111 and 94. The total numbers of sampled households are 321 and 278, while the corresponding numbers (n) of individuals are 797 and 712.

3.2 Benchmark Constraints, Range Restrictions and Common Weights per Household

The number (p) of BCs is four for each domain. They correspond to the demographic population counts for the four groups: age < 15, age ≥ 15, one person households, and households with two or more persons. The corresponding counts are 40696, 139047, 12746, and 48457 for Regina, and 42544, 139299, 20628, and 52059 for Saskatoon. Thus, the total numbers of households for the two domains are 61203 and 72687 respectively and the corresponding population sizes (N) are 179743 and 181843. The auxiliary x-variables here are indicators for the above four groups.

For Regina, (min, max) of g-weights are obtained as (−0.72, 2.74) and (0.19, 3.95) respectively for regression and raking methods. It is therefore of interest to make them nonnegative for regression and to reduce the high weights for raking. Two types of RRs are chosen: one has somewhat loose bounds with $L = 1/5$ and $U = 5$ and the other has somewhat tight bounds with $L = 2/5$ and $U = 5/2$. For Saskatoon, (min, max) of g-weights are obtained as (0.86, 1.08) and (0.87, 1.09) respectively for regression and raking methods. Note that both methods give g-weights close to 1, and therefore there is no real need for RRs. However, for the sake of illustration, we choose $L = 0.88$ and $U = 1.12$.

The initial sampling weights or h-weights of individuals in the same household are common and equal to the weight of that household. It is desirable that after calibration, all members of a household have the same $\boldsymbol{c}$-weights. This can be achieved by modifying the $\boldsymbol{X}$ matrix so that $\boldsymbol{x}_j$-values for each person in the same household are common and equal to the average value for the household, see, *e.g.*, Lemaître and Dufour (1987). We also perform an initial scaling on the h-weights so that they add up to N; this is similar to the Hájek modification of the Horvitz-Thompson estimator. This scaling essentially redefines $[L, U]$ to make them meaningful for calibration of h-weights.

3.3 Descriptive Measures for Comparison

For comparing various methods, we consider four types of descriptive measures:

(i) Summary statistics for the distribution of the g-weights,
(ii) Point estimates for several variables,
(iii) Estimated precision of the calibration estimates, and
(iv) Computational burden imposed by each method.

The first measure consists of a graphical summary using a box plot for $\boldsymbol{g}$-weights, and the standard deviation of g-weights, SD(g), defined as $[N^{-1} \sum_{k=1}^{n} h_k (g_k - 1)^2]^{1/2}$. Note

that the mean of g-weights, *i.e.*, $N^{-1}\sum_{k=1}^{n} h_k g_k$, is 1 in view of the fact that $\sum h_k = \sum c_k = N$, and the SD(g) also equals $[N^{-1}\sum_{k=1}^{n}(c_k - h_k)^2/h_k]^{1/2}$, the square root of a normalized chi-square type distance for measuring closeness between h- and c-weights. For comparing point estimates and their precision for estimating parameter for each variable y of interest, we compute relative difference (RD) and relative precision (RP) with respect to the MCS-u weights, *i.e.*, relative to the regression estimator. Denoting an estimator based on c-weights as a c-estimator, we have RD as (c-estimator minus regression estimator) divided by the regression estimator, and RP as SE(regression estimator) divided by SE(c-estimator). Note that for the numerical examples under consideration, variances are computed using jackknifing by deleting PSUs. Finally, the computational burden is expressed in terms of the number of iterations. Testing has shown that for all the restricted methods, each iteration takes a similar amount of time and hence a good comparison of their computational burden is the number of iterations required for convergence.

3.4 Specification of Other Parameters

We also need to specify some other parameters, namely, α, β for SMCS, and α, η for SM. Empirically, values of $\alpha = 0.67$, $\eta = 0.9$ and $\beta = 0.8$ are found to perform well. The tolerance levels ϵ for family I and δ for family II are set at 0.01, and v_{max} is set at 10.

