Survey Methodology, June 1994
Vol. 20, No. 1, pp. 33–43
Statistics Canada

Time Series EBLUPs for Small Areas Using Survey Data

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ABSTRACT

In estimation for small areas it is common to borrow strength from other small areas since the direct survey estimates often have large sampling variability. A class of methods called composite estimation addresses the problem by using a linear combination of direct and synthetic estimators. The synthetic component is based on a model which connects small area means cross-sectionally (over areas) and/or over time. A cross-sectional empirical best linear unbiased predictor (EBLUP) is a composite estimator based on a linear regression model with small area effects. In this paper we consider three models to generalize the cross-sectional EBLUP to use data from more than one time point. In the first model, regression parameters are random and serially dependent but the small area effects are assumed to be independent over time. In the second model, regression parameters are nonrandom and may take common values over time but the small area effects are serially dependent. The third model is more general in that regression parameters and small area effects are assumed to be serially dependent. The resulting estimators, as well as some cross-sectional estimators, are evaluated using bi-annual data from Statistics Canada's National Farm Survey and January Farm Survey.

KEY WORDS: Composite estimation; State space models; Kalman filter; Fay-Herriot estimator.

1. INTRODUCTION

There exists a considerable body of research on small area estimation using cross-sectional survey data in conjunction with supplementary data obtained from census and administrative sources. A good collection of papers on this topic can be found in Platek, Rao, Särndal and Singh (1987). Small area estimation techniques in use in U.S. federal statistical programs are reviewed by the Federal Committee on Statistical Methodology (1993). The basic idea underlying all small area methods is to borrow strength from other areas by assuming that different areas are linked via a model containing auxiliary variables from the supplementary data. It would also be important to borrow strength across time because many surveys are repeated over time. Recently time series methods have been employed to develop improved estimators for small areas; see Pfeffermann and Burck (1990) and Rao and Yu (1992). It is interesting to note that after the initiative of Scott and Smith (1974) on the application of time series methods to survey data, there has only lately been a resurgence of interest in developing suitable estimates of aggregates from complex surveys repeated at regular time intervals; see *e.g.*, Bell and Hillmer (1987), Binder and Dick (1989), Pfeffermann (1991), and Tiller (1992).

In this paper we consider some natural generalizations of the best linear unbiased predictor (BLUP) for small areas when a time series of direct small area estimates is available. An important example of the BLUP for small areas is the Fay-Herriot (FH) estimator, which entails smoothing of direct estimators by cross-sectional modelling of small area totals. The resulting estimators are composite estimators (*i.e.*, convex combinations of direct and synthetic estimators) and are called empirical BLUPs, or EBLUPs, whenever estimates of some variance components are substituted in the BLUPs. The work of Fay and Herriot (1979) represents an important milestone in the field of small area estimation because it is probably the first example of a large scale application of small area estimation by government agencies for policy analysis. With the use of structural models, we derive time series EBLUPs which combine both cross-sectional and time series data. The models underlying the time series EBLUPs were chosen on the basis of general heuristic considerations rather than formal model testing procedures. Formal testing of these types of models with survey data is very difficult and not very much is available. Instead, we begin with a regression model that is reasonable for the larger area, and then allow random small area effects to account for any local deviations from the global model. The regression parameters and random small area effects are allowed to evolve over time according to a state space model that was also formulated heuristically. We have not considered here the problem of mean squared error (MSE) estimation for our estimators. MSEs with respect to the motivating models could be defined and estimated for many of the estimators; however, the focus of this paper is on the performance of the estimators in a repeated sampling framework. MSE estimation is an important and difficult problem, and the availability of reliable MSE estimators could be an important consideration in the choice of estimators.

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The main purpose of this paper is to compare time series EBLUPs with cross-sectional estimators such as post-stratified domain, synthetic, FH and sample size dependent estimators. In the time series modelling of the direct small area estimates we assume that the survey errors are uncorrelated over time. When survey errors are correlated over time and can be modelled reasonably (*e.g.*, ARMA) the approach of Pfeffermann (1991) can be used to obtain time series EBLUPs via the Kalman filter. Rao and Yu (1992) obtain EBLUPs for a model, in which the Kalman filter cannot be applied, with survey errors having arbitrary correlation structure over time but being uncorrelated across areas. They also develop second order approximations to, and estimation of, the mean squared error under their model. When a model for the correlated survey errors is difficult to specify it may be possible, using a suitably modified Kalman filter, to get good sub-optimal estimators (Singh and Mantel 1991).

In this paper we report on an empirical study of the efficiency of time series EBLUPs. The study uses Monte Carlo simulations from real time series data obtained from Statistics Canada's biannual farm surveys. The main findings of the study are

(i) There can be reasonable gains in efficiency with time series EBLUPs over cross-sectional estimators.

(ii) Within the class of time series methods considered in this paper, introduction of serial dependence in the random small area effects is found to be beneficial.

(iii) Although any smoothed version of the direct small area estimator is expected to be biased, the time series EBLUPs exhibit less bias than cross-sectional smoothing methods.

Section 2 contains a description of various cross-sectional methods for small area estimation. Time series EBLUPs are described in Section 3 and the details and results of the Monte Carlo comparative study are given in Section 4. Finally, Section 5 contains concluding remarks.

2. METHODS BASED ON CROSS-SECTIONAL DATA

In this section we describe some well known small area estimation methods that use survey data from only the current time. Ghosh and Rao (1994) contains a good survey of various small area estimators.

Let $\underset{\sim}{\Theta}$ denote the vector of small area population totals Θ_k, $k = 1, \ldots, K$. In this section, which deals with methods based on cross-sectional data, we ignore the dependence of $\underset{\sim}{\Theta}$ on time t for simplicity.

2.1 Method 1 (Expansion Estimator for Domains)

This estimator is given by

$$g_{1k} = \sum_{j \in s_k} d_j y_j,$$

where d_j is the survey weight for sample unit j. For stratified simple random sampling, which is used for our simulation study in Section 4, we have

$$g_{1k} = \sum_{h} (N_h/n_h) \sum_{j \in s_{hk}} y_{hj}, \qquad (2.1)$$

where y_{hj} is the j-th observation in the h-th stratum, s_{hk} denotes the set of n_{hk} sample units falling in the k-th small area in the h-th stratum and n_h, N_h denote respectively the sample and population sizes for the h-th stratum. This estimator is often unreliable because n_{hk}, the random sample size in the small area, may be small in expectation and could have high variability. Conditional on the realized sample size n_{hk}, g_{1k} is biased. However, unconditionally, it is unbiased for Θ_k.

