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A New Method to Reduce Unwanted Ripples and Revisions in Trend-Cycle Estimates From X-11-ARIMA

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ABSTRACT

The estimation of the trend-cycle with the X-11-ARIMA method is often done using the 13-term Henderson filter applied to seasonally adjusted data modified by extreme values. This filter however, produces a large number of unwanted ripples in the final or "historical" trend-cycle curve which are interpreted as false turning points. The use of a longer Henderson filter such as the 23-term is not an alternative for this filter is sluggish to detect turning points and consequently is not useful for current economic and business analysis. This paper proposes a new method that enables the use of the 13-term Henderson filter with the advantages of: (i) reducing the number of unwanted ripples; (ii) reducing the size of the revisions to preliminary values and (iii) no increase in the time lag to detect turning points. The results are illustrated with nine leading indicator series of the Canadian Composite Leading Index.

KEY WORDS: Trend-cycle; X-11-ARIMA; Turning points; Leading economic indicators.

1. INTRODUCTION

The estimation of the trend-cycle with the X-11-ARIMA seasonal adjustment method (Dagum 1980, 1988) as well as the U.S. Bureau of the Census X-11 variant (Shiskin, Young and Musgrave 1967) is done by the application of linear filters due to Henderson (1916). These Henderson filters are applied to seasonally adjusted series where the irregulars have been modified to take into account the presence of extreme values. The length of the filters is automatically selected on the basis of specific values of noise to signal ratios (I/S) being the most commonly chosen the 13-term filter.

The problem of trend-cycle estimation has attracted the attention of several authors, among others, Rhoades (1980); Cholette (1981, 1982); Kenny and Durbin (1982); Castles (1987); Dagum and Laniel (1987); Cleveland, Cleveland, McRae and Terpenning (1990); Wallgren and Wallgren (1990); Gray and Thomson (1990); Findley and Monsell (1990); Scott (1990); and Kenny (1993). Nevertheless, most statistical agencies (excepted the Australian Bureau of Statistics) concentrate their publications on seasonally adjusted series and only very few provide some sort of information on the trend-cycle, usually under the form of graphs.

There are several reasons for limiting the publication of trend-cycle estimates. In the majority of the cases, the seasonally adjusted data are already smooth enough as to be able to provide a clear signal of the short-term trend. But for highly volatile series where further smoothing is required the main objections for trend-cycle estimation are: (1) the size of the revisions of the most recent values (generally much larger than for the corresponding seasonally adjusted estimates) and (2) the presence of short cycles or ripples (9 and 10 months cycles) in the final trend-cycle curve when the 13-term Henderson filter is applied. On this regard, Kenny (1993) has argued that the presence of ripples in the final estimates of the trend-cycle leads to a large number of false turning points, making the 13-term filter unsuitable for monitoring turning points. He has proposed the use of the 23-term Henderson filter with the object of obtaining a much smoother trend. However, it is well known that this longer filter is sluggish to detect turning points and, hence not useful for current economic and business analysis. For this latter viewpoint, the 13-term filter is preferable but it produces ripples which can be interpreted as false turning points (an unwanted property).

The main purpose of this study is to introduce a new method by which the 13-term Henderson filter can be used with the advantages of: (1) reducing the number of unwanted ripples, (2) reducing the size of the revisions made to the most recent estimates when new observations are added to the series, and (3) not increasing the time lag to detect turning points.

2. TREND-CYCLE CASCADE FILTERS

The 13-term Henderson filter is the most often selected and combined with the standard seasonal filters (5- and 7-term moving averages) produces a symmetric cascade filter for final or central values (at least four years from each end of the series) with a gain as exhibited in Figure 1.