3.5 Results: A Descriptive Analysis

3.5.1 Distribution of g-weights

We first consider the Regina data. Figure 1 gives a box plot of the distribution of g-weights with $L = 0.4$ and $U = 2.5$. Note that there are negative g-weights (and hence negative c-weights) for MCS-u and large g-weights (which produce large c-weights) for the MDI-u method. For MCS-u, the fraction of g-weights < 0 is 4.9%, the fraction < 0.4 is 5.9%, the fraction above 2.5 is 1.25% while above 3.5 is 0%. For MDI-u, the fraction below 0.4 is 4.9%, the fraction > 2.5 is 4.3% and above 3.5 is 1.25%. Thus, both methods yield c-weights which are out of bounds with respect to RRs with tight bounds. The range restricted methods all have median g-weights between 0.65 and 0.75; the SMCS g-weights show, however, the most clustering around the median. Table 1 shows that under loose bounds, the SD(g) for each restricted method is slightly higher (about 7%) than the regression method, but for tight bounds, the difference increases to about 15% for family I and about 10% for family II.

Now for the Saskatoon data, Figure 2 gives a box plot of g-weights with $L = 0.88$ and $U = 1.12$. For both regression and raking methods, about 5.6% are below L and 0% are above U. All methods have similar interquartile range for g-weights with medians slightly above 1. Also it is seen from Table 1 that SD(g) for all the methods (restricted and unrestricted) are about the same and quite small.

Table 1
Number of Iterations and SD(g)
($\alpha = .67$, $\beta = .8$, $\eta = .9$, $\epsilon = \delta = .01$, $v_{max} = 10$)

Method	Regina				Saskatoon	
	$L = 0.2$, $U = 5.0$ (Loose bounds)		$L = 0.4$, $U = 2.5$ (Tight bounds)		$L = 0.88$, $U = 1.12$	
	Number of iterations	SD(g)	Number of iterations	SD(g)	Number of iterations	SD(g)
Family I						
SMCS	2	0.647	3	0.702	2	0.071
SM	2	0.636	4	0.689	2	0.070
Family II						
MCS-r	2	0.628	3	0.654	1	0.069
MDI-r	3	0.642	3	0.660	1	0.069
GMDI	3	0.640	3	0.659	2	0.069

Note: For the unrestricted (or no bounds) case, the number of iterations and SD(g) are: for Regina MCS-u and MDI-u are (1,0.599) and (3,0.647) respectively; for Saskatoon MCS-u and MDI-u are (1,0.070) and (1,0.069) respectively.

3.5.2 Relative Difference of Point Estimates

Tables 2(a) and (b) show that for Regina, under loose bounds RD is small for all the methods for each of the variables. In fact, it is negligible except for the variable "owned dwelling" for which it is generally under 4%. However, under tight bounds, it increases somewhat but remains small with values ranging between 1% and 5%. For Saskatoon (Table 2c), under the given bounds RD is negligible for all the methods.

3.5.3 Estimated Relative Precision of Estimates

For Regina, under loose bounds, RP is generally within 5% (of the precision of the regression estimator) for all methods and all variables except for MDI-r with the variable "ladies' clothing" for which it is lower by 9%. However, under tight bounds, RP varies more and is now generally within 9% except for SMCS and SM with the variable "Men's clothing" (RP is lower by 20%) and MDI-r for the variable "Ladies' clothing" for which RP is lower by 11%. For Saskatoon (Table 2c), under the chosen bounds RP is close to 1 for all cases.

3.5.4 Computational Burden

For Regina (Table 1), under loose bounds each method takes two or three iterations. As the bounds are tightened, most of the methods require more iterations to converge. To see how tightly the bounds could be squeezed before encountering convergence problems, three more sets of bounds were used with [L, U] = [0.425, 2.35], [0.45, 2.22] and [0.475, 2.11]. These results are not shown in the table. With v_{max} as 10, the SM method does not converge for [0.425, 2.35]. The SMCS and GMDI methods do not converge for [0.45, 2.22] and the MCS-r and MDI-r finally have