2.2 Method 2 (Post-stratified Domain Estimator)

We will also refer to this estimator as the direct small area estimator. If the population size N_{lk} is known for some post-strata indexed by l, then the efficiency of the estimator g_{1k} could be improved by post-stratification. We define

$$g_{2k} = \sum_{l} N_{lk} \sum_{j \in s_{lk}} d_j y_j \Big/ \sum_{j \in s_{lk}} d_j = \sum_{l} N_{lk} \bar{y}_{lk}.$$

In our simulations our post-strata are the intersections of design strata with small areas which leads to

$$g_{2k} = \sum_{h} (N_{hk}/n_{hk}) \sum_{j \in s_{hk}} y_{hj} = \sum_{h} N_{hk} \bar{y}_{hk}. \qquad (2.2)$$

This estimator also may not be sufficiently reliable because of the possibility of n_{hk}'s being small in expectation. If $n_{hk} = 0$, the above estimator is not defined. It is conventional to replace $\bar{y}_{hk}$ by 0 when $n_{hk} = 0$. In the empirical study presented in this paper, we replaced $\bar{y}_{hk}$ by the synthetic estimate $(\bar{X}_{hk}/\bar{X}_h)\bar{y}_h$, where X is a suitable covariable, whenever $n_{hk} = 0$.

The estimator g_{2k} in (2.2) is conditionally (given $n_{hk} > 0$) unbiased and approximately unconditionally unbiased. Appendix A.1 gives details of estimation of the conditional mean squared error, v_k, of g_{2k}.

2.3 Method 3 (Synthetic Estimator)

It is possible to define a more efficient estimator by assuming a model which allows for "borrowing strength" from other small areas. This gives rise to synthetic estimators, see *e.g.*, Gonzalez (1973) and Ericksen (1974). Suppose different small area totals are connected via the auxiliary variable X_k by a linear model as

$$\Theta_k = \beta_1 + \beta_2 X_k, \; k = 1, \ldots, K, \qquad (2.3a)$$

or in matrix notation

$$\underset{\sim}{\Theta} = F\underset{\sim}{\beta}, \tag{2.3b}$$

where $F = (F_1, F_2, \ldots, F_K)'$, $F_k = (1, X_k)'$. Now consider a model for the direct small area estimators g_{2k}'s as

$$\underset{\sim}{g_2} = F\underset{\sim}{\beta} + \underset{\sim}{\epsilon},$$

where $\underset{\sim}{g_2} = (g_{21}, \ldots, g_{2K})'$, $\underset{\sim}{\epsilon} = (\epsilon_1, \ldots, \epsilon_K)'$, ϵ_ks are uncorrelated survey errors with mean 0 and variance v_k. Note that the g_{2k}s are uncorrelated over areas since they are conditionally (given n_{hk}) unbiased and the samples in different small areas are conditionally independent.

Denoting by $\hat{\underset{\sim}{\beta}}$ the weighted least squares (WLS) estimate of $\underset{\sim}{\beta}$, we obtain the regression-synthetic estimator of Θ_k under the assumed model as

$$\underset{\sim}{g_3} = F\hat{\underset{\sim}{\beta}}.$$

The above estimator could be heavily biased unless the model (2.3) is satisfied reasonably well. The above model may not be realistic because no random fluctuation or random small area effect (a_k, say) is allowed.

2.4 Method 4 (Fay-Herriot Estimator or EBLUP)

Using the empirical Bayes approach of Fay and Herriot (1979) or the more general best linear unbiased predictor approach (see *e.g.*, Battese, Harter and Fuller 1988, and Pfeffermann and Barnard 1991), the bias of the synthetic estimator can be reduced considerably by using a composite estimator; for an early reference on composite estimation see Schaible (1978). The composite estimator is obtained as a convex combination of $\underset{\sim}{g_2}$ and a modified $\underset{\sim}{g_3}$. For this purpose, it is assumed that

$$\underset{\sim}{\Theta} = F\underset{\sim}{\beta} + \underset{\sim}{a}, \tag{2.4}$$

where a_k's are uncorrelated random small area effects with mean 0 and variance w_k known up to a constant. In our empirical study later we take $w_k = w$. Thus we model $\underset{\sim}{g_2}$ as

$$\underset{\sim}{g_2} = F\underset{\sim}{\beta} + \underset{\sim}{a} + \underset{\sim}{\epsilon}. \tag{2.5}$$

Here $\underset{\sim}{a}$ is also assumed to be uncorrelated with $\underset{\sim}{\epsilon}$. The BLUP of $\underset{\sim}{\Theta}$ under the model defined by (2.4) and (2.5) is

$$\begin{aligned} \underset{\sim}{g_4} &= \underset{\sim}{g_3^*} + \Lambda(\underset{\sim}{g_2} - \underset{\sim}{g_3^*}) \\ &= \Lambda \underset{\sim}{g_2} + (I - \Lambda)\underset{\sim}{g_3^*}, \end{aligned} \tag{2.6}$$

where

$$\Lambda = (V^{-1} + W^{-1})^{-1} V^{-1} = WU^{-1},\ U \equiv V + W,$$

$$V = \mathrm{diag}(v_1, \ldots, v_K), \quad W = \mathrm{diag}(w_1, \ldots, w_K),$$

and $\underset{\sim}{g_3^*} = F\underset{\sim}{\beta}^*$, $\underset{\sim}{\beta}^*$ is the WLS estimate of $\underset{\sim}{\beta}$ under model (2.5). Here it is assumed that both the covariance matrices V and W are known in computing the BLUP.

The expression (2.6) follows from the general results on linear models with random effects, see *e.g.*, Rao (1973, p. 267) and Harville (1976). The BLUP or BLUE of $F\underset{\sim}{\beta}$ is $\underset{\sim}{g_3^*}$ and the BLUP of $\underset{\sim}{a}$ is $\Lambda(\underset{\sim}{g_2} - \underset{\sim}{g_3^*})$. It may be of interest to note that the structure of the BLUP does not change regardless of whether or not $\underset{\sim}{\beta}$ is known. However, its MSE does change as expected due to estimation of $\underset{\sim}{\beta}$.