Figure 1 also shows the gain functions of other filter convolutions, namely: (1) short seasonal filters with the 9-term Henderson filter and (2) long seasonal filters with the 23-term Henderson filter. It is apparent that cycles of 9 and 10 months (in the 0.08-0.16 frequency band) will not be suppressed by any of the cascade filters, particularly,

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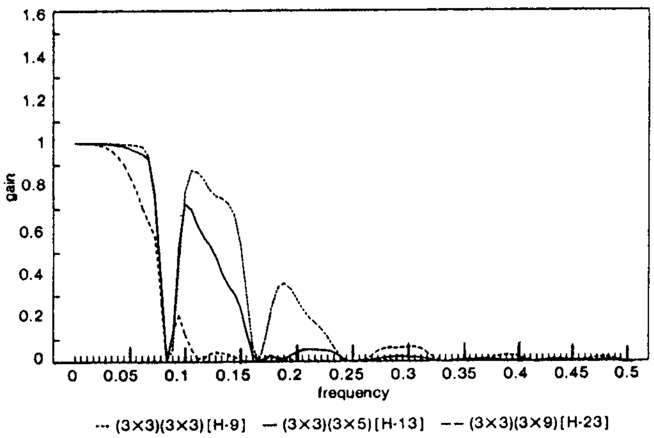

Figure 1. Trend-cycle symmetric cascade filters.

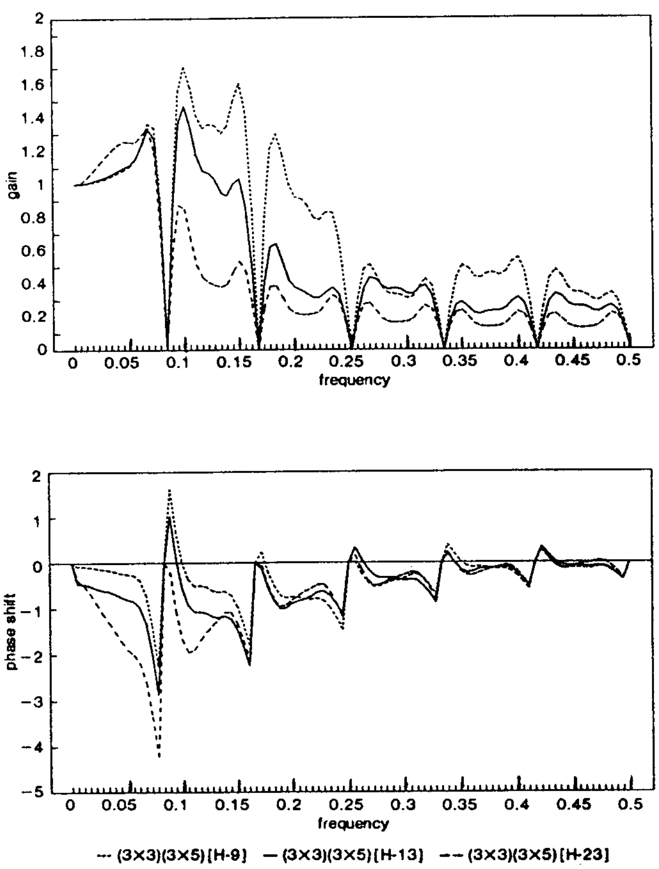

Figure 2. Trend-cycle concurrent cascade filters. Standard seasonal m.a. combined with three Henderson filters.

those using the 9- and 13-term Henderson filters. In fact, the symmetric trend-cycle cascade filter that results from the 9-term Henderson passes about 90% of the power of these short cycles; 72% and 21% are passed by the 13- and 23-term Henderson filters, respectively.

For the concurrent trend-cycle filters which are applied to the last available observation, the peak reached at the frequency band corresponding to 9 and 10 months cycles is even larger (see Figure 2). Furthermore, all these asymmetric filters introduce phase shift, being near to two months for the 23-term (the largest), one month for the 13-term, and one-half month for the 9-term filter.

