Robust Joint Modelling of Labour Force Series of Small Areas

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ABSTRACT

In this article we report the results of fitting a state-space model to Canadian unemployment rates. The model assumes an additive decomposition of the population values into a trend, seasonal and irregular component and separate autoregressive relationships for the six survey error series corresponding to the six monthly panel estimators. The model includes rotation group effects and permits the design variances of the survey errors to change over time. The model is fitted at the small area level but it accounts for correlations between the component series of different areas. The robustness of estimators obtained under the model is achieved by imposing the constraint that the monthly aggregate model based estimators in a group of small areas for which the total sample size is sufficiently large coincide with the corresponding direct survey estimators. The performance of the model when fitted to the Atlantic provinces is assessed by a variety of diagnostic statistics and residual plots and by comparisons with estimators in current use.

KEY WORDS: Design variance; Kalman filter; Panel survey; Rotation bias; State-space model.

1. INTRODUCTION

A time series model for survey data is the combination of two distinct models. The "census model" describing the evolution of the finite population values over time and the survey errors model representing the time series relationships between the survey errors of the survey estimators. There are at least four main reasons for wishing to model the raw survey estimators:

(a) The model based estimators of the population values resulting from the modelling process have in general smaller variances than the survey estimators, particularly in small areas where the sample sizes are small.

(b) The model we employ yields estimators for the seasonal effects and for the variances of these estimators as a by-product of the estimation process.

(c) The model can be used to forecast the population values, the trend and the seasonal components for time periods beyond the sample time period for which the direct survey estimators are available. Such forecasts are important when assessing the performance of the model and for policy decision making.

(d) The model can be used to detect turning points in the level of the series and assess their significance. (Work on this problem will be addressed in a separate article).

The methodology described in this article integrates the methodologies presented in Pfeffermann and Burck (1990) and Pfeffermann (1991) with some new modifications and extensions. The main features of the model are as follows:

1. The model decomposes the population values into the unobservable components of trend, seasonality and irregular terms. Smoothed predictors of the three components (and hence of the population values) based on all the available data, and standard errors of the prediction errors are obtained straightforwardly by application of the Kalman filter. The standard errors are modified to account for the extra variation induced by the use of estimated parameter values.

2. The model uses the distinct monthly panel estimators as input data. The use of the panel estimators has two important advantages over the use of the mean estimators: (i) It identifies better the time series model holding for the survey errors by analysing contrasts between the panel estimators, (ii) It yields more efficient estimators for the model parameters and hence better predictors for the unobservable model components.

3. The model accounts for changes in the variances of the survey errors over time and for possible rotation group effects.

4. The model can be applied simultaneously to the panel estimators in separate small areas. The census model is extended in this case to account for the cross-correlations between the unobservable components of the population values operating in these areas.

5. A modification to ensure the robustness of the small area estimators against possible model breakdowns is incorporated into the model equations. The modification consists of constraining the model based estimators of aggregates of the population values over a group of small areas for which the total sample size is sufficiently large to coincide with the corresponding aggregate survey estimators. As a result, sudden changes in the level of the series are reflected in the model based estimators with no time lag.

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The model and the robustness modifications are described in more detail in section 2. Empirical results obtained when fitting the model to the four Atlantic provinces of Canada are presented in section 3. Section 4 contains a short summary with suggestions for extension of the analysis.

Before concluding this section we mention that in the U.S., the state unemployment estimates are produced for most of the states based on time series models which have a similar structure to the model used in our study. See Tiller (1992) for details. A major difference between the two models is that in the U.S., the model postulated for the population values includes also explanatory variables so that the trend and the seasonal component only account for the trend and seasonal variations not accounted for the explanatory variables. The models fitted to the survey errors are like in our case of the ARIMA type and they likewise account for changes in the variances of the survey errors. They are otherwise different because of the very different sample rotation schemes used in the two countries. Another notable difference between the two models is that in the U.S., the models are fitted to each state separately and the input data consist of only the mean survey estimates, that is, one observation for every month. As a result, the models do not account for rotation group biases.

2. A STATE-SPACE MODEL FOR CANADA UNEMPLOYMENT SERIES

2.1 The Canadian Labour Force Survey

Data on unemployment are collected as part of the Labour Force Survey (LFS) carried out by Statistics Canada. The Canadian LFS is a rotating monthly panel survey by which every new sampled panel of households is retained in the sample for six successive months before being replaced by another panel from the same PSU's or strata. The PSU's are defined by geographic locations (city blocks or urban centers in the urban regions and groups of enumeration areas in the rural regions). The strata are homogeneous groups of PSU's defined by geographic locations such as city tracts, census subdivisions and enumeration areas. In the urban regions, (about 2/3 of the sample), every PSU is represented in only one panel. In the rural regions, the PSU's are represented in all the panels but with different enumeration areas in different panels. As a result, the separate panel estimators can be assumed to be independent, a property validated and utilized in other studies, see e.g. Lee (1990). For a recent report describing the design of the LFS and the construction of the direct survey estimators, the reader is referred to Singh et al. (1990).

2.2 The Census Model

In what follows we consider a single small area. In section 2.4 we consider joint modelling of the panel estimates in a group of small areas. The model postulated for the population values is the Basic Structural Model (BSM) which consists of the following set of equations.

$$Y_t = L_t + S_t + \epsilon_t; \quad L_t = L_{t-1} + R_{t-1} + \eta_{Lt};$$

$$R_t = R_{t-1} + \eta_{Rt}; \quad \sum_{j=0}^{11} S_{t+j} = \eta_{St}. \quad (2.1)$$

In (2.1) Y_t is the population value ("true" unemployment rate) at time t, L_t, is the trend level, R_t is the increment, S_t the seasonal effect and ϵ_t the irregular term assumed to be white noise with zero mean and variance σ_ϵ^2. Thus, the first equation in (2.1) postulates the classical decomposition of a time series into a trend, seasonal and irregular components. This decomposition is inherent in the commonly used procedures for seasonal adjustment, see e.g. Dagum (1980). Notice however that in the present case the series $\{Y_t\}$ is itself unobservable. The series $\{\eta_{Lt}\}$, $\{\eta_{Rt}\}$ and $\{\eta_{St}\}$ are independent white noise disturbances with mean zero and variances σ_L^2, σ_R^2 and $\sigma_S^2 \times g(t)$ respectively. Hence, the second and third equations of (2.1) define a local approximation to a linear trend whereas the last equation models the evolution of the seasonal effects such that the sum of every 12 successive effects fluctuates around zero. Notice that the variances of the error terms η_{St} are time dependent. The functions $g(t)$ are specified at the end of section 3.1.

The theoretical properties of the BSM in comparison to other models are discussed in Harrison and Stevens (1976), Harvey (1984) and Maravall (1985). Empirical results illustrating the performance of the model are shown in Harvey and Todd (1983), Morris and Pfeffermann (1984) and Pfeffermann (1991). Although more restricted than the family of ARIMA models, the BSM is now recognized as being flexible enough to approximate the behaviour of many diverse time series.

2.3 The Survey Errors Model

The model holding for the survey errors was identified initially by analyzing separately the pseudo error series $e_{t,p}^{(j)} = (y_t^{(j)} - \bar{y}_t)$, $t = 1, \ldots, N$, where $y_t^{(j)}$ is the estimator of Y_t based on j-th panel $j = 1, \ldots, 6$, (the panel surveyed for the j-th successive month) and $\bar{y}_t = \sum_{j=1}^{6} y_t^{(j)}/6$ is the mean estimator. Notice that $(y_t^{(j)} - \bar{y}_t) = (e_t^{(j)} - \sum_{j=1}^{6} e_t^{(j)}/6)$, where $e_t^{(j)} = (y_t^{(j)} - Y_t)$ are the true survey errors. Thus, the notable feature of the contrasts $(y_t^{(j)} - \bar{y}_t)$ is that they are functions of only the survey errors irrespective of the model holding for the population values.

