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Use of Capture-Recapture Techniques to Estimate Population Size and Population Totals when a Complete Frame is Unavailable

K.H. POLLOCK, S.C. TURNER and C.A. BROWN[1]

ABSTRACT

We present a formal model based sampling solution to the problem of estimating list frame size based on capture-recapture sampling which has been widely used for animal populations and for adjusting the US census. For two incomplete lists it is easy to estimate total frame size using the Lincoln-Petersen estimator. This estimator is model based with a key assumption being independence of the two lists. Once an estimator of the population (frame) size has been obtained it is possible to obtain an estimator of a population total for some characteristic if a sample of units has that characteristic measured. A discussion of the properties of this estimator will be presented. An example where the establishments are fishing boats taking part in an ocean fishery off the Atlantic Coast of the United States is presented. Estimation of frame size and then population totals using a capture-recapture model is likely to have broad application in establishment surveys due to practicality and cost savings but possible biases due to assumption violations need to be considered.

KEY WORDS: Incomplete frames; Capture-recapture sampling; Angler surveys; Telephone surveys; Access surveys.

1. INTRODUCTION

In classical sampling theory it is assumed that a complete frame exists. There is, at least conceptually, a complete list of population units. It is then possible to draw a probability sample from the population. Estimators of population parameters such as mean or total then have known properties and are easily studied theoretically or numerically. Books on sampling theory such as Cochran (1978) concentrate on this situation and give properties of estimators for common sampling designs such as simple random sampling, stratified random sampling and multistage (cluster) sampling.

In practice in surveys of establishments or businesses a complete frame may not exist. Lists of establishments kept by professional associations or government agencies are often incomplete. One approach to tackling this problem is to use the multi-frame approach originally developed by Hartley (1962, 1974). Examples of this approach are the National Agricultural Statistics Service (USDA) farm surveys (Vogel and Kott 1993). These surveys use an incomplete list frame of farms plus an area frame where all farms within a sample unit are enumerated. Therefore the list frame is incomplete while the area frame is conceptually complete. (There is a list of all area units and within each area unit theoretically all farms could be enumerated.)

There are some situations, however, where it may not be possible to use an area frame for practical reasons. All that the researcher may have available may be several incomplete list frames of establishments. The usual approach in this situation is to merge all the incomplete lists and ignore any remaining incompleteness. Depending on the degree of incompleteness remaining there could be serious negative bias on estimates of population size and population total.

Later we present a formal model based sampling solution to this problem based on capture-recapture sampling. Capture-recapture sampling models are widely used in sampling animal populations (Seber 1982) and also for adjusting the U.S. census for undercoverage (Feinberg 1992). In the simplest case of two incomplete lists we consider ''marked'' units to be those which occur on both lists and unmarked units to be those which do not occur on both lists. It is easy to estimate total frame size using the Lincoln-Petersen estimator (Seber 1982, p. 59). This estimator is model based with a key assumption being independence of the two lists. Once an estimator of the population size has been obtained it is possible to obtain an estimator of population total for some characteristic if a sample of units has that characteristic measured.

The usual estimator of a population total for simple random sampling without replacement is

$$\hat{Y} = N\bar{y}, \tag{1.1}$$

where N is known and $\bar{y}$ is the mean of the sample, see for example Cochran (1978, p. 21) . The variance of $\hat{Y}$ is given by

$$\mathrm{Var}(\hat{Y}) = N^2\mathrm{Var}(\bar{y}), \tag{1.2}$$

[1] K.H. Pollock, North Carolina State University, Raleigh, NC 27695; S.C. Turner and C.A. Brown, National Marine Fisheries Service, Miami, FL 33149, U.S.A.

where

$$\mathrm{Var}(\bar{y}) = \frac{S^2}{n}\left(\frac{N-n}{N}\right),$$

S^2 is the population variance and $(N - n/N)$ is called the finite population correction factor. The estimator (1.1) is also an unbiased estimator of the population total.