When V and W are replaced by estimates, the estimator $\underset{\sim}{g_4}$ is termed EBLUP. Note that the model (2.4) is more realistic than (2.3), and therefore, the performance of $\underset{\sim}{g_4}$ is expected to be quite favourable. The estimator $\underset{\sim}{g_4}$ approaches $\underset{\sim}{g_2}$ when the v_ks get small, *i.e.*, when the n_{hk}s become large. However, it remains biased, in general, conditional on $\underset{\sim}{\Theta}$, with bias tending to 0 as the v_ks get small.

2.5 Method 5 (Sample Size Dependent Estimator)

An alternative composite estimator is given by the sample size dependent estimator of Drew, Singh and Choudhry (1982). It is defined as

$$\underset{\sim}{g_5} = \Delta \underset{\sim}{g_2} + (I - \Delta)\underset{\sim}{g_3},$$

where $\Delta = \mathrm{diag}(\delta_1, \ldots, \delta_K)$,

$$\delta_k = \begin{cases} 1 & \text{if } \sum_{j \in s_k} d_j \geq \lambda N_k, \\ \sum_{j \in s_k} d_j / \lambda N_k & \text{otherwise} \end{cases} \tag{2.7}$$

and the parameter λ is chosen subjectively as a way of controlling the contribution of the synthetic component. The above estimator takes account of the realized sample size n_{hk}'s and if these are deemed to be sufficiently large according to the condition in (2.7), then it does not rely on the synthetic estimator. This property is somewhat similar to that of $\underset{\sim}{g_4}$; however, unlike $\underset{\sim}{g_4}$, the above estimator does not take account of the relative sizes of the within area and between area variation. Rao and Choudhry (1993) have demonstrated empirically how EBLUPs can sometimes outperform sample size dependent estimators, especially when the between area variation is not large relative to the within area variation. Särndal and Hidiroglou (1989) also proposed estimators similar to the above sample size dependent estimator.

3. METHODS BASED ON POOLED CROSS-SECTIONAL AND TIME SERIES DATA

Suppose information is available for several time points, $t = 1, \ldots, T$, in the form of direct small area estimators g_{2t}, where g_{2t} is the vector of estimates g_{2k} in (2.2) based on data from time t, and also the small area population totals for the auxiliary variable. We will now introduce some estimators which generalize the Fay-Herriot estimator g_{4T} in different ways by taking account of the serial dependence of the direct estimates $\{g_{2t} : t = 1, \ldots, T\}$. Recall that for the Fay-Herriot estimator, the model for θ_T has two components, namely, the structural component $F_T \beta_T$ and the area component a_T. The estimator g_{4T} borrows strength over areas for the current time T and is given by the sum of two components, each being EBLUP (BLUE) for the corresponding random (fixed) effect, *i.e.*,

$$g_{4T} = F_T \beta_T^* + a_T^*. \tag{3.1}$$

Methods based on time series data could, however, borrow strength over time as well. Here we introduce three estimators which are motivated from specific structural models for serial dependence. All three of these estimators are optimal under different special cases of a structural time series model for the direct small area estimates $\{g_{2t} : t = 1, \ldots, T\}$ specified by the following state space model. Let α_t denote $(\beta_t', a_t')'$ and H_t denote (F_t, I). Then we have

$$\begin{aligned} g_{2t} &= \theta_t + \epsilon_t, \\ \theta_t &= F_t \beta_t + a_t \equiv H_t \alpha_t \end{aligned} \tag{3.2a}$$

and

$$\alpha_t = G_t \alpha_{t-1} + \zeta_t, \tag{3.2b}$$

where

$$G_t = \begin{pmatrix} G_t^{(1)} & 0 \\ 0 & G_t^{(2)} \end{pmatrix}, \quad \zeta_t = \begin{pmatrix} \xi_t \\ \eta_t \end{pmatrix}, \tag{3.2c}$$

along with the usual assumptions about random errors, *i.e.*, ϵ_t, ζ_t are uncorrelated, ζ_t is uncorrelated with α_s for $s < t$, and that $\epsilon_t \sim (0, V_t)$, $\zeta_t \sim (0, \Gamma_t)$ where $\Gamma_t = \text{block diag}\{B_t, Q_t\}$. The covariance matrices V_t, B_t, and Q_t are generally diagonal. If $G_t^{(1)} = I$ and $G_t^{(2)} = I$ then β_t and a_t evolve according to a random walk.

This model is in the general class defined by Pfeffermann and Burck (1991) using structural time series models. The main purpose of their study was to show how accounting for cross-sectional correlations between neighbouring small areas (in addition to serial correlations) and inclusion of certain robustness modifications (to protect against model breakdowns) could improve the performance of time series model based estimators. They also used the maximum likelihood method under normality to estimate model parameters. The focus of this paper, on the other hand, is on the Monte Carlo evaluation of a special class of time series estimators (related to Fay-Herriot) chosen on the basis of heuristic considerations and not on the basis of model fitting. The methods considered could, therefore, be viewed as model assisted methods whose performance will be evaluated in a design based (*i.e.*, repeated sampling) framework by Monte Carlo simulation. Moreover, it will be seen later that, for the types of serial dependence considered, the model parameters can be estimated relatively simply by the method of moments, without making any distributional assumptions such as normality.

To find the optimal estimator (BLUP) of θ_T in (3.2) based on all the direct estimates up to time T, we first found the BLUP $\tilde{\alpha}_T$ of α_T from which the BLUP of θ_T is obtained as $H_T \tilde{\alpha}_T$. It is possible, albeit cumbersome, to get $\tilde{\alpha}_T$ directly from the complete data using the theory of linear models with random effects. However, since the α_Ts are connected over time according to the transition equation (3.2b), it is more convenient to compute it recursively using the Kalman filter (KF). Traditionally KF is viewed as a Bayesian technique in which at each time t, the posterior distribution of α_t given data up to $t - 1$ is updated to get the posterior distribution of α_t given data up to time t. Although it is instructive to view KF in this manner, it is not necessary under mixed linear models. Suppose $\tilde{\alpha}_{T|s}$ denotes the BLUP of α_T based on data up to time s, $s < T$. It is known (see Duncan and Horn 1972) that, for the special structure of serial dependence considered here, the BLUP $\tilde{\alpha}_T$ of α_T based on data up to time T is the same as the BLUP of α_T based on $\tilde{\alpha}_{T|s}$ and the last $T - s$ observations. In other words, information in the previous data can be condensed into an appropriate BLUP before augmenting more current data points. A good description of the Kalman filter is given in chapter 3 of Harvey (1989).