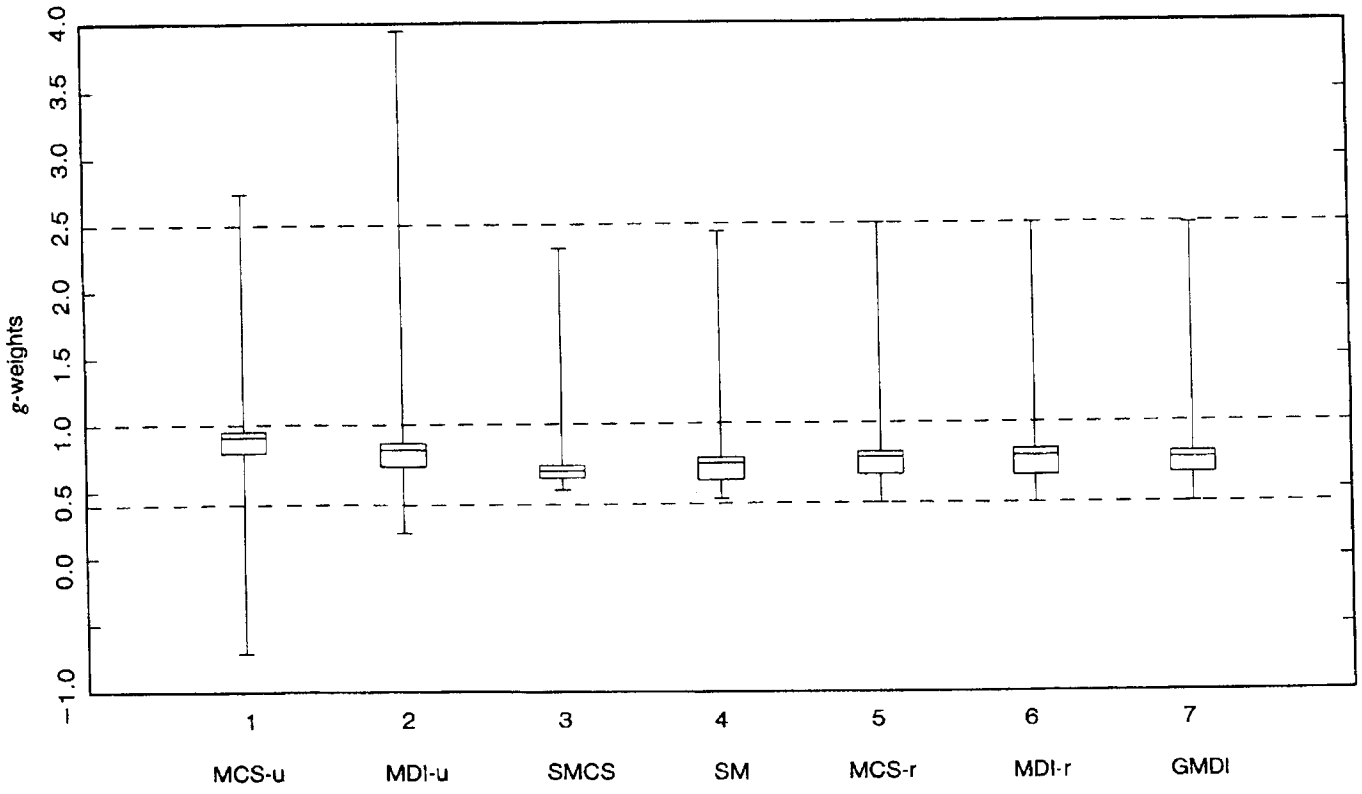

Figure 1. Box Plot: g-weights for Regina FAMEX data ($L = 0.4$, $U = 2.5$)