Here our estimator is

$$\hat{Y} = \hat{N}\bar{y}, \tag{1.3}$$

where $\hat{N}$ is obtained from the capture-recapture method.

This means the properties of the estimator (1.3) are more difficult to evaluate because both $\hat{N}$ and $\bar{y}$ are random variables unlike in estimator (1.1) where N is a known quantity. The estimated variance of $\hat{Y}$ here is given by

$$\widehat{\mathrm{Var}}(\hat{Y}) = (\hat{N})^2\,\widehat{\mathrm{Var}}(\bar{y}) + (\bar{y})^2\,\widehat{\mathrm{Var}}(\hat{N}) + \widehat{\mathrm{Var}}(\bar{y})\,\widehat{\mathrm{Var}}(\hat{N}), \tag{1.4}$$

assuming that $\bar{y}$ and $\hat{N}$ are independent and using an exact result due to Goodman (1960). The estimator (1.3) is only an unbiased estimator if $\hat{N}$ and $\bar{y}$ are unbiased estimators of the population size and population mean respectively which is not usually the case in practice. We discuss the estimator (1.3) in the large pelagic fishery survey example in Section 3.

The remainder of the paper is structured as follows. In Section 2 we review the capture-recapture literature to give an overview of the types of models available. In Section 3 we present an example of a sample survey of fishing boats. (We consider a boat analogous to a business establishment). While this example has some unique features we believe it has many features common to other establishment surveys. In the final discussion section we summarize the strengths and weaknesses of using the capture-recapture approach to estimating frame size in establishment surveys. Many of our ideas will require further research.

2. A BRIEF REVIEW OF CAPTURE-RECAPTURE MODELS

It is obviously beyond the scope of this manuscript to review the extensive capture-recapture literature. For more information we recommend Seber (1982), White *et al.* (1982), Pollock *et al.* (1990) and Pollock (1991). Pollock (1991) is a review paper and a good lead into the literature and our treatment in this section follows it very closely. The other references are books and monographs for the serious reader with more time.

Here we briefly discuss the Lincoln-Petersen model for two samples, more general closed population and open population models for more than two samples, and finally a method which combines closed and open population models in one sampling design. Pollock *et al.* (1990, p. 9) presents a flow chart which shows an overview of the models and how they relate to each other.

2.1 The Lincoln-Petersen Model

This is the oldest, simplest and best known capture-recapture model dating back to Laplace, who used it to estimate the population size of France. It was first used in fisheries by Petersen around the turn of the century. An excellent detailed discussion of this model is given by Seber (1982, Chapter 3).

In the original fisheries setting the method can be described as follows. A sample of M fish is caught, marked, and released. Later a second sample of n fish is captured, of which m are marked. An intuitive derivation of the estimator follows from equating the proportions marked in the sample and the population,

$$m/n = M/N, \tag{2.1}$$

which gives

$$\hat{N} = Mn/m. \tag{2.2}$$

A modified estimator with less bias in small samples is due to Chapman (1951) and is given by

$$\hat{N}_c = \left[(M+1)(n+1)/(m+1)\right] - 1. \tag{2.3}$$

An estimate of the variance of $\hat{N}_c$ is given by

$$\widehat{\mathrm{Var}}(\hat{N}_c) = \frac{(M+1)(n+1)(M-m)(n-m)}{(m+1)^2(m+2)}. \tag{2.4}$$

See for example Seber (1982, p. 60).

The crucial assumptions of this model are:

(a) The population is completely closed to additions and deletions,

(b) all the fish are equally likely to be captured in each sample, and

(c) marks are not lost or overlooked.

The assumption about closure can be weakened, but even for a completely open population, where this estimator does not apply, a modification of the Lincoln-Petersen estimator is used. The assumption of equal catchability causes problems in most applications. There may just be inherent variability (heterogeneity) in capture probabilities of individual animals due to age, sex or other factors. There may also be a response to initial capture (trap response). In the next section, we consider closed population models with more than two samples that allow for time variation as well as heterogeneity and trap responses in the animals' capture probabilities. The loss or overlooking of marks can be serious. One way to estimate mark loss is to use two marks (Seber 1982, p. 94).