There are two prior considerations in the choice of a model for the survey errors:

(a) The model should account for possible rotation group biases or more generally, allow for different means for the survey errors of different panels.

(b) The model should account for changes in the variances of the survey errors over time.

Rotation group biases may arise from providing different information on different rounds of interview, depending on the length of time that respondents are included in the sample, or on the method of data collection, say, whether by telephone or by home interview. (In the Canadian LFS, the first panel is interviewed by home visits, the other panels are interviewed by telephone). Another possible reason for differences between the panel survey error means is differences in the nonresponse patterns across the panels. See Pfeffermann (1991) for further discussion with references to earlier studies on this problem.

Changes in the variances of the survey errors over time occur when the variances are function of the level of the series. Indeed, as revealed by figure 1 in section 3, the estimates of the standard deviations of the survey errors are subject to seasonal effects with a seasonal pattern that follows the seasonal pattern of the population values. Another possible explanation for changes in the variances of the survey errors is changes in the sampling design. For example, the overall sample size of the Canadian LFS was reduced in 1985-1986 from 55,000 households to 48,000 households. This reduction in the sample size was associated with other changes in the design. See Singh et al. (1990) for details.

Application of simple model estimation and diagnostic procedures to the pseudo survey errors suggest a 3rd order autoregressive (AR) model for the standardized survey errors $\bar{e}_t^{(j)} = (e_t^{(j)} - \beta_j)/SD(e_t^{(j)})$, i.e.

$$\bar{e}_t^{(j)} = \phi_{j1} \bar{e}_{t-1}^{(j-1)} + \phi_{j2} \bar{e}_{t-2}^{(j-2)} + \phi_{j3} \bar{e}_{t-3}^{(j-3)}$$

$$+ u_t^{(j)}, \quad j = 1, \ldots, 6, \quad (2.2)$$

where $\beta_j = E(e_t^{(j)})$ are the rotation group biases, $SD(e_t^{(j)})$ are the design standard deviations and $u_t^{(j)}$ are independent white noise with mean zero and variances σ_j^2. It is assumed that $\sum_{j=1}^{6} \beta_j = 0$ which implies that the mean survey estimator, $\bar{y}_t$, is unbiased. See Pfeffermann (1991) for discussion on the need to constraint the bias coefficients. Subsequent analysis when fitting the combined model defined by (2.1) and (2.2) (see section 2.4) validates this model with the further observation that the coefficients $(\phi_{j1}, \phi_{j2}, \phi_{j3})$ can be assumed to be equal for $j = 4, 5, 6$. Furthermore, for the first panel an AR(1) model already gives a good fit whereas for the second and third panel an AR(2) model is appropriate although with different coefficients. These relationships hold for each of the four Atlantic provinces.

One of the referees of this article raised the question of whether the AR(3) model defined by (2.2) is flexible enough to account for the panel estimates correlations at high lags which are believed to be high because of "PSU effects". As mentioned in section 2.1, panels rotating out of the sample are replaced by panels from the same PSU's and it usually takes several years before a PSU is exhausted and replaced by a neighbouring PSU. Lee (1990) presents two sets of panel estimates correlations for the Canadian LFS. The first set, denoted by ρ_j, are the correlations between estimates produced from the same panel so that j ranges from 1 to 5. The second set, denoted by γ_j, are the correlations between estimates produced from a panel and its predecessor so that j ranges from 1 to 11. The ρ-correlations are generally high as expected but it should be emphasized that they are lower for the unemployment series than for the employment series, demonstrating the high mobility of the unemployment Labour Force. The γ-correlations are much smaller than the ρ correlations but as mentioned by the author, the computation of these correlations is much less reliable and their behavior is somewhat fuzzy showing occasionally an increasing trend. We computed the serial correlations based on the models (2.2) with the ϕ-coefficients replaced by their estimated values and found in general a close fit to the ρ-correlations at all the lags from 1 to 5. The correlations at higher lags are different from the corresponding γ-correlations but interesting enough, they are in most cases higher and always decrease as j increases.

Another question related to the model (2.2) raised by the referees is whether one could apply the log transformation to the raw data for stabilizing the survey error variances, rather than modelling the standardized errors. There are two main reasons for not using the log transformation in our case. Foremost, the use of this transformation would imply a multiplicative decomposition for the population unemployment rates which is counter to common practice of postulating an additive decomposition. In Statistics Canada the unemployment rates in the two larger provinces out of the four considered in our study are deseasonalized by postulating the additive decomposition. In the U.S. the models fitted to the state unemployment series likewise postulate an additive decomposition. See Tiller (1992). The second reason is that changes in the survey error variances may result from charges in the sampling design and in particular, from changes in the sample sizes. Such changes cause discrete shifts in the variances which cannot be handled effectively by the log transformation. As noted also by one of the referees, transforming the data has the drawback of producing nonlinearity in aggregating the estimates over the panels and/or the small areas.

The model defined by (2.2) satisfies the two prior considerations discussed above. The actual application of the model requires however two modifications:

1. For the first three panels there is not a long enough history to permit the fitting of an AR(3) model. For example, the survey error $e_t^{(1)}$ corresponds to the panel which is in the sample for the first time. In order to overcome this problem, we replace the missing survey errors by the survey errors corresponding to the panels previously selected from the same PSU's or strata. For example, the AR(2) model fitted to $\tilde{e}_t^{(2)}$ is

$$\tilde{e}_t^{(2)} = \phi_{21} \tilde{e}_{t-1}^{(1)} + \phi_{22} \tilde{e}_{t-2}^{(6)} + u_t^{(2)}. \quad (2.3)$$

Notice that the panel surveyed for the second time at month t replaces at time $(t - 1)$ the panel surveyed for the sixth time at month $(t - 2)$ so that both panels represent the same PSU's or strata. The use of surrogate survey errors in the case of the first three panels may explain the different models identified for these panels as compared to the model identified for the other three panels.

2. The true standard deviations of the survey errors are unknown whereas the survey estimates of the standard deviations are themselves subject to sampling errors. To overcome this problem, we use smoothed values of the estimated standard deviations, obtained by fitting the relationship

$$(\widetilde{SD})_t = \hat{\gamma} (\widehat{SD})_{t-1} + \hat{\gamma}_0 t + \sum_{i=1}^{12} \hat{\gamma}_i D_{it}, \quad (2.4)$$

with the γ-coefficients estimated by ordinary least squares. The notation $(\widehat{SD})_t$ defines the raw, unsmoothed estimate of the design standard deviation of the mean survey estimator, $\bar{y}_t$, at month t and $\{D_{it}\}$ are dummy variables accounting for monthly seasonal effects so that $D_{it} = 1$ when $t = 12k + i, k = 0, 1, \ldots, i = 1, \ldots, 12$ and $D_{it} = 0$ otherwise. The smoothed standard deviations of the panel survey errors are obtained as $\widetilde{SD}(e_t^{(j)}) = \sqrt{6}(\widetilde{SD})_t$. The latter estimates are used as surrogates for the true, unknown, standard deviations.

2.4 State-space Representation and Estimation of the Model Holding for the Survey Estimators

It follows from (2.1) that the panel estimators can be modeled as

$$y_t^{(j)} = L_t + S_t + \epsilon_t + e_t^{(j)}, \quad j = 1, \ldots, 6, \quad (2.5)$$

where

$$L_t = L_{t-1} + R_{t-1} + \eta_{Lt}; \quad R_t = R_{t-1} + \eta_{Rt};$$

$$\sum_{j=0}^{11} S_{t+j} = \eta_{St}, \quad (2.6)$$

with $\{\epsilon_t\}$, $\{\eta_{Lt}\}$, $\{\eta_{Rt}\}$ and $\{\eta_{St}\}$ defined as in (2.1). The separate models defined by (2.5), (2.6) and (2.2) can be cast into a compact state-space representation with $y_t' = (y_t^{(1)}, \ldots, y_t^{(6)})$ as the input data, similar to the representation in Pfeffermann (1991). Following that representation, the survey errors (and in the present study also the census irregular terms) are included as part of the state vector so that there are no residual terms in the observation equation defined by (2.5). Unlike in Pfeffermann (1991), however, the transition matrix and the Variance-Covariance (V-C) matrix of the state error terms are not fixed in time since they depend on the design variances of the survey errors which, as explained in section 2.3, change over time.