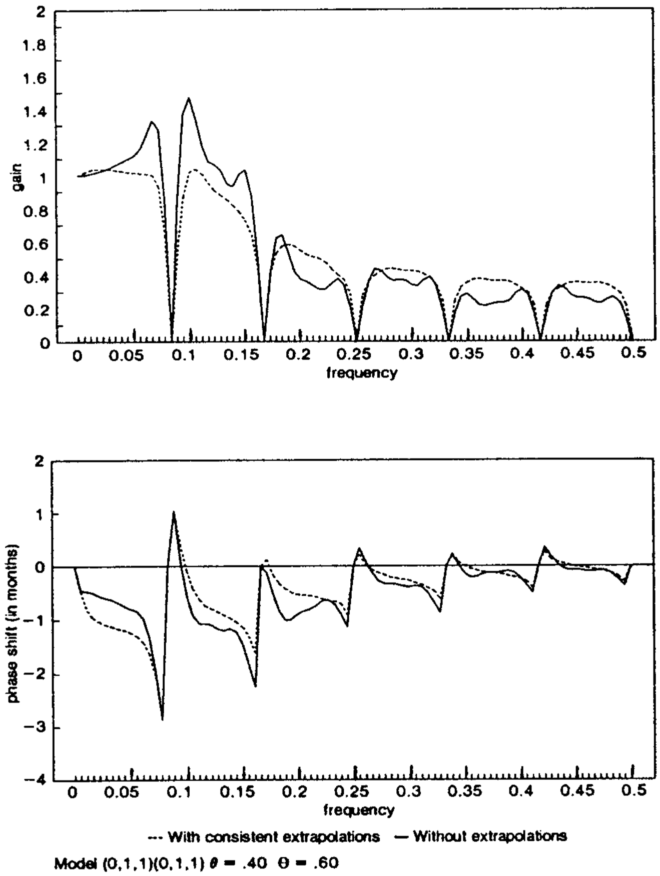

Figure 3. Trend-cycle concurrent cascade filters, (3 × 3) (3 × 5) [H − 13], with and without ARIMA extrapolations.

Figure 3 shows how the use of ARIMA extrapolations makes the gain of the concurrent cascade filters (using the 13-term Henderson) to resemble the symmetric one although at the expense of a small increase in phase shift. The extrapolations are from an ARIMA model (0,1,1) $(0,1,1)_S$ where the regular moving average parameter is $\theta = 0.40$ and the seasonal moving average parameter is $\Theta = 0.60$.

Although not shown for space reasons,the gain and phase shift of this trend-cycle concurrent filter fall between the other two combinations.

When ARIMA extrapolations are used, the gain of the concurrent filter converges very fast to that of the final. Dagum and Laniel (1987) show that after three more observations are added to the series, the gain of the asymmetric trend-cycle filter is very close to the symmetric one. The properties of these filters are also extensively discussed in Dagum, Chhab and Chiu (1993, 1996).

The presence of ripples in the final trend-cycle estimates will be produced by the 13-term Henderson filter only if some power is present in the input to the filter at the 0.08-0.16 frequency band. The input to the filter is the seasonally adjusted data with extreme values replaced.

In most empirical cases, the presence of unwanted ripples occurs in periods of high volatility when the observed data are mostly influenced by outliers which can be falsely interpreted as turning points. Although the seasonally adjusted series are modified by extreme values, there is a need for further smoothing which can be done either by applying a longer Henderson filter or by being stricter with the replacement of outliers. Since we want to keep the advantage of a short filter to detect turning points faster, the latter approach is the one followed here.

In the current procedure, the default sigma limits for the replacement of extreme values are ± 1.5 sigma and ± 2.5 sigma. Values greater than ± 2.5 sigma receive a zero weight and those smaller than ± 1.5 sigma a weight of one (full weight). Values falling within the boundaries are assigned a linearly graduated weight between zero and one.