2.2 Closed Population Models

Closed population models require the assumption that no births, deaths, or migration in or out of the population occur between sampling periods. Therefore, these models are generally used for studies covering relatively short periods of time (*e.g.*, trapping every day for 5 consecutive days). Capture histories for every animal caught are the data needed for obtaining estimates under these models. Important early references are Schnabel (1938) and Darroch (1958), who considered models that assumed equal catchability of animals in each sample.

A set of models that allow capture probabilities to vary due to heterogeneity, (h), trap response (b), time variation (t), (*i.e.*, capture probability for time i differs from that for time j) and all possible two- and three-way combinations of these factors is now available. The eight models $[M(o)$, $M(h)$, $M(b)$, $M(bh)$, $M(t)$, $M(th)$, $M(tb)$, $M(thb)]$ were first considered as a set by Pollock (1974) and were more fully developed by Otis *et al.* (1978), White *et al.* (1982), and Pollock and Otto (1983). Otis *et al.* (1978) provided a detailed computer program, CAPTURE, for use with their monograph. An updated version provides estimates for seven of the eight models and a model selection procedure that aids the biologist in choosing a model. The model selection procedure is based on a variety of goodness-of-fits tests. Recently, Menkins and Anderson (1988) have emphasized that the model selection procedure is poor for small populations, unless the capture probabilities are unrealistically high.

2.3 Open Population Models

In many capture-recapture studies, it is not possible to assume the population is closed to additions and permanent deletions. The basic open population model suitable for this situation is the Jolly-Seber model (Jolly 1965; Seber 1965; Seber 1982, p. 196). The Jolly-Seber model allows estimation of population size at each sampling time as well as estimation of survival rates and birth numbers between sampling times. Migration cannot be separated from the birth and death processes without additional information.

The Jolly-Seber model requires the following assumptions:

(a) Every animal present in the population at a particular sampling time has the same probability of capture,

(b) every marked animal present in the population immediately after a particular sampling time has the same probability of survival until the next sampling time,

(c) marks are not lost or overlooked,

(d) all emigration is permanent, and

(e) all samples are instantaneous, and each release is made immediately after the sample.

Assumptions (a), (c), and (e) were required under the basic Lincoln-Petersen model described in Section 2.1. Only marked animals are used to estimate survival rates so that, strictly, we do not need to assume equality of marked and unmarked survival rates. In practice however, the biologist will want to use the survival rate estimates to refer to the whole population. The Jolly-Seber model allows for some animals to be lost on capture and hence not returned to the population. The Jolly-Seber model also requires that all emigration is permanent. If animals emigrate and then return to the population this causes so called temporary emigration which is a serious assumption violation and causes major bias in population size estimates.

2.4 Combination of Closed and Open Models

Pollock (1982), Pollock *et al.* (1990) and Kendall (1992) discuss sampling methods which allow the use of closed and open models in one design. One advantage of these methods is that it is possible to allow for unequal catchability whereas in the traditional Jolly-Seber model it is not possible to allow for unequal catchability. They also have the advantage of allowing for temporary emigration of animals.

2.5 Applications of Capture-Recapture Models

Capture-recapture models have obviously been widely applied to wildlife and fishery populations. A variety of novel nonbiological applications of capture-recapture methods have also now appeared. Many authors have applied capture-recapture to estimating the census undercount. (See Feinberg (1992) for a complete bibliography). Cowan, Breakey, and Fischer (1986) used it to estimate the number of homeless people in a city. Greene (1983) has used the method to estimate demographic parameters on criminal populations. Wittes (1974) and Wittes, Colton, and Sidel (1974) have used capture-recapture to estimate numbers of people with illnesses from hospital and other lists. The sampling of elusive human populations using cluster sampling, network sampling, and capture-recapture sampling was discussed by Sudman, Sirken and Cowan (1988).