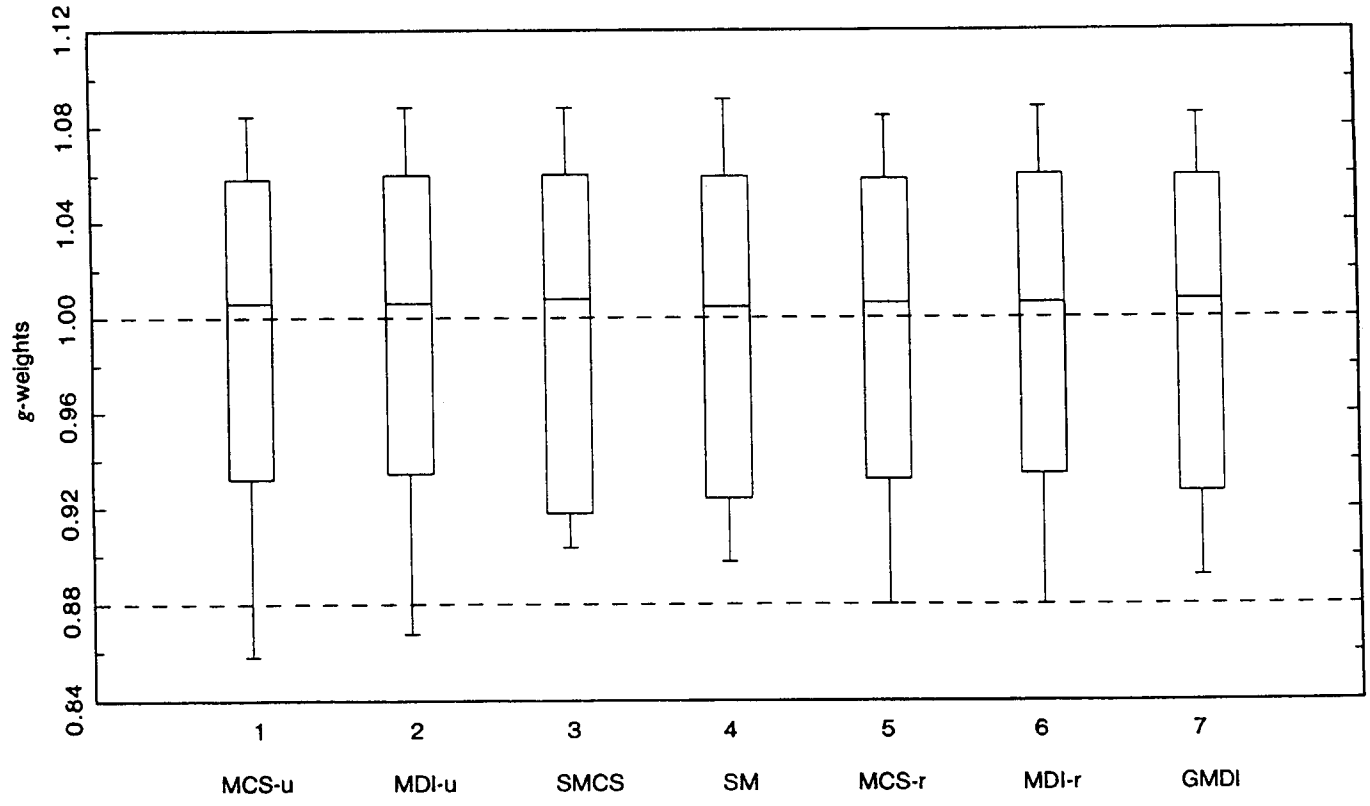

Figure 2. Box Plot: g-weights for Saskatoon FAMEX data ($L = 0.88$, $U = 1.12$)

Table 2a
Difference in Point Estimates and Precision Relative to Regression Estimator ($\alpha = .67, \beta = .8, \eta = .9, \epsilon = \delta = .01, \nu_{max} = 10$)
Regina: $L = 0.2$, $U = 5.0$ (Loose Bounds)

	Owned Dwelling		Furniture\Equipment	
	RD	RP	RD	RP
Family I				
SMCS	-0.043	1.047	0.001	1.032
SM	-0.036	1.032	-0.002	1.040
Family II				
MCS-r	-0.032	1.035	0.002	1.034
MDI-r	-0.033	0.991	-0.008	1.037
GMDI	-0.037	0.999	-0.004	1.041
	Ladies' Clothing		Men's Clothing	
Family I				
SMCS	0.015	0.931	0.009	0.952
SM	0.010	0.951	0.006	0.968
Family II				
MCS-r	0.011	0.950	0.008	0.964
MDI-r	0.007	0.911	-0.001	0.961
GMDI	0.009	0.940	0.002	0.968

Notes:
1. RD and RP denote respectively "relative difference" and "relative precision".
2. For the unrestricted (or no bounds) case, the corresponding measures for the raking (MDI-u) method relative to regression are (-0.034, 1.005), (-0.008, 1.049), (0.004, 0.968) and (0.002, 0.980) for the four study variables respectively.

Table 2b
Difference in Point Estimates and Precision Relative to Regression Estimator ($\alpha = .67, \beta = .8, \eta = .9, \epsilon = \delta = .01, \nu_{max} = 10$)
Regina: $L = 0.4$, $U = 2.5$ (Tight Bounds)

	Owned Dwelling		Furniture\Equipment	
	RD	RP	RD	RP
Family I				
SMCS	-0.056	1.100	0.012	1.000
SM	-0.055	0.992	0.017	0.919
Family II				
MCS-r	-0.048	1.073	0.008	0.952
MDI-r	-0.045	1.087	0.012	0.965
GMDI	-0.047	1.077	0.009	1.006
	Ladies' Clothing		Men's Clothing	
Family I				
SMCS	0.024	0.917	0.038	0.808
SM	0.025	0.917	0.024	0.801
Family II				
MCS-r	0.020	0.904	0.012	0.922
MDI-r	0.025	0.888	0.012	0.922
GMDI	0.021	0.938	0.018	0.917

Note: During the jackknifing procedure, the SM method failed to converge in ten iterations for four pseudo-replicates (out of a total of 111).