3.1 Method 6 (Time Series EBLUP-I)

For the first estimator, we let β_t evolve over time (*e.g.*, according to a random walk), but assume that a_t is serially independent. The equations for the state space model for this case are similar to (3.2) except that the serial independence of the a_ts implies $G_t^{(2)} = 0$. This will give rise to a composite estimator

$$g_{6T} = F_T \tilde{\beta}_T + \tilde{a}_T. \tag{3.3}$$

Note that $\tilde{\beta}_T$ in (3.3) would now be based on all the small area estimates up to time T and therefore would be different from β_T^* of (3.1) which is based on only direct estimates at time T. The estimator $\tilde{a}_T$, as a result, would also be different from the corresponding component a_T^* of (3.1).

In the simulation study described later we take $G_t^{(1)} = I$, $B_t = \mathrm{diag}(\gamma_1^2, \gamma_2^2)$, corresponding to a random walk model, and $Q_t = \tau^2 I$. Appendix A.2 illustrates the method of moments estimation of the parameters γ_1^2, γ_2^2, and τ^2. The KF may then be run, with initial values for $\tilde{\alpha}_1$ and its MSE obtained from the *FH* estimator at $t = 1$, to obtain the EBLUP of $\tilde{\alpha}_T$. Then $H_T\tilde{\alpha}_T$ is the time series EBLUP-I estimator g_{6T} at time T.

As pointed out by a referee, when the number of small areas is quite large, or when the variation in $\underset{\sim}{\beta}_t$ over t is relatively large, there is little difference between g_{6T} and g_{4T}. Indeed, there is little difference between the performances of these two estimators in our simulation study described in Section 4.

3.2 Method 7 (Time Series EBLUP-II)

For the second estimator, we let $\underset{\sim}{\beta}_t$ be fixed (it may or may not be common for different time points) and let the area effects $\underset{\sim}{a}_t$ be serially dependent according to, for example, a random walk. This time series generalization could be viewed as an analogue of the model proposed by Rao and Yu (1992). The resulting composite estimator will have the same form as (3.1), *i.e.*,

$$\underset{\sim}{g}_{7T} = F_T \bar{\underset{\sim}{\beta}}_T + \bar{\underset{\sim}{a}}_T, \tag{3.4}$$

but the component estimates $\bar{\underset{\sim}{\beta}}_T$ and $\bar{\underset{\sim}{a}}_T$ would be different. We have two cases.

3.2.1 Case 1: Suppose the $\underset{\sim}{\beta}_t$s are fixed and time-invariant but the $\underset{\sim}{a}_t$s are serially dependent. Then, in (3.2), $G_t^{(1)} = I$ and $B_t = 0$. If Q_t is taken as $\tau^2 I$, then the only unknown parameter τ^2 can be estimated by the method of moments; see Appendix A.2. We will denote by $\underset{\sim}{g}_{7T}$ the EBLUP obtained in this case when the parameter estimate is substituted.

3.2.2 Case 2: Here we assume that $\underset{\sim}{\beta}_t$s are fixed but different for different time points. The area effects $\underset{\sim}{\alpha}_t$ evolve over time as in Case 1. In (3.2) we have $G_t^{(1)} = 0$ and $B_t = mI$ where m is a large number. The expressions for $\tilde{\alpha}_T$ and its MSE obtained from the KF in this case give the correct formulas as $m \to \infty$ (see Sallas and Harville 1981). The KF updating equations for $\bar{\underset{\sim}{a}}_t$ in this case take the special form

$$\bar{\underset{\sim}{\beta}}_t = (F_t' A_t^{-1} F_t)^{-1} F_t' A_t^{-1} (g_{2t} - G_t^{(2)} \bar{\underset{\sim}{a}}_{t-1});$$

$$\bar{\underset{\sim}{a}}_t = G_t^{(2)} \bar{\underset{\sim}{a}}_{t-1} + P_{t|t-1} A_t^{-1} (g_{2t} - G_t^{(2)} \bar{\underset{\sim}{a}}_{t-1} - F_t \bar{\underset{\sim}{\beta}}_t);$$

$$P_t = P_{t|t-1} - P_{t|t-1} A_t^{-1} (A_t - F_t (F_t' A_t^{-1} F_t)^{-1} F_t') A_t^{-1} P_{t|t-1},$$

where $A_t = P_{t|t-1} + V_t$, P_t is the MSE of $\bar{\underset{\sim}{a}}_t$ about $\underset{\sim}{a}_t$, and $P_{t|t-1} = G_t^{(2)} P_{t-1} \{G_t^{(2)}\}' + Q_t$ is the MSE of $G_t^{(2)} \bar{\underset{\sim}{a}}_{t-1}$ as an estimator of $\underset{\sim}{a}_t$. The time series EBLUP in this case will be denoted by $\underset{\sim}{g}_{7T}^*$.

3.3 Method 8 (Time Series EBLUP-III)

For the third estimator, we let both $\underset{\sim}{\beta}_t$ and $\underset{\sim}{a}_t$ evolve over time. This will have more complex serial dependence than either (3.3) or (3.4). Its form will be similar to (3.1) and can be represented as

$$\underset{\sim}{g}_{8T} = F_T \tilde{\tilde{\underset{\sim}{\beta}}}_T + \tilde{\tilde{\underset{\sim}{a}}}_T. \tag{3.5}$$

As before, if $B_t = \mathrm{diag}\{\gamma_1^2, \gamma_2^2\}$ and $Q_t = \tau^2 I$, then the model parameters τ^2, γ_1^2, γ_2^2 can be estimated by the method of moments as in Appendix A.2. The resulting EBLUP of $\underset{\sim}{\theta}_T$ will be denoted by $\underset{\sim}{g}_{8T}$.

It may be of interest to note that many of the estimators considered so far are optimal under special cases of the model underlying $\underset{\sim}{g}_{8T}$. As has been shown, the time series EBLUPs of methods 6 and 7 result from making restrictions on the matrices G_t and Γ_t. The cross-sectional Fay-Herriot estimators of Section 2.4 result from restricting the data to a single time point. The synthetic estimators of section 2.3 are special cases of the Fay-Herriot estimators with zero variance for the random small area effects, and the direct (post-stratified) estimator is obtained in the limit as the variance of the small area effects goes to infinity.

A further generalization that could be useful is to allow correlations between neighbouring small area effects. This can be accomplished by allowing the matrix Q_t in (3.2) to be non-diagonal; however, it is not clear what would be an appropriate correlation structure in Q_t.