3. USE OF CAPTURE-RECAPTURE MODELS IN THE LARGE PELAGIC SURVEY

The Large Pelagic survey is an angler survey conducted by the National Marine Fisheries Service using a telephone-access survey design. A sample of fishing boat owners on a list are telephoned to obtain fishing effort (*i.e.*, number of fishing trips in a period) information. Catch per unit effort (*i.e.*, catch per trip) information is obtained from a second sample of boat owners at access points at completion of their fishing trips. The information from the two surveys is combined to estimate total effort and total catch of important species such as Bluefin Tuna.

A serious problem with this survey is that the list of boat owners used in the telephone survey is very incomplete. Therefore, classical sampling theory which assumes a complete frame of known size (N) is inadequate and has to be modified. The current method of estimating the size of the fishing boat list frame involves combining two lists, (a telephone list with a dockside list) and using the Lincoln-Petersen model. There are questions about whether this is the best approach. For example, it might be possible to combine more than two lists and if so then we could use the closed or open population models reviewed in Sections 2.2 and 2.3. However, we defer those questions and begin by reviewing and evaluating the current method as an example to illustrate the potential usefulness of the approach to other establishment surveys.

3.1 The Lincoln-Petersen Model

3.1.1 Estimation of Frame Size (N)

Under the current method the "marked" boats (M) are those on the master list which is primarily derived from previous telephone interviews. The recapture sample is carried out dockside at gas pumps and the total number of boats intercepted (n) is checked to see which ones are "marked" (m) (*i.e.*, on the original master list). Equation 2.3 can then be used to provide an estimator of the frame size (N). Let us now consider the assumptions of this model and what effect violations might have on the bias of the estimator of N.

Closure

This assumption is likely to be violated. Fishing boats may be on the master list and then no longer take part in the fishery (losses). New fishing boats may join the fishery while it is in progress (gains). Ideally a separate estimate of frame size should be obtained for each two week time period. The advantage of using the Lincoln-Petersen closed model estimator is its simplicity and practicality. Biases in the estimator due to lack of closure could be either positive or negative.

Currently it is not known how the fishing fleet size is likely to change during the fishing season. A multiple capture-recapture sampling design would allow use of the Jolly-Seber model to estimate the fleet size during each period. Examination of these estimators and the survival rate and recruitment number estimators will enable us to evaluate the validity of the closure assumption. At the moment we can only make conjectures.

Equal Catchability

Violation of the assumption of equal catchability may be due to either inherent heterogeneity of capture probabilities between individuals or "trap response" where individuals that are marked have higher or lower capture probabilities than unmarked individuals. In either situation when the individuals on the lists are fishing boats we believe there is a potential for heterogeneity of capture probabilities among fishing boats. If heterogeneity is operating across both samples, individuals "caught" on the first list will tend to be those with high capture probabilities and therefore they will more likely to be "caught" again on the second list. This means that the proportion marked in the second sample (list) will be too high and the estimator of N will be negatively biased. Note that this intuitive argument makes clear it is not heterogeneity per se which is the problem but the positive correlation of capture probabilities between the two samples. Another way of stating the equal catchability assumption is that capture probabilities in the two samples are independent. One method of attempting to achieve independence of the capture probabilities in the two samples is to use totally different sampling schemes for the two samples. This is why we recommended earlier that one sample list be based on the telephone interviews and the other on dockside interviews. However, we do suspect that there is still another heterogeneity and lack of independence in capture probabilities. We believe that fishing boats which take a very active part in the fishery are more likely to be on any lists gathered (telephone or dockside). This heterogeneity will cause a negative bias on the estimate of frame size but we have no idea of the degree of this negative bias. A more complete discussion of heterogeneity and independence of samples is given by Seber (1982, p. 86).

Marks Lost or Overlooked

The situation here is a little confusing. At first one might think that in this application there is not a way that a mark could be lost or overlooked. However, this assumes that all boats have distinct names or that if boats do have the same name there is additional information like captain's name which makes all individuals on the lists unique. If there is any problem with lack of uniqueness it may not be clear whether a marked boat has been recaptured or not. Another related point is that agents may make errors in the records which make it hard to match up a recapture with the original record. A standard operating procedure is being developed and documented to minimize these kinds of errors in the future.