Table 2c
Difference in Point Estimates and Precision Relative to Regression Estimator ($\alpha = .67, \beta = .8, \eta = .9, \epsilon = \delta = .01, \nu_{max} = 10$)
Saskatoon: $L = 0.88$, $U = 1.12$

	Owned Dwelling		Furniture\Equipment	
	RD	RP	RD	RP
Family I				
SMCS	-0.001	1.001	-0.001	0.999
SM	-0.000	1.001	-0.000	0.999
Family II				
MCS-r	0.000	0.999	0.000	1.000
MDI-r	0.002	0.997	0.002	0.994
GMDI	-0.000	1.007	-0.000	0.990
	Ladies' Clothing		Men's Clothing	
Family I				
SMCS	0.000	1.013	-0.001	0.999
SM	-0.000	1.002	-0.000	0.998
Family II				
MCS-r	0.000	0.990	0.000	0.994
MDI-r	0.002	1.001	0.002	0.983
GMDI	0.000	0.977	-0.000	0.990

Notes:
1. For the unrestricted (or no bounds) case, the corresponding measures for the raking (MDI-u) method relative to regression are (0.002, 1.000), (0.002, 1.000), (0.002, 1.002) and (0.002, 0.995) for the four study variables respectively.
2. During the jackknifing procedure, the SM method failed to converge in ten iterations for two pseudo-replicates (out of a total of 94).

convergence problems for [0.475, 2.11]. For Saskatoon (Table 1), under the chosen bounds each method takes only one or two iterations. With ν_{max} as 10, as bounds are tightened to [0.92, 1.08], SM does not converge. At [0.93, 1.07], SMCS, MCS-r, and MDI-r have convergence problems, and finally at [0.96, 1.06], GMDI has problems.

4. DISCUSSION

Although numerical results for a few variables for two different domains considered in this paper are quite limited to draw general conclusions, the results based on a descriptive analysis are nevertheless interesting and may provide some indications which might be useful in practice. These can be summarized in the following observations. For loose bounds, all the restricted methods seem to perform almost at par with the regression method. However, for tight bounds, there seem to be a difference in point estimates and especially in estimated precision. This observation clearly needs further study in light of the fact that all methods are asymptotically equivalent to the regression method. A simulation study in this regard would be desirable. The recent study of Stukel, Hidiroglou, and Särndal (1996) sheds some light on this issue. Moreover, for tight bounds, there may not be convergence

under the specified number of iterations even if a solution exists. This problem may be more apparent in dealing with jackknife replicates. Therefore, caution should be exercised in choosing the maximum number of iterations for tight bounds. Finally, in practice, it is possible that even with minimal requirements on BCs and RRs, none of the calibration estimators converge within a reasonable number of iterations. In this situation, it would be of interest to investigate whether the (asymptotic) design consistency of calibration estimators could be preserved while allowing deviation from BCs. The idea of using ridge regression by Bardsley and Chambers (1984), although not in the design-based context, may be useful for this purpose. This problem is currently being investigated in collaboration with J.N.K. Rao.

APPENDIX

Here we provide computational algorithms for all seven methods of weight adjustment. These algorithms were used to write computer programs in GAUSS software for the numerical examples presented in this paper.

In all the methods, some form of the following expression denoted by the $\boldsymbol{n}$-vector $\boldsymbol{f}^{(\nu)}$, is used repeatedly for computing $c_k^{(\nu)}$ for $\nu = 1, 2, \ldots$.

$$\boldsymbol{f}^{(\nu)} \equiv \boldsymbol{X}(\boldsymbol{X}'\Gamma_{\nu-1}\boldsymbol{X})^{-1}(\tau_x - \hat{\tau}_x^{(\nu-1)}) \qquad (1)$$

where $\Gamma_{(\nu-1)}$ is an $n \times n$ diagonal matrix defined below in the algorithm for each method. Initially $\Gamma_0 = \text{diag}(\boldsymbol{h})$ and $\hat{\tau}_x^{(0)} = \sum \boldsymbol{x}_k h_k$.