The state-space representation of the model permits us to update, smooth or predict the state vectors and hence the seasonal, trend and population values at any given month t by means of the Kalman filter. Denote by α_t the state vector corresponding to month t. The state vector comprises the trend level, increment and seasonal effects, the rotation group biases and the survey errors. See Pfeffermann (1991) for details. By "updating" we mean estimation of α_t at month t based on all the data until and including month t. "Smoothing" refers to the estimation of α_t based on all the available data for all the months before and after month t. Smoothing is required for improving past estimates as, for example, when estimating the seasonal effects or when estimating changes in the population values or the trend levels. "Prediction" of state vectors corresponding to postsample months is important for policy making. Predictions within the sample period allow to assess the performance of the model, e.g. by comparing the forecasted panel estimates as derived from the predicted state vectors with the actual estimates. See section 3 for details. The theory of state-space models and the Kalman filter is developed in numerous publications, see Pfeffermann (1991) for the filtering and smoothing equations with references. Notice that the filtering and the smoothing equations not only yield the three sets of estimators for any given month t but also the V-C matrices of the corresponding estimation errors.

The actual application of the Kalman filter requires the estimation of the unknown model parameters and the initialization of the filter, that is, the estimation of the initial state vector α_0 and the corresponding V-C matrix of the estimation errors. For a single small area, the unknown model parameters are the four variances of the error terms in the census model (2.1) and the eight

autoregression coefficients and six residual variances in the panel survey error models (2.2). (The rotation group means are included in the state vectors as fixed, time invariant coefficients). In order to reduce the number of free parameters in the combined state-space model, we assume $\sigma_j^2 = \sigma^2 \times \tilde{\sigma}_j^2, j = 1, \ldots, 6$, where $\{\sigma_j^2\}$ are the residual variances in (2.2) and $\tilde{\sigma}_j^2$ are the estimates of the residual variances obtained by fitting the autoregression equations to the pseudo survey errors $e_{t,p}^{(j)}$ defined in section 2.3. This assumption reduces the number of unknown parameters from 18 to 13. (The estimates $\tilde{\sigma}_j^2$ are very close for $j = 4, 5, 6$ and have been set equal).

Assuming that the error terms in the census and survey error models have a normal distribution, the unknown model parameters can be estimated by maximization of the likelihood. See Pfeffermann and Burck (1991) for a brief description of the application of the method of scoring maximization algorithm and for the initialization of the filter. That article includes references to more rigorous discussions.

2.5 Adjustments to Account for the Use of Estimated Parameter Values

Once the unknown model parameters have been estimated, the Kalman filter equations can be applied with the true parameter values replaced by the parameter estimates. As noted in section 2.4, the Kalman filter not only produces estimates for the state vectors but also the V-C matrices of the corresponding estimation errors. A possible problem arising from the use of these V-C matrices, however, is that they ignore the extra variation implied by parameter estimation, thus resulting in underestimation of the true variances.

Formally, let $\hat{\alpha}_t(\hat{\lambda})$ define the estimator of α_t at month t, based on all the data available until some given month n, where $\hat{\lambda}$ represents the estimators of the unknown model parameters. The estimation error can be decomposed as

$$[\hat{\alpha}_t(\hat{\lambda}) - \alpha_t] = [\hat{\alpha}_t(\lambda) - \alpha_t] + [\hat{\alpha}_t(\hat{\lambda}) - \hat{\alpha}_t(\lambda)], \quad (2.7)$$

which is the sum of the error if λ were known plus the error due to estimation of λ. The two terms in the right-hand side of (2.7) are uncorrelated. A simple way to verify this property is by noting that $\hat{\alpha}_t(\lambda) = E(\alpha_t | Y, \lambda)$ where Y represents all the available data. By conditioning on Y and λ, $[\hat{\alpha}_t(\hat{\lambda}) - \hat{\alpha}_t(\lambda)]$ is nonstochastic whereas $E\{[\hat{\alpha}_t(\lambda) - \alpha_t] | Y, \lambda\} = 0$. It follows therefore from (2.7) that

$$Q_t = E\{[\hat{\alpha}_t(\hat{\lambda}) - \alpha_t][\hat{\alpha}_t(\hat{\lambda}) - \alpha_t]'\}$$

$$= E\{[\hat{\alpha}_t(\lambda) - \alpha_t][\hat{\alpha}_t(\lambda) - \alpha_t]'\}$$

$$+ E\{[\hat{\alpha}_t(\hat{\lambda}) - \hat{\alpha}_t(\lambda)][\hat{\alpha}_t(\hat{\lambda}) - \hat{\alpha}_t(\lambda)]'\}$$

$$= A_t + B_t. \quad (2.8)$$

In order to estimate A_t and B_t we condition on Y and follow the approach proposed by Hamilton (1986). By this approach, realizations $\lambda_{(k)}, k = 1, \ldots, K$ are generated from the asymptotic normal posterior distribution of λ, that is, from a $N(\hat{\lambda}, \hat{\Lambda})$ distribution where $\hat{\lambda}$ is the maximum likelihood estimator of λ and $\hat{\Lambda}$ is the asymptotic V-C matrix of $\hat{\lambda}$. (Both $\hat{\lambda}$ and $\hat{\Lambda}$ are obtained from the method of scoring). The Kalman filter is then applied with each of these realizations yielding estimates $\hat{\alpha}_t(\lambda_{(k)})$ with V-C matrices $P_t(\lambda_{(k)})$. The matrices A_t and B_t are estimated as

$$\hat{A}_t = \frac{1}{k} \sum_{k=1}^{K} P_t(\lambda_{(k)});$$

$$\hat{B}_t = \frac{1}{k} \sum_{k=1}^{K} [\hat{\alpha}_t(\lambda_{(k)}) - \hat{\alpha}_t(\hat{\lambda})][\hat{\alpha}_t(\lambda_{(k)}) - \hat{\alpha}_t(\hat{\lambda})]'.$$

$$(2.9)$$

Ansley and Kohn (1986) propose an estimator for B_t based on first order Taylor series approximation. The use of their estimator is computationally less intensive but the procedure proposed by Hamilton is somewhat more flexible in terms of the assumptions involved and it enables a better insight into the sensitivity of the Kalman filter output to errors in the parameter estimators.

2.6 Joint Modelling in Several Small Areas

The model considered so far refers to a single area. When the sample sizes in the various areas are small, more efficient estimators can often be derived by modelling in addition the cross-sectional relationships between the area population values. Clearly, the increase in efficiency resulting from such joint modelling depends on the sample sizes within the small areas and the closeness of the behaviours of the area population values over time.

The survey errors are independent between the areas so that any joint modelling of the survey estimators applies only to the census model. For modelling the unemployment rates in the four Atlantic provinces, we follow Pfeffermann and Burck (1990) and allow for nonzero contemporary correlations between corresponding error terms of the census models operating in these provinces. Thus, if $y_{t,a}' = (\epsilon_t^{(a)}, \eta_{Lt}^{(a)}, \eta_{Rt}^{(a)}, \eta_{St}^{(a)})$ denotes the vector of error terms at time t associated with the census model operating in area a, it is assumed that $C_{a,b} = E(y_{ta} y_{tb}')$ is diagonal but with possibly non zero covariances on the main diagonal. The actual implication of this assumption is that if, for example, there is a significant increase in the trend level in one province, similar increases can be expected to occur in other provinces.

The resulting joint model holding for the four provinces (or more generally for a group of areas) can again be cast into a state-space form, see equations (2.7) and (2.8) in

Pfeffermann and Burck (1990). A major problem with the fitting of this model, however, is the joint estimation of all the unknown parameters which is computationally too intensive in terms of computer time and storage space. (The computer program written for the application of the method of scoring uses numerical first order derivatives so that each derivative requires a separate sweep through all the data. Each sweep involves the computation of the Kalman filter equations for each month included in the sample period).