3. A NEW METHOD

The new method here proposed, basically consists of: (1) extending a smoothed seasonally adjusted series (modified by extreme values with zero weight) with ARIMA extrapolations, and (2) applying the 13-term Henderson filter to the extended series using stricter sigma limits for the identification and replacement of extreme values.

Experimentation with real data showed that the power spectrum of the seasonally adjusted series at the 0.08-0.16 frequency band was drastically reduced only when strict sigma limits such as ± 0.7 sigma and ± 1.0 sigma were used. Hence, when applying the 13-term Henderson filter, the trend-cycle curve did not exhibit unwanted ripples while still maintaining its good property of rapid detection of turning points. Under the assumption of normality, these new sigma limits imply that 48% of the irregulars will be modified, 32% will get zero weight and will be replaced by the mean value and 16% will get graduated weights from zero to one.

The extension of the smoothed seasonally adjusted series with ARIMA extrapolations is needed to reduce the size of the revisions for the most recent estimates of the trend-cycle.

The implementation of this new procedure in the context of the X-11-ARIMA and X-11 methods must be done in two steps as follows:

(1) Produce the best seasonally adjusted series selecting appropriate options for the estimation of the components, that is, seasonality, trend-cycle, trading-day variations and Easter effects plus permanent or temporary priors, if applicable. The seasonally adjusted values are printed in Table D11. The seasonally adjusted series is modified by extreme values with zero weights using the default sigma limits and printed in Table E2. When the estimates of the published seasonally adjusted series for the current year are modified according to some revision practices, then this published revised series should be resubmitted to the X-11-ARIMA program to obtain the corresponding output shown in Table E2.

(2) The output from Table E2 is extended with one year of extrapolations from an ARIMA model. The ARIMA model found adequate with many real series is the (0,1,1) (0,0,1) model. Although the output from Table E2 does not contain seasonality, the seasonal moving average parameter (often of very small value) is needed to correct for some sort of seasonal autocorrelation in the data. The extended series is then run with the X-11-ARIMA program using the Summary Measures option and requesting strict sigma limits ($\pm 0.7\sigma$ and $\pm 1.0\sigma$) and the 13-term Henderson filter. The new trend-cycle estimates are printed in Table D12.

4. EMPIRICAL RESULTS

The new method for trend-cycle estimation is tested with nine leading indicator series of the Canadian Composite Leading Index. In the so called "filtered" version of the Canadian Composite Leading Index published by Statistics Canada, each of the components series as well as the Index itself are smoothed applying to the seasonally adjusted data asymmetric filters based on ARMA models developed by Rhoades (1980). The spectral properties of these ARMA trend-cycle filters are similar to those of the end point of the 9- 13- and 23-term Henderson filters depending on the ARMA model chosen (see Cholette 1982). (Although a comparison with the ARMA filters is not done in this paper, it is likely that the new approach will also give improved results.) Most of the series are highly volatile and all lead at turning points in the business cycle. The series are:

TSE300 Stock Price Index (TSE300)
House Spending Index (HSI)
Money Supply (M1)
Business and Personal Services Employment (BPSE)
Average Workweek in Manufacturing (AWM)
Retail Sales of Furniture and Appliances (RSFA)
Retail Sales of Durable Goods (RSDG)
New Orders for Durable Goods (NODG)
Shipments to Inventories Ratio (SIR).

The advantages of the new procedure versus the currently available in X-11-ARIMA are evaluated as follows.

4.1 Reduction of Ripples in the Final Trend-Cycle Estimates

To calculate the reduction of ripples we first introduce the definition of a turning point within the context of trend-cycle data. A turning point is generally defined as a point in time t when a series, say Y_t is larger (smaller) than or equal to the preceding k and subsequent m observations of the series. That is,

$$Y_{t-k} \leq \ldots \leq Y_{t-1} > Y_t \geq Y_{t+1} \geq \ldots \geq Y_{t+m}$$

defines a downturn and

$$Y_{t-k} \geq \ldots \geq Y_{t-1} < Y_t \leq Y_{t+1} \leq \ldots \leq Y_{t+m}$$

defines an upturn.