4. MONTE CARLO STUDY

The cross-sectional and time series methods were compared empirically by means of a Monte Carlo simulation from a real time series obtained from Statistics Canada's biannual farm surveys, namely, the National Farm Survey (in June) and the January Farm Survey. Due to the redesign after the census of Agriculture in 1986, the survey data for the six time points starting with the summer of 1988 were employed to create a pseudo-population for simulation purposes. To this, data from the census year 1986 was also added. Thus information at one more time point was available although this resulted in a 3-point gap in the series. The missing data points, however, can be easily handled by time series methods. It may be noted that although the data series is short, it is nevertheless believed to be adequate for illustrative purposes. The parameter of interest was taken as the total number of cattle and calves for each crop district (defined as the small area) at each time point. For simplicity, independent stratified random samples were drawn for each occasion from the pseudo-population, though the farm surveys use rotating panels over time. The dependence of direct small area estimates over time was modelled by assuming that the underlying small area population totals are connected according to some random process. The auxiliary variable used in the model was the ratio-adjusted census 1986 value of the total cattle and calves for each small area. This showed high correlations with the corresponding variable over time at the farm level. Specific details of the empirical study are described below.

4.1 Design of the Simulation Experiment

First we need to construct a pseudo-population from the survey data over six time points (June 1988, January 1989, ..., January 1991). The actual design involves two frames (list and area) with a one stage stratified sampling from the list frame and a two stage stratified sampling from the area frame, for details see Julien and Maranda (1990). We decided to use survey data from the list frame only because the list frame corresponds to farms existing at the time of Census 1986 and the chosen auxiliary variable for model building was based on Census 1986 information. Moreover, we chose to use the data from the province of Quebec because its area sample is only a minor component of the total sample and the estimated coefficient variation for the twelve crop-districts (*i.e.*, small areas of interest) of this province showed a wide range for the livestock variables. It was decided to avoid variability due to changes in the underlying population over time by retaining only those farms which responded to all the six occasions. Also, farm units who belonged to a multiholding arrangement in any one of the seven time points (including the census) were excluded because of the problems in finding individual farm's data from the multiholding summary record and changes in their reporting arrangement over time.

The various exclusions described above were motivated from considerations of yielding a sharper comparison between small area estimators. The total count of farm units after exclusions was found to be 1,160 out of a total of over 40,000 farms on the list frame. For the pseudo-population, we replicated the 1,160 farm units proportional to their sampling weight so that the total size N of the pseudo-population was 10,362, which was manageable for micro-computer simulation.

The pseudo-population was stratified into four take-some and one take-all strata using Census 1986 count data on cattle and calves as the stratification variable. Although we did not consider alternative stratifications or sample sizes in our simulation study, there is no reason to think that our conclusions would alter significantly if we were to do so. The sigma-gap rule (Julien and Maranda 1990) was used for defining the take-all stratum. To apply the sigma-gap rule we look at the smallest population value greater than the population median where the distance to the next population value, in order of size, is at least one population standard deviation; all units above this point are placed into the take-all stratum. The algorithm of Sethi (1963) was used for determining optimal stratification boundaries for take-some strata. Neyman's optimum allocation was used for sample sizes for strata in order to optimize the precision of the provincial estimate of total count. This resulted in, from a total sample size of 207 (2% sampling rate), allocations of 51, 62, 48 and 35 from takesome strata with 5,001, 3,188, 1,850 and 312 farms, respectively, and the size of the take all stratum was 11. The expected number of sample farms in each small area varied from 4.6 in area 9 up to 27.5 in area 6, with an average of 17.3. The expected number of sample farms with some cattle and calves varied from 3.6 in area 9 to 18.8 in area 3, and the average over the small areas was 11.7. A total of 30,000 simulations were performed. For each simulation, samples were drawn independently for each time point using stratified simple random sampling without replacement. The 30,000 simulations were conducted in 15,000 sets of 2 simulations where each set corresponds to a different vector of realized sample sizes in the twelve small areas within each stratum. This was required to compute certain conditional evaluation measures as described in the next subsection, see also Särndal and Hidiroglou (1989).

4.2 Evaluation Measures

Suppose m simulations are performed in which m_1 sets of different vectors of realized sample sizes in domains (h,k) are replicated m_2 times. The following measures can be used for comparing performance of different estimators at time T. Let i vary from 1 to m_1 and j from 1 to m_2.

(i) Absolute Relative Bias for area k:

$$\mathrm{ARB}_k = \left| m^{-1} \sum_i \sum_j (\mathrm{est}_{ijk} - \mathrm{true}_k)/\mathrm{true}_k \right| . \quad (4.1)$$

The average of ARB_k over areas k will be denoted by AARB. We take the absolute relative bias since our primary interest in this study is in an overall measure like AARB; however, in other contexts the actual biases for individual small areas may also be of considerable interest.

The following measure is motivated by a desire to evaluate the conditional performance of estimators, conditional on the vectors of realized sample sizes in domains. It is conventional to measure performance conditional on fixed domain sample sizes; here we consider the standard deviation of the conditional bias, B_{ik}, as a simple summary measure. If this standard deviation is small then the method is robust to variations in the realized sample sizes. Note that the expected value of B_{ik} is just the unconditional bias which is estimated by ARB_k. Let B_k^2 denote the unconditional expected value of B_{ik}^2. We define the following Monte Carlo measure:

(ii) Standard Deviation of Conditional Relative Bias for area k:

$$\mathrm{SDCRB}_k = \left\{ m_1^{-1} \sum_i (\hat{B}_{ik}^2 - \hat{C}_{ik}) \Big/ \mathrm{true}_k - \mathrm{ARB}_k^2 \right\}^{1/2};$$

$$\hat{B}_{ik} = m_2^{-1} \sum_j \mathrm{est}_{ijk} - \mathrm{true}_k, \qquad (4.2)$$

$$\hat{C}_{ik} = m_2^{-1}(m_2 - 1)^{-1} \left(\sum_j \mathrm{est}_{ijk}^2 - \left(\sum_j \mathrm{est}_{ijk} \right)^2 \Big/ m_2 \right).$$

The correction term $\hat{C}_{ik}$ adjusts for bias in $\hat{B}_{ik}^2$, as an estimate of B_{ik}^2, due to m_2 being finite. $\hat{B}_{ik}^2 - \hat{C}_{ik}$ is conditionally unbiased for B_{ik}^2; it is also unconditionally unbiased for B_k^2. The Monte Carlo average $m_1^{-1} \sum_i (\hat{B}_{ik}^2 - \hat{C}_{ik})$ converges to B_k^2 with probability 1 as $m_1 \rightarrow \infty$. $\hat{B}_{ik}^2 - \hat{C}_{ik}$ may be negative for some i, due to finite m_2. For large m_1 the average over i is usually very close to B_k^2; whenever the average is less than ARB_k^2 we set SDCRB_k to 0. ASDCRB will denote the average of SDCRB_k over areas k.