To deal with this problem, we first fitted the models defined by (2.5), (2.6) and (2.2) separately for each of the provinces. We also postulated equal correlations between the corresponding error terms of the separate census models across the provinces so that

$$\phi_{a,b} = C_{a,a}^{-\frac{1}{2}} C_{a,b} C_{b,b}^{-\frac{1}{2}} = \phi \quad 1 \le a,b \le 4, \quad (2.10)$$

where $C_{a,a} = E(y_{ta} y_{ta}')$. The four correlations maximizing the likelihood of the joint model were determined by a grid search procedure with the other model parameters held fixed at their previously estimated values.

The assumption of equal correlations reduces the number of unknown parameters considerably. It can be justified also by the small number of areas considered for this study implying that no other pre-imposed structure on these correlations can be safely detected. More substantively, a simple breakdown of the Labour Force by industry (Table 1 of Section 3) shows very similar relative frequencies in the four provinces suggesting a high degree of homogeneity in their economies.

2.7 Modifications to Protect Against Model Failures

The use of a model for the production of official statistics raises the question of how to protect against possible model failures. As discussed below, testing the model every time that new data becomes available is not feasible requiring instead the development of a built-in mechanism to ensure the robustness of the estimators when the model fails to hold.

For modelling the Labour Force series in small areas we employed the modification proposed by Pfeffermann and Burck (1990). By this modification, the updated state vector estimates at any given time t, are constraint to satisfy the condition

$$\sum_{a=1}^{A} w_{ta} \hat{Y}_{ta} = \sum_{a=1}^{A} w_{ta} \bar{y}_{ta} \quad t = 1, 2, \ldots, \quad (2.11)$$

where $\hat{Y}_{ta}$ is the model based estimator of the population value Y_{ta} in area a, $\bar{y}_{ta} = 1/6 \sum_{j=1}^{6} y_{ta}^{(j)}$ is the corresponding survey estimator and $w_{ta} = M_{ta}/M_t$ is the relative size of the Labour Force in that area so that $M_t = \sum_{a=1}^{4} M_{ta}$ and $\sum_{a=1}^{4} w_{ta} = 1$. Notice that $\sum_{a=1}^{A} w_{ta} \hat{Y}_{ta}$

and $\sum_{a=1}^{A} w_{ta} \bar{y}_t$ are correspondingly the model based estimator and the direct survey estimator of the aggregate population value in the group of areas considered. The condition 2.11 can be written alternatively as $\sum_{a=1}^{A} w_{ta} \bar{e}_{ta} = 0$ where $\bar{e}_{ta} = \sum_{j=1}^{6} e_{ta}^{(j)}/6$ is the mean survey error for state a. Pfeffermann and Burck (1990) show how to modify the Kalman filter equations so that it produces the constrained state vector estimator and its correct V-C matrix under the model (without the constraint), for every month t.

The rationale behind the modification is simple. It assumes that the total sample size in all the areas is sufficiently large and hence that the aggregate survey estimators can be trusted. This assumption in fact dictates the level of aggregation required, see below. By constraining the aggregate model based estimators to coincide with the aggregate survey estimators, the analyst ensures that any real change in the population values reflected in the survey estimators will be likewise reflected in the model based estimators. Notice that without constraining the estimators, sudden changes in the level of the series, for example, will be reflected in the model based estimators only after several months because these estimators depend not only on current data but also on past data. On the other hand, if no substantial changes occur, the model based estimators can be expected to satisfy approximately the constraints even without imposing them explicitly. Thus, the constrained estimators should perform almost as well as the unconstrained estimators in regular time periods.

The assumption that the total sample size in all the areas is large and hence that the aggregate survey estimator is sufficiently close to the corresponding population value is critical. It guarantees (in high probability) that the modification will only occur when there are real changes in the population values and not as a result of large sampling errors. Admittedly, and as noted by one of the referees, in the application of the method to the Atlantic provinces described in section 3, the aggregate estimator is based on only four provinces so that its standard error is about 50 percent of the standard errors of the province survey estimators, depending on the province sample sizes. (The province survey estimators are independent, conditional on the corresponding population province values). Thus, if the constraints are to be used in practice, the aggregation should be carried out over a larger set of provinces or other small areas.

The following two alternative approaches have been suggested for dealing with the robustness problem:

(i) Perform a time series outlier detection as proposed for example in Chang, Tiao and Chen (1988).

(ii) Model the time series of proportions $\{ \hat{\pi}_{ta} = \bar{y}_{ta} / \sum_{a=1}^{A} \bar{y}_{ta}, a = 1, \ldots, (A - 1) \}$ if these time series exhibit smoother behavior than the series $\{ \bar{y}_{ta} \}$.

The detection of outliers is an important aspect of any modelling exercise but the question remaining is how to modify the population value estimates once observations (survey estimates) are detected as outliers. Notice in this respect that our main concern is with current estimates that is, the most recent available estimates. In Chang, Tiao and Chen (1988), the motivation for the outlier detections is to *remove* their effect from the observations so as to better understand the underlying structure of the series and improve the estimation of the model parameters. But if the cause of an outlier observation is a real shift in the level of the population values, this shift should not be removed but rather accounted for in the model based estimators. Harrison and Stevens (1976) propose to account for such changes by modifying the prior distribution of the state vectors, *e.g.* by increasing the variances of the state vector errors so as to allow for more rapid changes in the state vector estimators. See Morris and Pfeffermann (1984) for an example. Our approach of constraining the model based estimators to coincide with aggregate survey estimators provides a more automatic procedure that does not require timingly prior information.

The second approach suggested for dealing with the robustness problem is appealing since abrupt changes in the population values can be expected to cancel out in the ratios $\hat{\pi}_{ta}$. The main disadvantage of the use of this approach is that the model holding for the 'true' ratios π_{ta} is naturally very different from the model holding for the population values Y_t as defined by (2.1) and in particular, it no longer provides estimates for the trend and the seasonal effects which, as mentioned in the introduction, is one of the major uses of our approach. It is also not clear how to extract the estimates for the population values Y_t from the model holding for the ratios $\hat{\pi}_{ta}$, without some additional assumptions, like, for example, our assumption that the aggregate survey estimator is sufficiently close to the corresponding population value.

The use of constraints of the form (2.11) was previously considered by Battese, Harter and Fuller (1988) and by Pfeffermann and Barnard (1991) for analyzing cross-sectional surveys. Pfeffermann and Burck (1990) present empirical results illustrating the good performance of the modified estimators in abnormal time periods. See also section 3.

3. FITTING THE MODEL TO THE ATLANTIC PROVINCES, EMPIRICAL RESULTS

The model defined by (2.2), (2.5) (2.6) and (2.10) was fitted to the monthly panel estimators in the four Atlantic provinces in two stages. In the first stage the model defined by (2.2), (2.5) and (2.6) was fitted to each of the provinces separately. In the second stage, the correlations defining the matrix ϕ of (2.10) were estimated using a grid search procedure. (See section 2.6). The estimators obtained are, $\text{Diag}(\phi) = (0.5, 0.25, 0.80, 0.0)$. The data used for estimation of the model cover the years 1982-1988. Data for 1989 were used for model diagnostics by comparing the results within and outside the sample period.

3.1 Preliminary Analysis

Table 1 shows a breakdown of the Labour Force in the four provinces by industry. The figures in the table refer to March 1991. The (expected) sample sizes of the LFS are also shown. As can be seen, the percentage breakdowns in the four provinces are very similar justifying the assumption of equal correlations between the error terms of the census models across the provinces. The similarity of the percentage breakdowns suggests also possible improvements in the efficiency of the model based estimators derived from the joint model over estimators which ignore the cross-sectional correlations between the province population values.