From the viewpoint of seasonally adjusted series and trend-cycle data, there is no general consensus for what values of k and m, a turning point has occurred. Rhoades (1980) defines a turning point for $k = 1$ and $m = 0$; Wecker (1979) defines a turning point to be the second of two (or more) successive declines or increases, *i.e.*, for $k = 2$ and $m = 2$; Zellner, Hong and Min (1991), LeSage (1991) and Pfeffermann and Bleuer (1992) have chosen $k = 3$ and $m = 0$. These definitions do not necessarily correspond to those of cyclical turning points for business cycle analysis but any one can be useful to calculate the number of unwanted ripples as long as two turning points (a downturn and an upturn) occur within a period of ten months or less. We use here the turning point definition for which $k = 3$ and $m = 0$ given the smoothness of the trend-cycle data.

Table 1 shows the number of ripples present in the trend cycle estimates from the standard and the modified 13-term Henderson filter for the period January 1981-December 1993.

Table 1

Number of Unwanted Ripples in the Trend-Cycle Data Using the 13-Term Henderson Filter for the Period 1981-1993

Series	Standard Procedure	Modified Procedure
NODG	9	2
HSI	8	4
RSDG	8	4
BPSE	8	5
AWM	7	1
SIR	5	1
TS300	4	2
M1	4	2
RSFA	4	0

The results show that the reduction is larger for those series with a large number of ripples and significant in all cases.