(iii) Mean Absolute Relative Error for area k:

$$\mathrm{MARE}_k = m^{-1} \sum_i \sum_j \mid \mathrm{est}_{ijk} - \mathrm{true}_k \mid / \mathrm{true}_k \qquad (4.3)$$

and AMARE denotes the average of MARE_k over areas.

(iv) Mean Squared Error for area k:

$$\mathrm{MSE}_k = m^{-1} \sum_i \sum_j (\mathrm{est}_{ijk} - \mathrm{true}_k)^2 \qquad (4.4)$$

and AMSE as before denotes the average over areas.

(v) Relative Root Mean Squared Error for area k:

$$\mathrm{RRMSE}_k = \{\mathrm{MSE}_k\}^{1/2} / \mathrm{true}_k. \qquad (4.5)$$

Again, ARRMSE denotes the average over areas.

The precision (*i.e.*, the Monte Carlo standard error) of each measure depends on m_1, m_2. For all measures except (ii), the optimal choice of m_1, m_2 under the restriction that $m_2 > 1$ is $m_1 = m/2$, $m_2 = 2$, since this minimizes the Monte Carlo standard error. To see this, let A be the average of an evaluation measure from m_2 samples all with the same sample configuration (set of random sample sizes in domains) which we call C. Then the expected value of A conditional on C is a function of C, say $E(C)$, and the conditional variance of A is proportional to m_2^{-1}, say $V(C)/m_2$. The unconditional variance of A is then $V\{E(C)\} + E\{V(C)\}/m_2$, and the overall Monte Carlo variance of an evaluation measure based on m_1 sample configurations replicated m_2 times is $V\{E(C)\}/m_1 + E\{V(C)\}/m_1 m_2$ which is minimized, since $m = m_1 m_2$ is fixed, by taking m_1 as large as possible. For the second measure, the appropriate choice of m_1, m_2 is less straightforward. In the simulation study, m was chosen as 30,000 and the corresponding values of m_1, m_2 were set at 15,000 and 2.

4.3 Estimators Used in the Comparative Study

There were nine estimators included in the study, namely, g_1 to g_8 and g_7^*, all calculated for time $T = 10$. We used a simple linear regression model for the synthetic component with the auxiliary variable defined as

$$X_{kt} = (\hat{\Theta}_t / \Theta_1) \Theta_{k1}, \qquad (4.6)$$

where Θ_{k1}, Θ_1 respectively denote the population totals for small area k and the province at $t = 1$, *i.e.*, at Census 1986. The estimator $\hat{\Theta}_t$ denotes the post-stratified estimator of Θ_t from the farm survey at time t at the province level. Thus X_{kt} is simply a ratio-adjusted synthetic variable. The variances of error components in the regression model were assumed to be constant over areas. For time series models, it was assumed that the serial dependence was generated by a random walk. The above type of model assumptions have been successfully used in many applications and the main reason for our choice was simplicity. It was hoped, however, that the chosen models might be adequate for our purpose and might illustrate the differential gains with different types of model assisted small area estimators, *i.e.*, both cross-sectional and time series smoothing methods.

Since the Census 1986 data was included in the time series, the direct estimate $\underset{\sim}{g}_{21}$ corresponds to Census 1986 and therefore the survey error $\underset{\sim}{\epsilon}_1$ would be identically $\underset{\sim}{0}$. Moreover, from the definition of X_{kt}, it follows that a reasonable choice of (β_{11}, β_{21}) would be (0,1) which implies that $\underset{\sim}{a}_1$ must be $\underset{\sim}{0}$. Thus the covariance matrices B_t and W_t at $t = 1$ are null and, therefore, the distribution of $\underset{\sim}{\alpha}_t$ at $t = 1$ would not require estimation. The above modification in the initial distribution of $\underset{\sim}{\alpha}_t$ is natural in view of the extra information available from the census. Moreover, since the direct estimates $\underset{\sim}{g}_{2t}$ were not available for $t = 2, 3, 4$, equations for estimating model variance components in Appendix A.2 were modified accordingly.

For method 7 (case 1), $\underset{\sim}{\beta}_t$ was assumed to have a common fixed value only for $t \geq 2$ because at $t = 1$, $\underset{\sim}{\beta}_t = (0,1)'$. For the sample size dependent estimator g_5 the parameter λ was taken to be 1.

4.4 Empirical Results

The main findings were listed in Section 1. Here we give some detailed comparisons and some possible explanations. We do not show separate results for g_7^* which performs slightly worse than, though overall similarly to, g_7. The estimators are summarized in Table 1. Figures 1 to 3 and Tables 2 to 4 present some of the empirical results. We have not shown the Monte Carlo standard errors but they were all found to be quite negligible.

Table 1

Summary of Estimators

g_1 - Expansion	g_6 - Time Series EBLUP-I, βs evolve over time, αs independent over time
g_2 - Post-stratified	
g_3 - Synthetic	g_7 - Time Series EBLUP-II, αs evolve over time, fixed common β
g_4 - Fay-Herriot	
g_5 - Sample Size Dependent	g_8 - Time Series EBLUP-III, βs and αs evolve over time

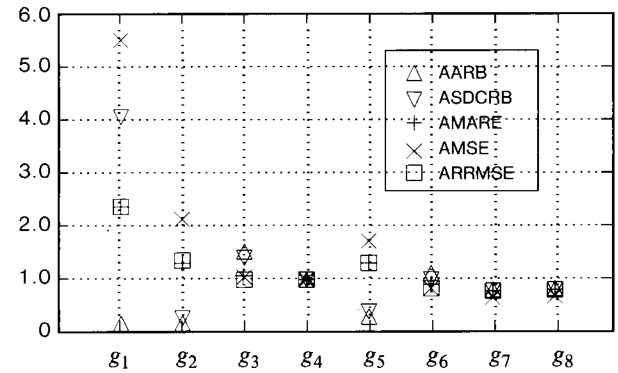

Figure 1. Evaluation Measures Relative to Fay-Herriot
Note: Relative ASDCRB for g_1 (= 18.98) not shown.