Table 1

Labour Force by Industry in the Atlantic Provinces, March 1991

| Sample size	Nova Scotia		New Brunswick		Newfoundland		Prince-Edward Island	
	4,409		3,843		2,970		1,421	
	Thousands	%	Thousands	%	Thousands	%	Thousands	%
Agriculture	7	1.7	7	2.3	0.5	0.2	6.0	9.8
Other primary industry	18	4.4	13	4.2	18.0	7.7	4.0	6.6
Manufacturing	44	10.7	37	11.9	23.0	9.9	6.0	9.8
Construction	24	5.9	21	6.8	18.0	7.7	4.0	6.6
Transp. and communication	35	8.6	30	9.6	20.0	8.6	5.0	8.3
Trade and Commerce	81	19.8	61	19.6	41.0	17.6	10.0	16.4
Finance	20	4.9	12	3.9	6.0	2.6	0.5	0.8
Services	143	35.0	107	34.4	83.0	35.6	19.0	31.1
Public Administration	36	8.8	22	7.0	23.0	9.9	6.0	9.8
Unclassified	1	0.2	1	0.3	0.5	0.2	0.5	0.8
Total	409	100.0	311	100.0	233.0	100.0	61.0	100.0
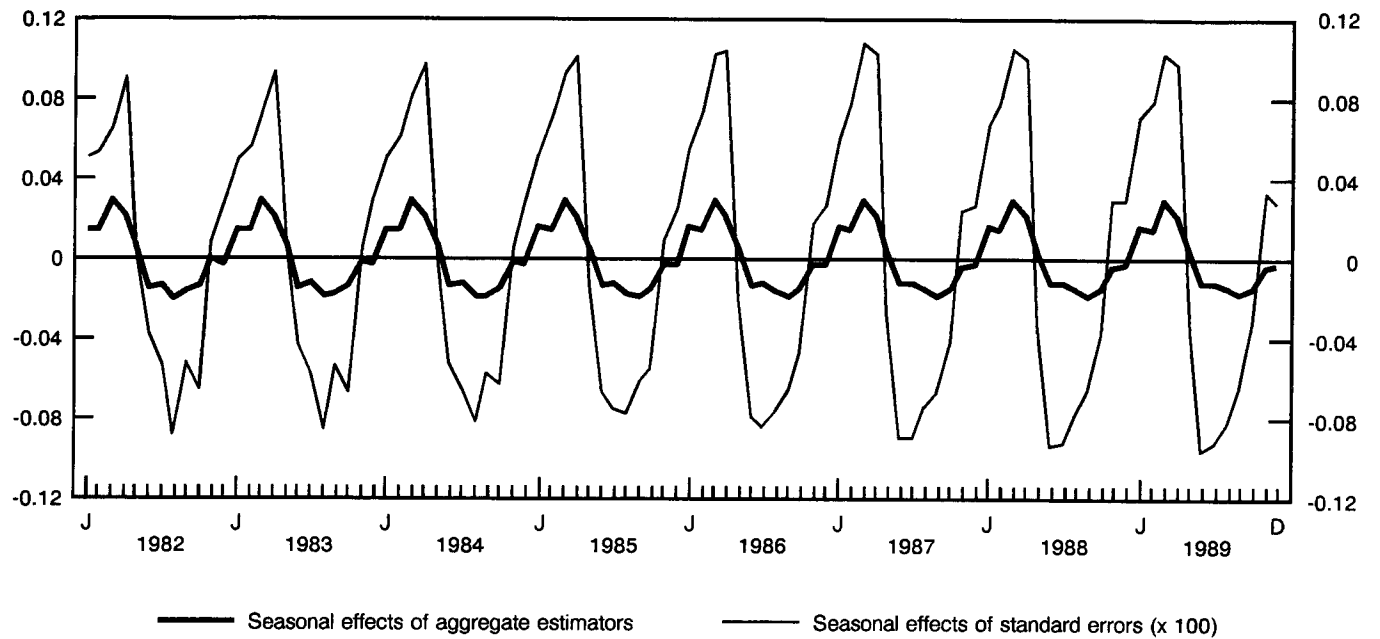
Figure 1. Seasonal Effects of Aggregate Survey Estimators and of Standard Errors of Aggregate Survey Estimators ($\times$ 100)
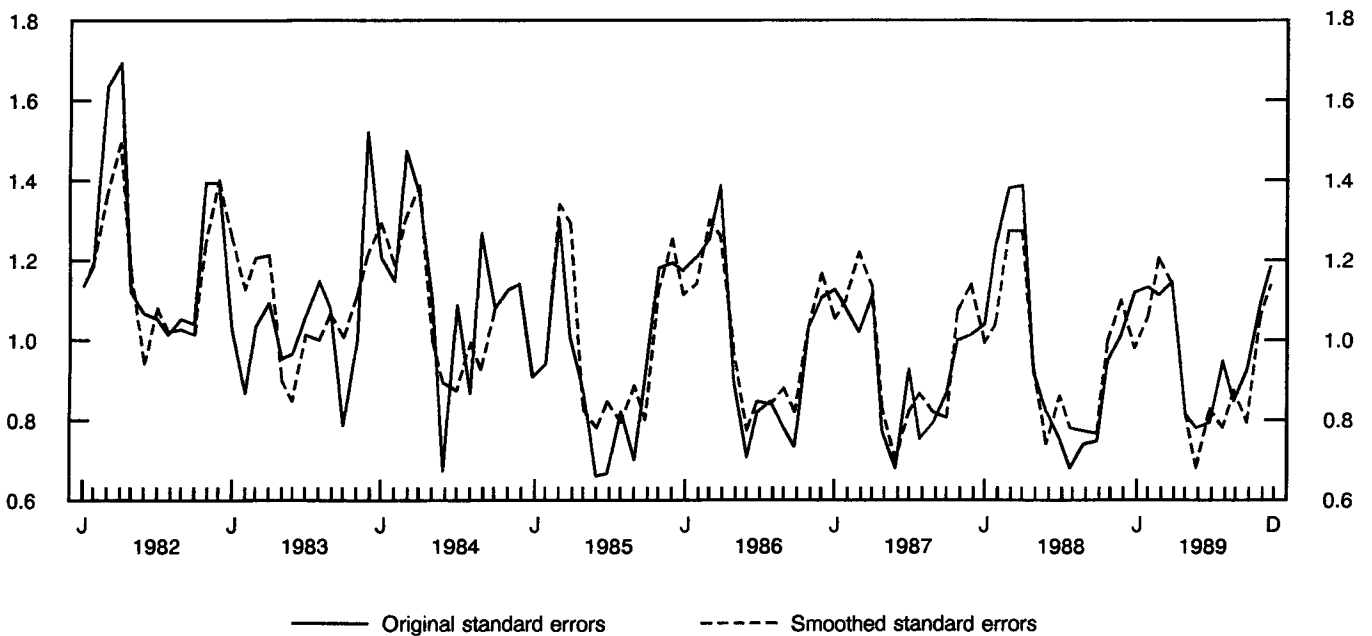

Figure 2. Original and Smoothed Standard Errors of Survey Estimators ($\times$ 100) for P.E.I. Province

Two other prior considerations mentioned in section 2.3 are that the model should account for possible rotation group effects and for changes in the variances of the survey errors over time. In order to obtain initial estimates for the rotation group effects, we averaged the pseudo survey errors, $e_{t,p}^{(j)} = (y_t^{(j)} - \bar{y}_t), j = 1, \ldots, 6$ over all the months in the sample period. We then divided the averages by the conventional estimates of the standard errors. (The errors $e_{t,p}^{(j)}$ are correlated over time but the correlations are small because except for lags 6, 12 etc. the data of any given panel refer to different PSU's in the urban areas and different enumeration areas in the rural areas. See section 2.1). Notice that in the absence of rotation group effects, $E(e_{t,p}^{(j)}) = 0$ for all j and t irrespective of the model postulated for the population values.

This preliminary (model free) analysis yields similar results to the results obtained under the full model, presented in Table 2 of section 3.3.

Next consider the variances of the survey errors.