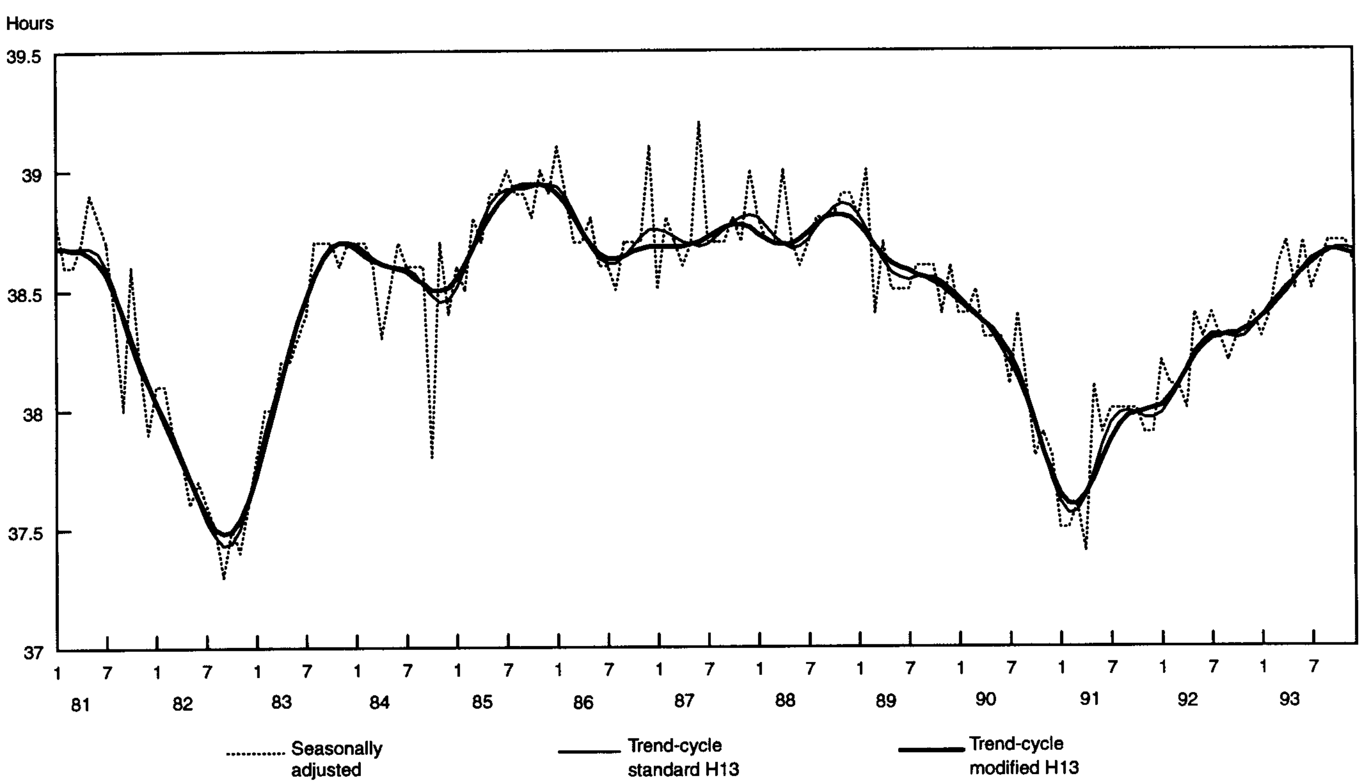

Figure 4. Average work week manufacturing.

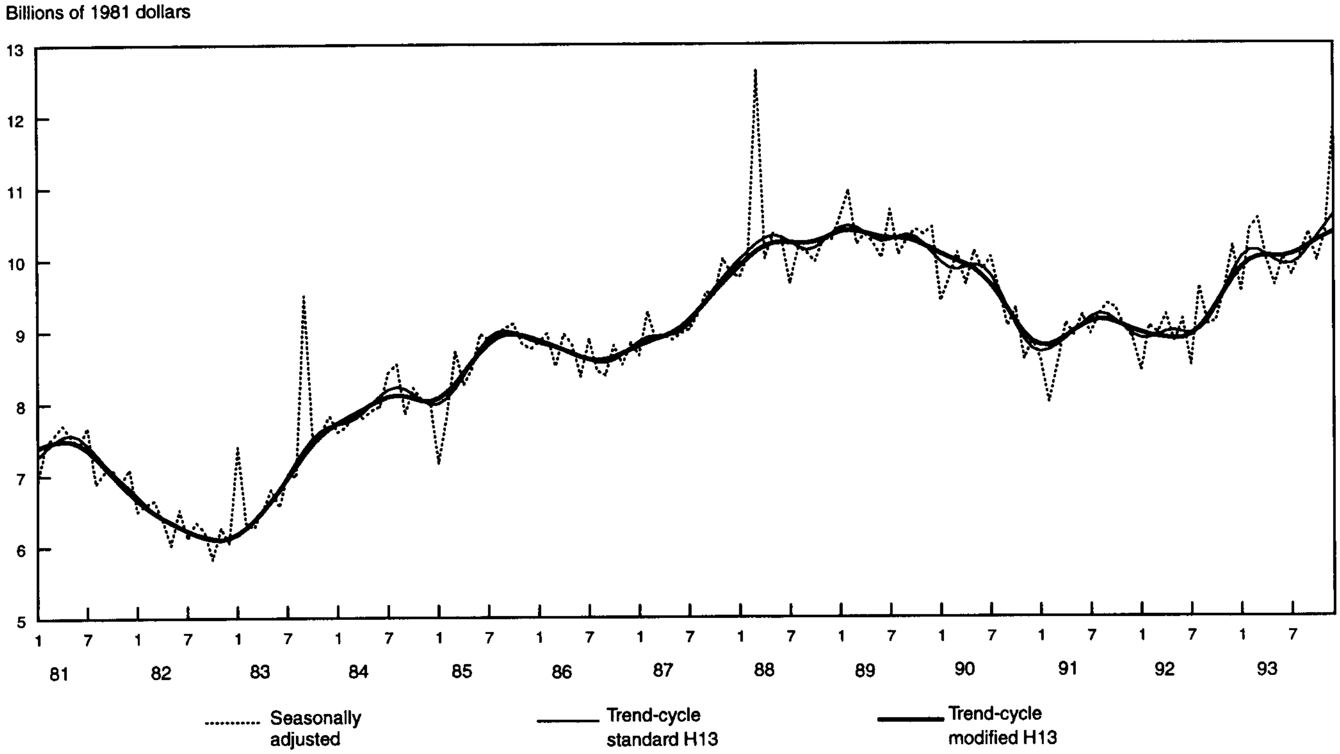

Figure 5. New orders for durable goods.