A1. METHOD 1 (MCS-u)

The solution is non-iterative and is given in two steps as follows.

(i) Compute $f_k^{(1)}$, $k = 1$ to n from (1) by setting $\Gamma_{(\nu-1)} = \Gamma_0$.

(ii) Compute g_k as $1 + f_k^{(1)}$ and then $c_k^{\text{MCS-u}}$ as $h_k g_k$.

A2. METHOD 2 (MDI-u)

The solution is obtained iteratively by the following steps for $\nu = 1, 2, \ldots$.

(i) Set the tolerance level $\delta \geq 0$ for meeting BCs at some small value.

(ii) For the ν-th iteration, compute $f_k^{(\nu)}$, $k = 1$ to n, from (1) by setting $\Gamma_{\nu-1} = \text{diag}(c_k^{(\nu-1)})$.

(iii) For $\nu = 1, 2, \ldots$ compute $g_k^{(\nu)}$ as $g_k^{(\nu-1)}\exp(f_k^{(\nu)})$, $g_k^{(0)} = 1$ and then $c_k^{(\nu)}$ from $h_k g_k^{(\nu)}$.

(iv) Repeat steps (ii)-(iii) until the BCs are met up to the tolerance level δ or the number of iterations is at its maximum, ν_{max}. The last iteration gives $c_k^{\text{MDI-u}}$.

A3. METHOD 3 (SMCS)

The solution is obtained iteratively as follows.

(i) Set the RRs, *i.e.*, choose L and U, $L < 1 < U$.

(ii) Set the tolerance level $\epsilon \geq 0$ at a small value for meeting the RRs.

(iii) Choose a parameter α between 0 and 1 (*e.g.* 2/3) and set $L' = \alpha L + 1 - \alpha$, $U' = \alpha U + 1 - \alpha$. The default value of 1 for α is also allowed in which case $L' = L$, $U' = U$.

(iv) For the ν-th iteration with $g_k^{(0)} = 1$, define $\xi_k^{(\nu-1)} = (g_k^{(\nu-1)} - 1)/(L' - 1)$ if $g_k^{(\nu-1)} \leq 1$; $(g_k^{(\nu-1)} - 1)/(U' - 1)$ otherwise.

(v) Choose another parameter β between 0 and 1 (*e.g.*, 4/5). Set $q_k^{(\nu-1)} = 1$ if $\xi_k^{(\nu-1)} < 1/2$; $1 - \beta(\xi_k^{(\nu-1)} - 1/2)^2$ if $1/2 \leq \xi_k^{(\nu-1)} < 1$; $(1 - \beta/4)/\xi_k^{(\nu-1)}$ if $\xi_k^{(\nu-1)} \geq 1$ and then define for $\nu = 1, 2, \ldots$, $q_k^{[\nu-1]} = q_k^{(0)} \ldots q_k^{(\nu-1)}$ where $q_k^{(0)} = 1$. Note compounding of q-factors in defining $q_k^{[\nu-1]}$.

(vi) Compute $f_k^{(\nu)}$ from (1) by setting $\Gamma_{\nu-1} = \text{diag}(h_k q_k^{[\nu-1]})$, and $\hat{\tau}_x^{(\nu-1)} = \hat{\tau}_x^{(0)}$ for all ν.

(vii) Find $g_k^{(\nu)}$ as $1 + q_k^{[\nu-1]} f_k^{(\nu)}$ and then $c_k^{(\nu)}$ as $h_k g_k^{(\nu)}$.

(viii) Repeat steps (iv)-(vii) until the RRs are met up to the tolerance level ϵ or $\nu = \nu_{max}$. The last iteration gives c_k^{SMCS}. The value of β should remain the same at each iteration.

A4. METHOD 4 (SM)

This method consists of the following steps performed iteratively.

(i)-(ii) Same as in Method 3.

(iii) Choose parameters α, η, $0 < \alpha \leq \eta \leq 1$, (*e.g.*, $\alpha = 2/3$, $\eta = 9/10$) and define

$$L' = \alpha L + (1 - \alpha),\ U' = \alpha U + (1 - \alpha)$$
$$L'' = \eta L + (1 - \eta), U'' = \eta U + (1 - \eta).$$

The default option for α and η is 1 in which case $L' = L'' = L$, $U' = U'' = U$.