Figure 1 plots the seasonal effects of the aggregate survey estimators in the four provinces along with the seasonal effects of the standard errors of these estimators (multiplied by 100). Denote as before by w_{ta} the relative labour force size in province a at time t. The aggregate survey estimator is defined as $y_t^* = \sum_{a=1}^4 w_{ta} \bar{y}_{ta}$ (Equation 2.11). The standard error of y_t^* is $(SD^*)_t = [\sum_{a=1}^4 w_{ta}^2 (\widehat{SD})_{ta}^2]^{1/2}$. The seasonal effects were estimated by application of the additive model of X-11 so as not to bind them to any particular model. We chose the additive model since we assume the additive decomposition for the survey estimators. (As revealed from Figure 4, the seasonal effects of the aggregate survey estimators produced by X-11 are very close to the seasonal effects obtained under the model).

Figure 1 shows that the standard errors are influenced by seasonal variations with a seasonal pattern that follows closely the seasonal pattern of the survey estimators and hence of the corresponding population values.

As discussed in section 2.3, rather than using the original estimates of the design standard errors in the models fitted to the panel survey errors we use smoothed values, thus reducing the effect of the sampling errors on the former estimators. Figure 2 plots the two sets of estimators for Prince Edward Island (P.E.I.) province which is the smallest province in the Atlantic region and hence has the smallest sample sizes. As can be seen, the effect of the smoothing is to trim the extreme raw estimates but otherwise the smoothed values behave similarly to the raw estimates. The plots for the other provinces show a similar pattern but the differences between the raw and the smoothed estimates are smaller because of the larger sample sizes in these provinces.

We conclude this section by specifying the models postulated for the seasonal effects in the four provinces. Our initial model assumed fixed variances for the error terms $\eta_{St} = \sum_{j=0}^{11} S_{t+j}, t = 1, 2, \ldots$ (see equation 2.1). The predicted errors $\hat{\eta}_{st} = \sum_{j=0}^{11} \hat{S}_{t+j}$ obtained under that model were found to decrease in absolute value as a function of time in three out of the four provinces and increase in time in the remaining province. Notice that under the model defined by (2.1), with constant variances of the state error terms, the Kalman filter converages to a steady state by which the V-C matrices of the state vector estimators and hence of $\hat{\eta}_{st}$ are constant. Thus, we modified the initial model such that $VAR(\eta_{st}) = \sigma_s^2 \times g(t)$ where for the provinces of Nova Scotia, Newfoundland and P.E.I. $g(t) = t^{(-3/2)}$ whereas for New Brunswick $g(t) = t^{1/2}$.

3.2 Results

3.2.1 Rotation Group Biases

Table 2 shows the rotation group Biases (RGB) and their estimated standard errors (SE) in the four provinces as obtained under the full model defined by (2.3), (2.5), (2.6) and (2.10).

Table 2

Rotation Group Biases and Standard Errors
in the Four Provinces ($\times$ 100)

Panels	Nova Scotia		New Brunswick		Newfound-land		Prince Edward Island	
	RGB	SE	RGB	SE	RGB	SE	RGB	SE
1	-0.20	0.10	-0.02	0.11	-0.47	0.13	0.32	0.17
2	0.18	0.09	0.40	0.10	0.42	0.12	0.18	0.15
3	0.32	0.08	0.24	0.09	0.47	0.12	0.31	0.15
4	0.06	0.07	0.01	0.09	0.18	0.12	0.03	0.15
5	-0.03	0.08	-0.15	0.10	-0.10	0.13	-0.25	0.16
6	-0.34	0.08	-0.50	0.11	-0.50	0.14	-0.60	0.16

The RGB behave fairly consistently across the provinces. Thus, the biases for the 3rd and 6th panel are all highly significant using the conventional t-statistic, having a positive sign for the 3rd panel and a negative sign for the 6th panel. The biases for the 4th and 5th panels have again the same sign in all the provinces and they are all non-significant.

For the 2nd panel all the biases are positive but the bias in P.E.I. is not significant. (P.E.I. is the province with the smallest sample size). It is also in P.E.I. that the sign of the bias for the 1st panel is different from the signs in the other provinces.

As discussed in section 2.3, there is more than one possible reason for the existence of RGB but the results emerging from the Table provide a strong indication that whatever the reason is, the biases found for some of the panels are real and not just the outcome of sampling errors. A drawback of the present analysis, however, is that the RGB are assumed to be fixed over time. Section 4 proposes a more flexible model.

3.2.2 Goodness of Fit

A. TESTING FOR NORMALITY

Let $I_{ta}^{(j)} = (y_{ta}^{(j)} - y_{ta|(t-1)}^{(j)})$ define the innovation when predicting the j-th panel estimator one month ahead and denote $I_{ta}' = (I_{ta}^{(1)}, \ldots, I_{ta}^{(6)})$. The use of maximum likelihood estimation in this study assumes that the vectors I_{ta} are normal deviates (see section 2.4). To test this assumption, we computed the empirical distribution of the standardized innovations $\{ (SI)_{ta}^{(j)} = [I_{ta}^{(j)}/\widehat{SD}(I_{ta}^{(j)})], t = (k + 1), \ldots, N\}$ and compared it to the standard normal distribution using the Kolmogorov-Smirnov test statistic. This test statistic was computed for each of the six panels in the four provinces yielding P-values larger than 0.15 in 21 out of the 24 cases. (The tests were performed using PROC UNIVARIATE of the SAS package. By this procedure, if the sample size is greater than fifty as it is in our case, the data are tested against a normal distribution with mean and variance equal to the sample mean and variance). Applying the same test procedure to the standardized innovations $\{ (SI)_{ta} = [I_{ta}/\widehat{SD}(I_{ta})], t = (k + 1), \ldots, N\}$ where $I_{ta} = [\sum_{j=1}^{6} I_{ta}^{(j)}/6]$ yields P-values larger than 0.15 in all the four provinces.

The estimators of the standard deviations of the innovations used for the tests are those produced by the Kalman filter, without accounting for the variance component resulting from parameter estimation (see section 2.5). The

latter component is negligible even in P.E.I. which has the smallest samples sizes among the four provinces. We come back to this finding in section 3.4.

B. PREDICTION ERRORS WITH DIFFERENT PREDICTORS

Table 3 contains summary statistics comparing the behaviour of the prediction errors (innovations) in the four provinces as obtained for three different sets of estimators of the state vectors: (1) The estimators obtained under the separate models (SM) defined by (2.2), (2.5) and 2.6; (2) the estimators obtained under the joint model (JM) defined by (2.2), (2.5), (2.6) and (2.10); (3) the estimators obtained by imposing the robustness constraints (2.11) on the joint model (ROB). Below we define the summary statistics using as before the notation $I_{ta}^{(j)} = (y_{ta}^{(j)} - \hat{y}_{ta|(t-1)}^{(j)})$ for the prediction error when predicting the j-th panel estimator one month ahead.

$MB_a = \sum_{t=k+1}^{N} (\sum_{j=1}^{6} I_{ta}^{(j)}/6)/(N - k)$ – mean bias in predicting the mean survey estimator $\bar{y}_{ta} = \sum_{j=1}^{6} y_{ta}^{(j)}/6$.

$MAB_a = \sum_{j=1}^{6} |\sum_{t=k+1}^{N} I_{ta}^{(j)}/(N - k)|/6$ – mean absolute bias in predicting the panel estimators.

$SQRE_a = \{ \sum_{t=k+1}^{N} [1/6 \sum_{j=1}^{6} I_{ta}^{(j)}/\bar{y}_{ta}]^2/(N - k) \}^{1/2}$ – square root of mean square relative prediction error in predicting the mean survey estimator.

The above summary statistics are shown separately for the sample period of July 1983 – December 1988 and for the postsample period of January 1989 – December 1989. In the latter case, the data were added one data point at a time so that for predicting the survey estimator of February 1989 for example we used the data observed until January 1989 and so forth.