For illustrative purposes, Figures 4 and 5 for AWM and NODG respectively, exhibit the seasonally adjusted values and the trend-cycle data of both the standard and modified procedures. It is apparent that the new method reduces the ripples in the trend-cycle data with respect to those shown by the standard procedure. In fact, the modified trend-cycle data resembles that of the 23-term Henderson filter but with larger penetration into peaks and troughs of cycles of long duration.

4.2 Turning Point Detection

It is important that the reduction of ripples in the final estimates of the trend-cycle is not achieved at the expense of increasing the lag in detecting turning points which is the main limitation of the 23-term Henderson filter.

To study the revision path of the trend-cycle for any given point in time, the estimates were computed for all end points and previous time points. The revision path of the modified trend-cycle values showed that the identification of cyclical turning points is done with an average lag similar to the standard approach. Depending on the series, the lag was either equal or plus minus one month. For illustrative purposes, Figures 6a. exhibits the revision path of the modified trend-cycle values of New orders for durable goods for the cyclical turning point of February 1991. Successive updates are carried out using data up to March 1991, April 1991 and so on. The turning point is recognized in April, after 2 months whereas it takes

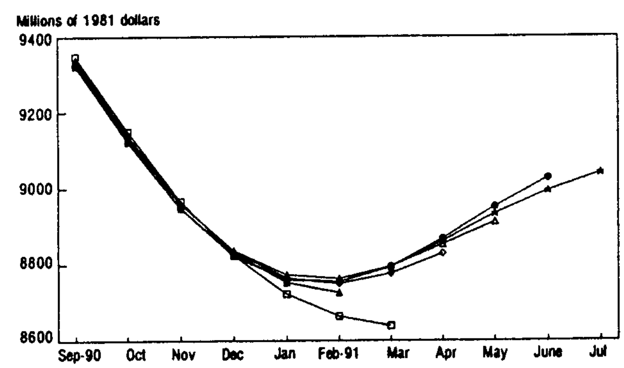

Figure 6a. New orders for durable goods. Trend-cycle modified H13 revisions path.

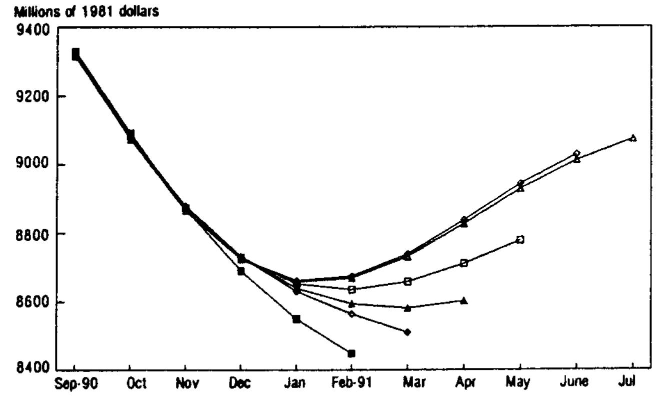

Figure 6b. New orders for durable goods. Trend-cycle standard H13 revisions path.

3 months for the standard procedure as exhibited in Figure 6b. Furthermore, it is shown that successive revisions of the trend-cycle estimates keep generally very close to the final values. The lines which protude, indicating a large revision, can be explained in terms of the underlying data which seem to indicate an increasing decline contradicted by the following values.