3.1.2 Estimation of Total Effort and Total Catch

Total Effort (E) (*i.e.*, the total number of fishing trips taken in a defined period) is estimated by

$$\hat{E} = \hat{N}\bar{e}, \tag{3.1}$$

where $\hat{N}$ is the frame size (Fleet Size) estimate and $\bar{e}$ is the mean fishing effort (*i.e.*, average number of fishing trips taken) obtained from the telephone sample. The evaluation of the properties of this estimator is more difficult than when N is known because both $\hat{N}$ and $\bar{e}$ are random variables. We suspect that $\bar{e}$ is biased high because fishing boats that do not fish much are less likely to be on the list. Unfortunately we cannot say that $\hat{N}$ will always be biased high or low. All three of the assumption violations discussed in 3.1.1 could be important (closure, heterogeneity, and mark loss) and it is not clear what direction the overall bias on $\hat{N}$ would take. The only possible approach is to use simulation with a variety of different scenarios for assumption violations. Using equation (1.4) the estimated variance of $\hat{E}$ is given by

$$\widehat{\text{Var}}(\hat{E}) = (\hat{N})^2\,\widehat{\text{Var}}(\bar{e}) + (\bar{e})^2\,\widehat{\text{Var}}(\hat{N}) + \widehat{\text{Var}}(\bar{e})\,\widehat{\text{Var}}(\hat{N}). \quad (3.2)$$

Total catch (C) is estimated by $\hat{C} = \hat{E}\bar{c}$ where $\hat{E}$ is the estimated total fishing effort and $\bar{c}$ is the average catch per unit effort calculated from the dockside interviews. Properties of this equation are likely to be subject to similar concerns as equation (3.1) and again simulation could be very useful.

3.1.3 Illustration of the Method

In this section we present the frame size estimates and total effort estimates for the Virginia Bluefin tuna fishery in part of 1992. These estimates are a part of a larger survey which covered the east coast of the U.S. from North Carolina to Massachusetts. The estimates are separate for charter boats and private boats.

Frame Size Estimates

Lists of unique private boats and charter boats were compiled mainly by telephone interviews from previous seasons. During the current 1992 season "marked" and "unmarked" boats were captured at gas pumps before or after fishing trips.

For private boats the list size was $M = 335$ boats before the season. A sample of $n = 374$ boats were contacted at gas pumps and of those $m = 49$ were marked. The Chapman estimator is $\hat{N}_c = 2{,}519$, $\widehat{SE}(\hat{N}_c) = 303.08$ and relative $\widehat{SE} = 0.12$.

For charter boats the list size was $M = 47$ before the season. A sample of $n = 31$ boats were contacted at gas pumps and of those $m = 13$ were marked. The Chapman estimator is $\hat{N}_c = 109$ with $\widehat{SE}(\hat{N}_c) = 17.88$ and relative $\widehat{SE} = 0.16$.

Total Effort Estimates

Total effort and total catch were estimated in weekly waves. Here we just illustrate the calculations for the week of the 8th to the 14th of June 1992 for total effort.

Total Effort - Private Boats

$\hat{N}_c = 2{,}519$ boats, $\widehat{\text{Var}}(\hat{N}_c) = 91{,}856.4706$, $\bar{e} = 0.15108$ trips per interview, $\widehat{\text{Var}}(\bar{e}) = 0.001242$ and $\widehat{SE}(\bar{e}) = 0.0352$. Using these estimates we obtain

$$\hat{E} = \hat{N}_c \times \bar{e} = 2{,}519 \times 0.15108 = 380.57 \text{ trips},$$

$$\widehat{\text{Var}}(\hat{E}) = \widehat{\text{Var}}(\bar{e})(\hat{N}_c^2) + \widehat{\text{Var}}(\hat{N}_c)(\bar{e})^2 + \widehat{\text{Var}}(\hat{N}_c)\,\widehat{\text{Var}}(\bar{e}) = 10{,}091.6633, \text{ and}$$

$$\widehat{SE}(\hat{E}) = 100.45.$$

It is useful to also calculate the variance of total effort assuming that the frame size were known. In this case it is $\widehat{\text{Var}}(\hat{E}) = 7{,}780.9384$ with $\widehat{SE}(\hat{E}) = 88.77$ and this shows that 89% of the standard error of the Total Effort estimate is due to variation in average effort and only 11% is due to estimation of frame size.