Table 3

Prediction Errors in the Four Provinces,
Summary Statistics ($\times$ 100)

	Nova Scotia			New Brunswick			Newfoundland			Prince Edward Island		
	SM	JM	ROB	SM	JM	ROB	SM	JM	ROB	SM	JM	ROB
					7.83 – 12.88							
MB	−.11	−.07	−.06	−.12	−.09	−.06	−.25	−.18	−.08	.06	.14	.15
MAB	.12	.11	.10	.14	.12	.11	.29	.24	.20	.20	.23	.23
SQRE	5.76	5.62	5.70	5.48	5.47	5.47	7.03	6.91	6.96	9.34	9.13	9.17
					1.89 – 12.89							
MB	.14	.11	.04	.47	.47	.46	.36	.33	.17	.84	.85	.86
MAB	.32	.32	.30	.51	.51	.50	.39	.37	.29	.84	.85	.86
SQRE	6.39	6.27	6.82	6.25	6.25	6.32	5.92	5.90	5.61	9.45	9.26	9.30

The main conclusions from Table 3 are as follows:

(1) The results obtained for the three sets of predictors are in general very similar, indicating that for the data analyzed the use of the joint model improves only slightly over the use of the separate models and that there are no abrupt changes in the level of the series in the years considered.

(2) The errors when predicting the survey estimators are small both within and outside the sample period, suggesting a good fit of the model. Notice that except in P.E.I., the relative prediction errors as measured by the statistics $SQRE_a$ are all less than 7%.

(3) The biases of the prediction errors in the postsample period are larger than in the sample period with relatively large differences in New Brunswick and P.E.I. This outcome by itself could suggest some model failure in the year 1989. Inspection of the monthly panel prediction errors in the four provinces for this year, (not shown in the Table), indicates however that although the errors are in general mostly positive, the relatively large biases are mainly the result of one or two extreme errors which, with only 12 data points, has a large effect on the average summary statistics. It should be noted also that the estimated unemployment

rates in the four provinces in the year 1989 are between 0.11 and 0.18 so that a prediction bias of .005 or even .009 as obtained for P.E.I. is not high. Clearly, the model can be modified to account for these biases if they persist with additional data. On the other hand, notice that the discussion above refers only to the bias of the prediction errors since the bias of the model based estimators of the concurrent population values is controlled by the robustness constraints (2.11).

In view of the very similar results obtained for the three sets of predictors considered and in order to highlight the performance of the robustness constraints, we deliberately deflated the unemployment rates in the period March 1985 to March 1987 by 33%, deflated the rates in the period April 1987 – November 1988 by 25% and inflated the rates in the period December 1988 – December 1989 by 33%. The effect of these operations is to introduce sudden drifts in the data in the months $t = 39$, $t = 64$ and $t = 84$. Figure 3 displays the aggregate, one step ahead prediction errors (APE), $I_t^a = \sum_{a=1}^{4} w_{ta} [\sum_{j=1}^{6} (y_{ta}^{(j)} - \hat{y}_{ta|(t-1)}^{(j)})/6]$ as obtained for the joint model with and without the robustness constraints, and for the separate models.

The clear conclusion from Figure 3 is that by imposing the constraints, the APE in the periods following the three months with sudden drifts are smaller than the APE
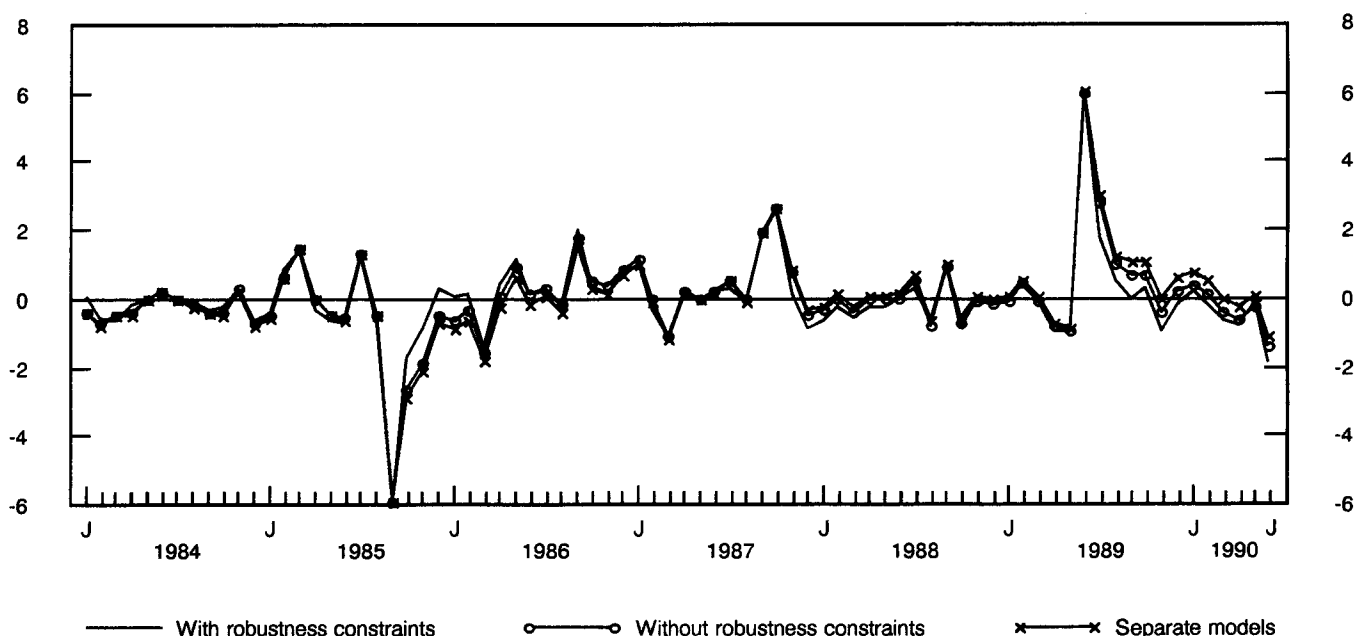