Table 2

Average Evaluation Measures

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
AARB	.001	.007	.097	.065	.018	.070	.053	.053
ASDCRB	.282	.016	.016	.015	.023	.010	.010	.010
AMARE	.269	.147	.115	.108	.136	.097	.087	.088
ARRMSE	.339	.192	.137	.137	.176	.120	.109	.111
AMSE (1,000's)	72,979	27,596	13,382	12,898	22,760	10,603	8,610	8,829

Table 2 gives the five evaluation measures averaged over small areas, Figure 1 shows plots of the averaged evaluation measures relative to the Fay-Herriot (g_4) value. There is a clear pattern in the behaviour of various measures across different estimators. The direct estimator g_2 does very well with respect to the bias measure (AARB) but does somewhat poorly with respect to the other measures. The cross-sectional smoothing method g_3 (synthetic) does quite poorly with respect to the bias measures. The Fay-Herriot method g_4 performs somewhat better than post-stratified on average with respect to the MSE measure but is much worse in terms of bias. The sample size dependent method g_5 is quite similar to g_2, slightly worse with respect to the bias measures and slightly better with respect to the other measures. The time series methods g_7 and g_8 perform quite well overall, though they are somewhat worse than g_2 with regard to bias. The performance of the time series estimator g_6 is generally between that of Fay-Herriot and the time series estimators g_7 and g_8. For all of the estimators (including the synthetic g_3) the standard deviation of the conditional relative bias (ASDCRB) is appreciable; however, it is smallest for the time series methods. As expected, the expansion estimator g_1 does well with respect to the unconditional bias measure, AARB, but its conditional performance (ASDCRB) is quite poor.

Figure 2 plots averages of RRMSE_k for three size groups, namely small, medium and large small areas, based on the ranking of their true population totals at time T. They are divided up into these three groups because the relative errors of estimation would be expected to be larger for the smaller totals, and the plots do not contradict this expectation. Again, the time series methods g_7 and g_8 perform best. Note that the time series method g_6, which assumes the small area effects to be independent over time, does not do as well. The unaveraged values of RRMSE_k are given in Table 3. RRMSE_9 is relatively large because the total number of cattle and calves for area 9 is less than half that of any other small area. Areas 6 and 8 stand out within the medium size small areas as being most difficult to estimate by the smoothing methods. The reason for this is that, while there was an overall decline of about 16% in the total number of cattle and calves in the pseudo-population from June 1986 to January 1991, the decreases for areas 6 and 8 were the furthest from the average at 33% and 1%, respectively, so the ratio adjusted covariate would be least appropriate for those areas. Nevertheless, the time series methods g_7 and g_8 performed significantly better than the post-stratified estimator for areas 6 and 8. This is because the random walk model for the small area effects is able to track small areas which, like areas 6 and 8, progressively deviate from the model.

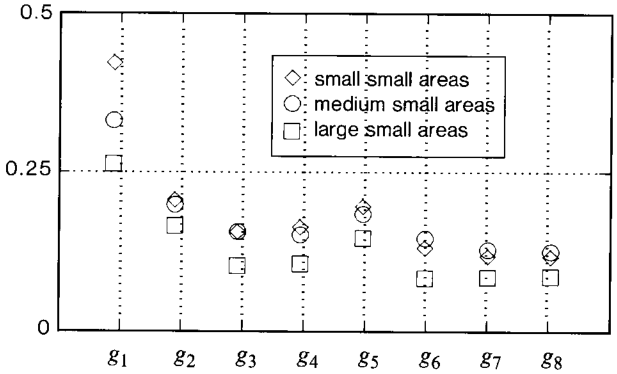

Figure 2. Relative Root Mean Squared Errors: Averaged within Size Groups

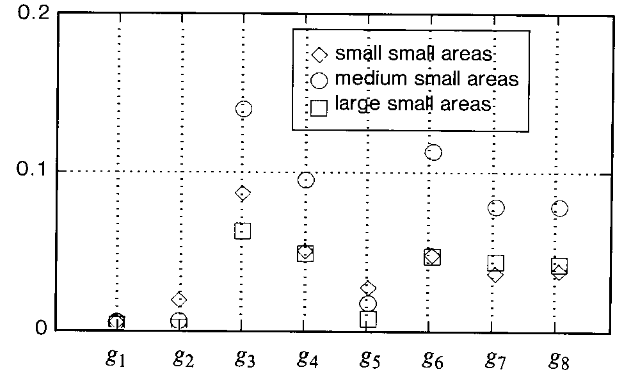

Figure 3. Absolute Relative Biases: Averaged within Size Groups

Table 3

Relative Root Mean Squared Errors and True Total Cattle and Calves for Small Areas

	Area	True Values	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
Small Size	9	8,502	.580	.277	.342	.275	.277	.199	.160	.174
	10	18,990	.360	.196	.078	.113	.175	.097	.103	.104
	11	18,776	.339	.122	.122	.103	.112	.096	.086	.087
	12	19,819	.409	.237	.076	.152	.212	.123	.117	.117
	Average	16,522	.422	.208	.154	.161	.194	.129	.116	.120
Medium Size	1	27,595	.312	.206	.117	.130	.185	.120	.100	.102
	6	29,012	.306	.241	.256	.216	.224	.224	.168	.172
	7	23,600	.341	.121	.107	.094	.110	.088	.092	.092
	8	23,627	.383	.250	.155	.165	.219	.155	.146	.144
	Average	25,959	.336	.205	.159	.151	.185	.147	.126	.127
Large Size	2	35,592	.268	.171	.113	.110	.156	.096	.089	.088
	3	40,582	.241	.151	.087	.090	.137	.070	.072	.073
	4	42,396	.256	.160	.099	.103	.144	.080	.088	.089
	5	35,996	.270	.176	.091	.097	.160	.088	.085	.088
	Average	38,642	.259	.164	.098	.100	.149	.083	.083	.084