Figures 7a. and 7b. for the Average work week in manufacturing reveal that the turning point February-March 1991 is detected three months later by both procedures.

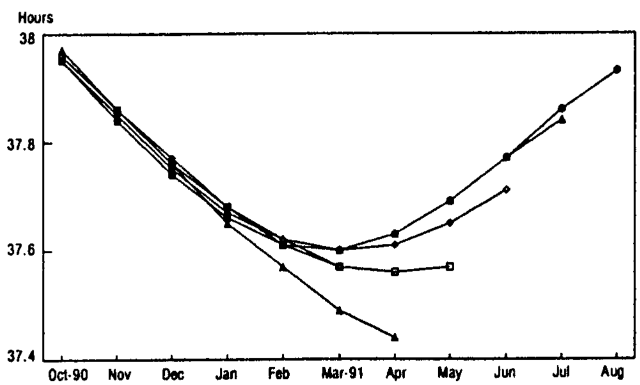

Figure 7a. Average work week manufacturing. Trend-cycle modified H13 revisions path.

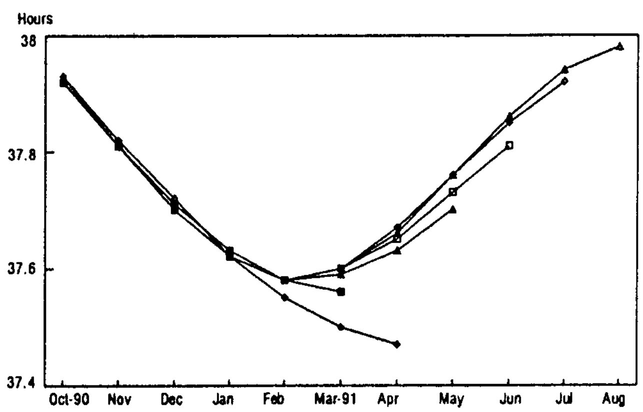

Figure 7b. Average work week manufacturing. Trend-cycle standard H13 revisions path.

4.3 Reduction of Revisions of Concurrent Trend-Cycle Estimates

Another important aspect to take into consideration is to reduce the total revision of the most recent estimate of the trend-cycle which is of preliminary character. Theoretically, the final trend-cycle value is obtained after the series is extended with four years of data but the size of the revisions is negligible after three more months.

Table 2 shows the mean absolute percent revision of the concurrent trend-cycle estimates over a four year period from January 1988 untill December 1991. The results indicate that for six of the nine cases analyzed the total revisions of the concurrent trend-cycle values using the modified procedure are much smaller compared to the standard, only for two series they are slightly larger.

Table 2

Mean Absolute Percent Total Revision of Concurrent Trend-Cycle Values Using the 13-Term Henderson Filter

Series	Standard Procedure (1)	Modified Procedure (2)	Ratio (2)/(1)
NODG	1.55	1.10	0.73
RSFA	0.62	0.47	0.76
RSDG	0.77	0.62	0.80
SIR	0.87	0.70	0.80
AWM	0.13	0.12	0.92
TS300	1.12	1.07	0.95
M1	0.35	0.35	1.00
HSI	2.09	2.20	1.05
BPSE	0.40	0.42	1.05

5. CONCLUSION

This paper introduced a new method for trend-cycle estimation which enables the use of the 13-term Henderson filter with the advantages of: (i) reducing the number of unwanted ripples in the final trend-cycle curves, (ii) reducing the size of the revisions to preliminary concurrent values, and (iii) not increase the time lag in turning point detection.

The new method basically consists of extending a smoothed seasonally adjusted series (modified by extreme values with zero weight) with one year of ARIMA extrapolations, and then applying the 13-term Henderson filter using strict sigma limits for the identification and replacement of outliers.

The procedure is illustrated with nine leading indicator series of the Canadian Composite Leading Index and the results are highly satisfactory.

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