Total Effort - Charter Boats

For charter boats $\hat{E} = 59.95$ trips with $\widehat{\text{Var}}(\hat{E}) = 512.5100$ and $\widehat{SE}(\hat{E}) = 22.64$.

The variance of the Total Effort estimate assuming the frame size is known is $\widehat{\text{Var}}(\hat{E}) = 404.8926$ with $\widehat{SE}(\hat{E}) = 20.12$. Again 89% of the standard error of the Total Effort estimate is due to variation in average effort and only 11% is due to estimation of frame size.

3.2 More Than Two Lists

In Section 2 we indicated that there are a lot more modeling possibilities if one has multiple (greater than 2) lists. Here we consider closed and open population models for the more general case. We foresee the sampling scheme as follows. Before the start of the fishing season there would be a preliminary sample to establish a list (either telephone or dockside). During each time period (say two weeks) there would be an additional list compiled using a telephone or dockside survey. Now each individual boat would have a capture history which would indicate which lists it appeared on. (Suppose we have five time periods then a capture history of 1 1 1 0 1 would indicate a boat appeared on the lists in all except the fourth time period).

The structure of the sample and the population would therefore be as in Table 1.The first question that has to be addressed is whether we need to use closed or open population models. The obvious way to proceed is to fit the Jolly-Seber open population model first and use it to evaluate the closure assumption.

Table 1

Structure of the Population Under an Open Population Model*

Period	Pre-season List	Season Lists (*e.g.*, every two weeks)					
	0	1	2	3	.	.	k
Marked Population Sizes	M_0	M_1	M_2	M_3	.	.	M_k
Total Population Sizes	N_0	N_1	N_2	N_3	.	.	N_k

* Marked and Total Population Sizes are shown for the whole study.

3.2.1 Open Population Models

Under the Jolly-Seber model previously discussed in Section 2.3 the following parameters are identifiable (Table 2). Notice that it is possible to estimate the number of fishing boats in the fleet at each time in the season except the last (*i.e.*, $\hat{N}_k$ cannot be estimated). One advantage of applying the model in this fashion with a preseason list is that any concerns with the preseason list due to it being out of date are taken care of by the model allowing for additions and deletions before the season begins. One disadvantage of the Jolly-Seber Model is increased complexity. Now each time period has its own frame size and there are also survival and recruitment parameters to estimate. Sometimes these parameter estimates have poor precision unless sample sizes are large. Another disadvantage of the Jolly-Seber model is that it does require the assumption of equal catchability.

Table 2

Structure of the Jolly-Seber Open Population Model*

Period	Preseason	Season						
	0	1	2	3	.	.	*k*-1	*k*
Marked Population	$(M_0 \equiv 0)$	$\hat{M}_1$	$\hat{M}_2$	$\hat{M}_3$	.	.	$\hat{M}_{k-1}$	-
Total Population	-	$\hat{N}_1$	$\hat{N}_2$	$\hat{N}_3$	.	.	$\hat{N}_{k-1}$	-
Survival Rate	$\hat{\phi}_0$	$\hat{\phi}_1$	$\hat{\phi}_2$		.	$\hat{\phi}_{k-2}$		-
Recruitment No.		$\hat{B}_1$	$\hat{B}_2$		.	$\hat{B}_{k-2}$		

* Identifiable parameter estimators are shown for Marked Population Sizes, Total Population Size, Survival Rate and Recruitment Number.