(iv) (Shrinkage). The $c_k^{(\nu)}$ from the ν-th iteration is shrunk to obtain $c_k^{(\nu)*}$ according to $c_k^{(\nu)*} = L'h_k$ if $c_k^{(\nu)} < L''h_k$; $U'h_k$ if $c_k^{(\nu)} > U''h_k$; $c_k^{(\nu)}$ otherwise. For $\nu = 0$, $c_k^{(0)} = c_k^{(0)*} = h_k$.

(v) (Minimization). Find $f_k^{(\nu)}$ from (1) by setting $\Gamma_{\nu-1} = \text{diag}(c_k^{(\nu-1)*})$ and $\hat{\tau}_x^{(\nu-1)} = \hat{\tau}_x^{(\nu-1)*}$.

(vi) Compute $g_k^{(\nu)}$ as $g_k^{(\nu-1)*}(1 + f_k^{(\nu)})$ where $g_k^{(\nu-1)*} = c_k^{(\nu-1)*}/h_k$ and then $c_k^{(\nu)}$ from $h_k g_k^{(\nu)}$.

(vii) Repeat steps (iv)-(vi) until the RRs are satisfied up to tolerance ϵ or $\nu = \nu_{max}$. The last iteration gives c_k^{SM}.

A5. METHOD 5 (MCS-r)

The iterative algorithm consists of the following steps.

(i) Set L and U.

(ii) Set the tolerance level $\delta \geq 0$ for meeting the BCs.

(iii) Compute $f_k^{(\nu)}$ from (1) by setting $\Gamma_{\nu-1} = \text{diag}(h_k a_k^{(\nu-1)})$ where $a_k^{(\nu-1)} = 1$ if $g_k^{(\nu-1)}$ was truncated to L or U, and 0 otherwise.

(iv) Set $g_k^{(0)} = 1$ and compute $g_k^{(\nu)}$ as $g_k^{(\nu-1)} + f_k^{(\nu)}$ if $L \leq g_k^{(\nu)} \leq U$; otherwise truncate $g_k^{(\nu)}$ to L or U as the case may be, and then $c_k^{(\nu)}$ as $h_k g_k^{(\nu)}$.

(v) Repeat steps (iii)-(iv) until BCs are met at the tolerance level δ or $\nu = \nu_{max}$. The last iteration gives $c_k^{\text{MCS-r}}$.

A6. METHOD 6 (MDI-r)

The iterative algorithm consists of the following steps.

(i)-(ii) Same as in Method 5.

(iii) Compute $f_k^{(v)}$ from (1) by setting $\Gamma_{v-1} = \text{diag}(c_k^{(v-1)} a_k^{(v-1)})$ where $a_k^{(v-1)}$ is defined as in Step (iii) of Method 5.

(iv) Set $g_k^{(0)} = 1$ and compute $g_k^{(v)} = g_k^{(v-1)} \exp(f_k^{(v)})$ if $L \le g_k^{(v)} \le U$; otherwise truncate $g_k^{(v)}$ to L or U as the case may be, and then $c_k^{(v)}$ as $h_k g_k^{(v)}$.

(v) Repeat steps (iii)-(iv) until BCs are satisfied at tolerance δ or $v = v_{max}$. The last iteration gives c_k^{MDI-r}.

A7. METHOD 7 (GMDI)

The iterative algorithm consists of the following steps.

(i)-(ii) Same as in Method 5.

(iii) Compute $f_k^{(v)}$ from (1) by setting $\Gamma_{v-1} = \text{diag}(h_k d_k^{(v-1)})$ where $d_k^{(v-1)}$ is analogous to $d_k^{(1)}$ of Section 2.7.

(iv) Using $\boldsymbol{x}_k' \boldsymbol{\lambda}^{(v)} = \boldsymbol{x}_k' \boldsymbol{\lambda}^{(v-1)} + f_k^{(v)}$, find $g_k^{(v)}$ from the formula for g_k given in Section 2.7, and then $c_k^{(v)}$ as $h_k g_k^{(v)}$.

(v) Repeat steps (iii)-(iv) until BCs are met at tolerance δ or $v = v_{max}$. The last iteration gives c_k^{GMDI}.

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