Table 4

Absolute Relative Biases and True Total Cattle and Calves for Small Areas

	Area	True Values	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
Small Size	9	8,502	.002	.047	.232	.139	.085	.099	.061	.069
	10	18,990	.002	.002	.006	.007	.003	.015	.026	.025
	11	18,776	.002	.009	.090	.052	.021	.062	.039	.037
	12	19,819	.000	.007	.019	.011	.007	.023	.024	.023
	Average	16,522	.001	.016	.087	.052	.029	.050	.037	.039
Medium Size	1	27,595	.001	.003	.093	.063	.007	.078	.044	.045
	6	29,012	.000	.001	.239	.157	.023	.195	.120	.123
	7	23,600	.000	.005	.088	.053	.014	.058	.062	.061
	8	23,627	.002	.008	.143	.106	.024	.124	.093	.091
	Average	25,959	.001	.004	.141	.095	.017	.114	.080	.080
Large Size	2	35,592	.000	.000	.095	.071	.009	.068	.049	.047
	3	40,582	.000	.001	.047	.041	.005	.029	.026	.025
	4	42,396	.001	.002	.066	.056	.008	.044	.057	.056
	5	35,996	.000	.000	.045	.029	.005	.048	.035	.039
	Average	38,642	.000	.001	.063	.049	.006	.047	.042	.042

Figure 3 and Table 4 are identical to Figure 2 and Table 3 in format, but show relative biases instead of relative root mean squared errors. The biases for both the expansion estimator g_1 and the post-stratified g_2 are negligible. For the smoothing methods the average absolute relative biases for medium size small areas are relatively large, mainly because of areas 6 and 8 for which the covariate is least appropriate. Among smoothing methods, the sample size dependent g_5 has the least bias because it is usually very close to the direct g_2; however, it also gains very little over g_2 with respect to mean squared error. Of the remaining smoothing methods the time series estimators g_7 and g_8, which had the smallest mean squared error, also have the smallest bias. Nevertheless, the relative bias of these methods can be quite large, as in areas 6 and 8. In practice it would not be possible to estimate these biases; however, the possible size of the bias could be assessed using simulated sampling from a variety of plausible populations.

5. CONCLUDING REMARKS

It was seen by means of a simulation study that small area estimation methods obtained by combining both cross-sectional and time series data can perform better than those based only on cross-sectional data, with respect to both bias and mean squared error. However, the cost in terms of bias could still be substantial. A question of obvious importance is whether it is possible in practical situations to judge if the gains from any type of smoothing would outweigh the costs, and how to make this judgement.

The models for the simulation study were chosen on general considerations. However, in practice, suitable diagnostics similar to those employed in Pfeffermann and Barnard (1991) should be developed for survey data before any model-assisted method can be recommended. It should also be noted that the small area estimators could be modified to make them robust to mis-specification of the underlying model as suggested by Pfeffermann and Burck (1990), see also Mantel, Singh and Bureau (1993). Finally, modification and further extension of the methods presented in this paper to the more realistic case of correlated sampling errors should be investigated in the future.

ACKNOWLEDGEMENT

We would like to thank Jon Rao, Danny Pfeffermann and M.P. Singh for useful discussions and comments on earlier versions of this paper. The comments and suggestions of an anonymous referee and an Associate Editor are also very much appreciated. The first author's research was supported in part by a grant from the Natural Sciences and Engineering Research Council of Canada held at Carleton University.

APPENDIX

A.1 Variance Estimation for g_{2kt}

Let v_{kt} denote the conditional (given n_{hkt}) variance of g_{2kt} in (2.2). Then v_{kt} is given by (whenever $n_{hkt} > 0$ for all h at time t),

$$v_{kt} = \sum_h N_{hkt}^2 \left(n_{hkt}^{-1} - N_{hkt}^{-1} \right) \sigma_{hkt}^2, \tag{A.1}$$

where σ_{hkt}^2 is the population variance for the intersection of the h-th stratum with the k-th small area at time t. The variance σ_{hkt}^2 can be estimated by the usual estimator s_{hkt}^2 for $n_{hkt} \geq 2$. Note that the estimate of the conditional variance v_{kt} also provides an estimate of the unconditional variance of g_{2kt}.

If $n_{hkt} = 1$, then we can use a synthetic value as an estimate of σ_{hkt}^2 which can be defined as $\sum (n_{hkt} - 1) s_{hkt}^2 / \sum (n_{hkt} - 1)$, the summation being over all k for which $n_{hkt} \geq 2$ within each (h,t). If $n_{hkt} = 0$, v_{ht} of (A.1) is of course not defined. With the synthetic value of $\bar{y}_{hkt}$ used in this case, we need a synthetic value of its mean squared error. For each (h,t), it can be defined as

$$(\bar{X}_{hkt}/\bar{X}_{ht})^2 (n_{ht}^{-1} - N_{ht}^{-1}) s_{ht}^2 + (\widehat{\text{bias}})^2,$$

where $(\widehat{\text{bias}})^2$ will be taken as

$$\sum_{n_{hlt}>0} ((\bar{X}_{hlt}/\bar{X}_{ht})\bar{y}_{ht} - \bar{y}_{hlt})^2 / m_{ht},$$

where m_{ht} is the number of small areas with sample in stratum h at time t.

A.2 Estimation of Variance Components

Using the notation of (3.2), we here illustrate the method of moments for estimating variance components for the model of Section 3.1 in the special case when there is only one auxiliary variable X_{ht}, $Q_t = \tau^2 I$ and $\underset{\sim}{\beta}_t$ follows a random walk, *i.e.*, $G_t^{(1)} = I$. Let $F_t = (F_{1t}, \ldots, F_{Kt})'$, $F_{kt} = (1, X_{kt})'$, $\underset{\sim}{\beta}_t = (\beta_{1t}, \beta_{2t})'$, and $B_t = \text{diag}(\gamma_1^2, \gamma_2^2)$. The parameter τ^2 is estimated by the solution of

$$\sum_{t=1}^{T} \sum_{k=1}^{K} (g_{2kt} - F_{kt}' \hat{\underset{\sim}{\beta}}_t)^2 / (v_{kt} + \tau^2) = T(K-2).$$

If there is no positive solution, we set $\hat{\tau}^2 = 0$. Here $\hat{\underset{\sim}{\beta}}_t$ denotes the WLS estimate of $\underset{\sim}{\beta}_t$ based on only the cross-sectional data at t. This is analogous to the method used in Fay and Herriot (1979) for cross-sectional data. An estimate of γ_i^2 can be obtained by solving (for $i = 1,2$)

$$\sum_{t=2}^{T} (\hat{\beta}_{i,t} - \hat{\beta}_{i,t-1})^2 / (\gamma_i^2 + d_{ii}^{(t)}) = T - 1,$$

where $d_{ii}^{(t)}$ is the (i,i)-th element of $(F_{t-1}' U_{t-1}^{-1} F_{t-1})^{-1} + (F_t' U_t^{-1} F_t)^{-1}$.

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