Another important question about the use of the Jolly-Seber model is what is called "temporary emigration." A fishing boat might leave the fishery for some periods and then return. The Jolly-Seber model makes the assumption that fishing boats which leave do not return. This issue needs further investigation. Use of the robust design (*i.e.*, combination closed and open models) allows for temporary emigration. This would necessitate having two lists obtained close together in each period.

3.2.2 Closed Population Models

If the Jolly-Seber model estimates of "survival" and "recruitment" suggest population closure (*i.e.*, N constant) then the general closed population models reviewed in Section 2.2 could be applied. The advantages are increased precision of $\hat{N}$ due to the use of more lists and increased robustness of $\hat{N}$ to unequal catchability. The disadvantage is primarily an increase in complexity.

4. DISCUSSION

4.1 Methods of Dealing with Incomplete List Frames

(i) Complete the List Frame

The advantage is that the survey researcher has a complete frame and does not have to generalize results for an estimated frame size. The disadvantage is the cost and possible impracticality of completing the list frame.

(ii) Use an Area Frame

The advantage is that one only has to enumerate the establishments in the areas to be sampled. The disadvantage is possible inefficiency if businesses are sparse in each large area.

(iii) Using List and Area Frame (Multi-Frame Approach)

The advantages are obviously increased precision and having all establishments covered. The disadvantage could be expense and impracticality.

(iv) Use of Capture-Recapture to Estimate List Frame Size

The advantage is having a practical method of lower expense than the first three approaches listed above. The disadvantages are potential bias if the assumptions of the capture-recapture method are violated and having to include variation due to frame size estimation in variance estimates of population total estimates.

4.2 Capture-Recapture Estimation of Frame Size

In this section we consider model assumptions, precision of estimates, estimation of population totals and the special problems in more complex sampling designs when the capture-recapture approach to frame size estimation is used.

Model Assumptions

(i) Closure

Can the frame size be considered constant so that the closed population models be used? This will depend on whether the survey is just a snapshot at a single time point or whether a series of surveys over time are required. It will also depend on how quickly establishments go out of business and how quickly new ones arise. We suspect there will be the need for use of closed and open population models depending on the establishments being studied.

There is also the question of temporary emigration where establishments go out of the frame and then come back in again. This was considered a potential problem in the fishing boat example because boats could go inactive and then become active again. This may also be a problem in some other establishment surveys if establishments go in and out of business frequently and keep the same name when they come back into business.

(ii) "Unequal Catchability" and Independence of Lists

As we discussed earlier ideally the lists used should be independent so that the estimates of frame size are unbiased. In practice it may not be easy to find two or more independent lists.

(iii) Mark Loss-Unique Identification of Establishment

Establishment names need to be unique and unmistakable or matches on different lists may be missed or mistaken. This was a problem in the fishing boat example in earlier years. We suspect this will not be such a big problem in most establishment surveys.

Precision of Estimates

The lists used need to be of sufficient size that the precision of the frame size estimate ($\hat{N}$) is adequate. Seber (1982, p. 96) discusses the Lincoln-Petersen estimate in detail and presents graphics of sample sizes required for various levels of precision. Pollock *et al.* (1990) presents sample size information for the open population models.

Estimation of Population Totals

Once the estimate of frame size is obtained then that estimate will often be combined with a sample mean to obtain an estimate of a population total ($\hat{Y} = \hat{N}\bar{y}$). The estimate of population total is subject to possible bias and additional variance because $\hat{N}$ is estimated. The estimate may also be biased because $\bar{y}$ is not based on a random sample of the complete frame.

More Complex Sampling Designs

In this paper we have emphasized estimation of frame size in simple random sampling using the capture-recapture method. Further questions arise if more complex sampling designs are used. For example in stratified designs the question would arise of whether to estimate frame size in each stratum separately or to estimate the total frame size and then apportion it to the strata assuming equal probabilities of different strata on the incomplete lists. There is also the more complex question of how to estimate frame size in multi-stage sampling designs. This is obviously an area that needs future research.

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