Figure 3. Aggregate One-Step Ahead Prediction Errors of the Three Sets of Predictors ($\times$ 100) for Contaminated Data
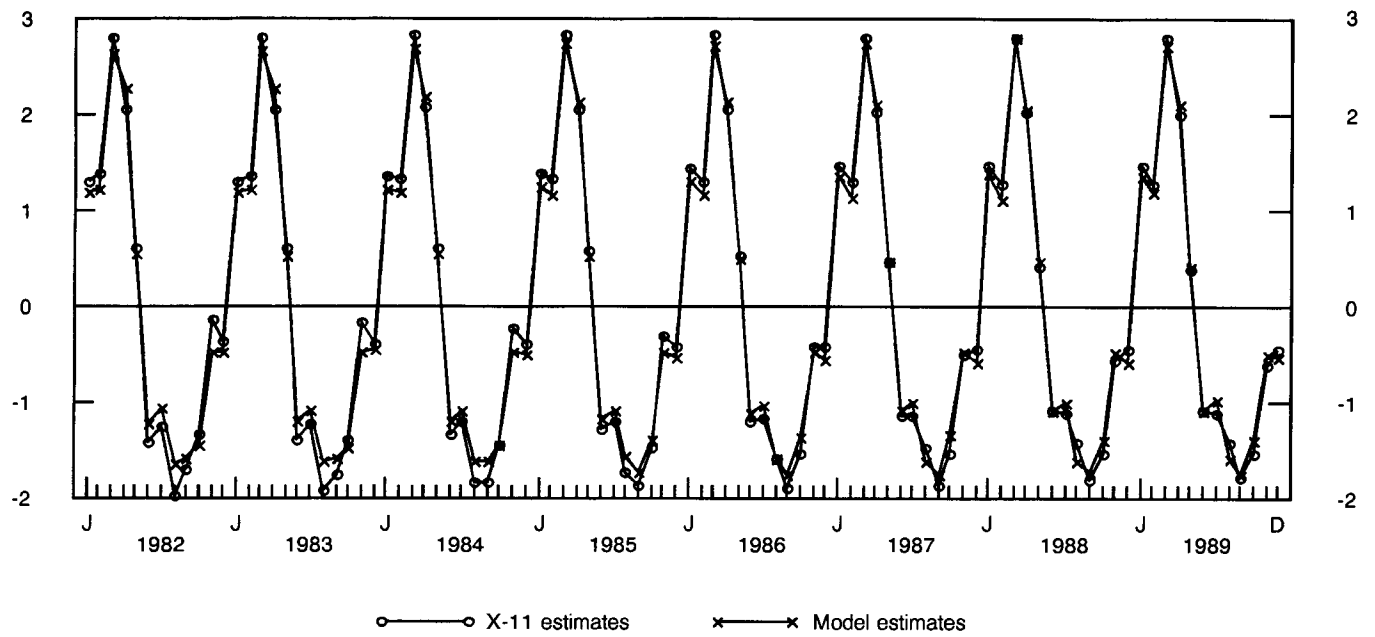
Figure 4. Weighted Averages of Seasonal Effects as Obtained by X-11 and Under the Model ($\times$ 100)
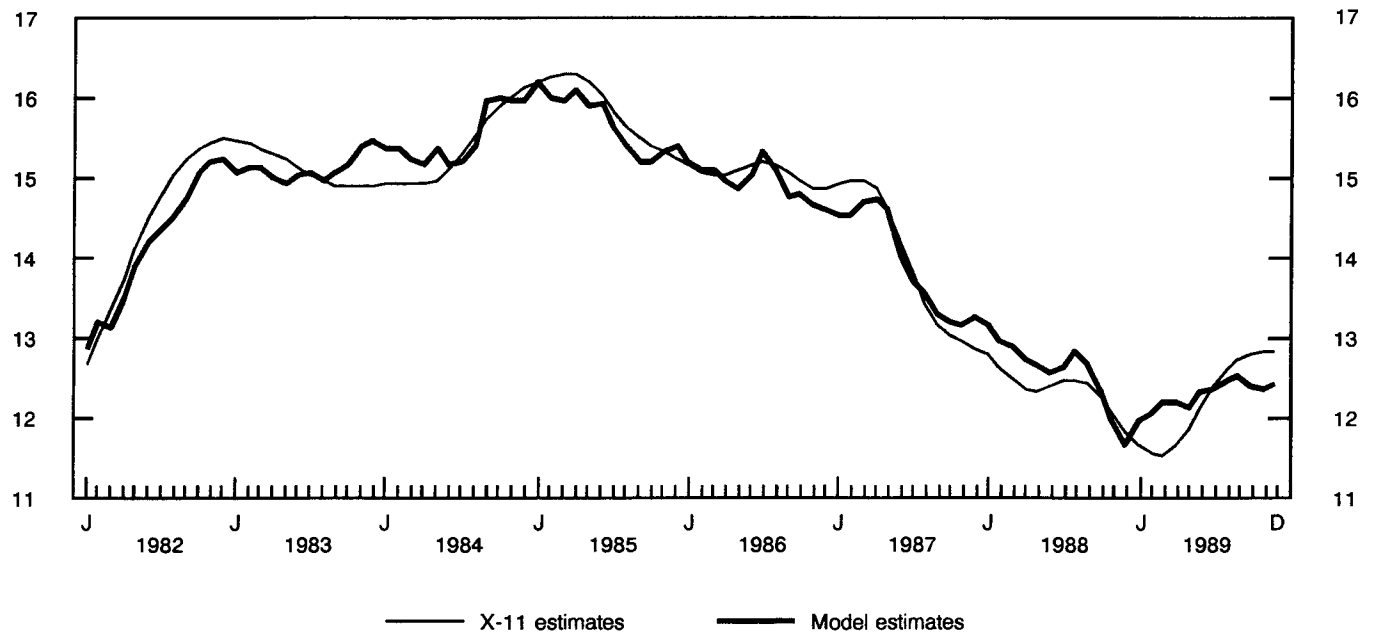

Figure 5. Weighted Averages of Trend Levels as Obtained by X-11 and Under the Model

obtained without the constraints. Thus, in March 1985 for example, $t = 39$, the APE are very large in absolute value both with and without the constraints which is obvious since the predictors use only the data until February 1985. The APE corresponding to the robust predictors return however, to their normal level much faster than the APE of the nonrobust predictors. A similar behaviour is seen to hold in the other two periods. Another notable result featured in the graph is that in the periods following the months with the sudden drifts, the joint model performs better than the separate models even without imposing the robustness constraints. Thus, by borrowing information from one province to the other, the joint model adapts itself more rapidly to the new level of the series. For more illustrations of the performance of the robustness constraints see Pfeffermann and Burck (1990).

C. COMPARISONS WITH ESTIMATORS PRODUCED BY X-11

As a final assessment of the appropriateness of the model, we compare the estimates of the seasonal effects and the trend levels as obtained under the model, with the estimates produced by the X-11 procedure (Dagum 1980). The latter is known to be less dependent on specific model assumptions. This procedure is the commonly used method for seasonal adjustment throughout the world. Figure 4 displays the average seasonal effects for the four provinces as obtained by X-11 and under the model. Figure 5 displays the corresponding trend level estimates. The averages are computed using the weights (w_{ta}) employed in previous analyses. The model based estimates shown in the two figures are the smoothed estimates which, like X-11, employ all the data in the sample period.

As can be seen, the seasonal effects produced by the two approaches are very close. The trend level estimates are also close but the X-11 trend curve is smoother than the model curve. Similar close correspondence between X-11 and the model is obtained for each of the four provinces separately, including, in particular, P.E.I. with its relatively small sample sizes.

3.3 Comparison of Design Based and Model Dependent Estimators

We mention in the introduction that one of the major reasons for wishing to model the raw survey estimators is that the model produces estimates for the population values which, at least in small areas, are more accurate (when the model holds) than the survey estimators. We computed the two sets of estimates for the four provinces and found that as expected, the estimates produced by the two approaches behave very similar but the design based estimators are less stable, having in general higher peaks and lower troughs. An important aspect when comparing

the two sets of estimates is their performance in estimating year to year changes of the population values. Such comparisons are free of the obscuring effects of seasonality. Figure 6 displays the results obtained for P.E.I.. The model dependent estimates are the smoothed values of the joint model which use all the data in all the months. As can be seen, the estimates produced by the model are much more stable and vary only mildly from one month to the other compared to the design based estimates. Figure 7 displays the standard errors (S.E.) of the unemployment rates estimators in P.E.I. as computed under the design, (smoothed values, see Figure 2), and under the joint model. Also shown are the S.E. when fitting the separate model defined by (2.2), (2.5) and (2.6) and the corresponding S.E. after accounting for the use of parameter estimates instead of the unknown parameter values. See section 2.5 for details. (The latter have been computed only for the separate model to save in computing time).

There are three notable features emerging from the graphs:

(1) The S.E. of the model dependent estimators under the joint model are only mildly smaller than the S.E. obtained for the separate model but considerably smaller than the S.E. of the survey estimators.

(2) The S.E. of the model dependent estimators behave similarly to the S.E. of the survey estimators, a direct consequence of accounting for the changes in the variances of the survey errors over time in the model. See section 2.3 for details.

(3) Accounting for the use of estimated parameter values in the computation of the S.E. of the model dependent estimators has only a marginal effect on the computed S.E. Recall that P.E.I. is the province with the smallest sample sizes. The effect of accounting for the use of parameter estimates in the other provinces is even smaller.

4. SUMMARY

This article illustrates that data collected by a complex sampling design, consisting of several stages of selection with rotating panels, can be successfully modelled by a relatively simple model. The model consists of two parts: the census model holding for the population values and the survey errors model describing the time series relationship between the survey errors. The use of the model yields more accurate estimators for the population values and their components like trend and seasonality and it permits estimating the S.E. of these estimators in a rather simple way. The model equations can be modified to secure the robustness of the model-dependent estimators against possible model failures.
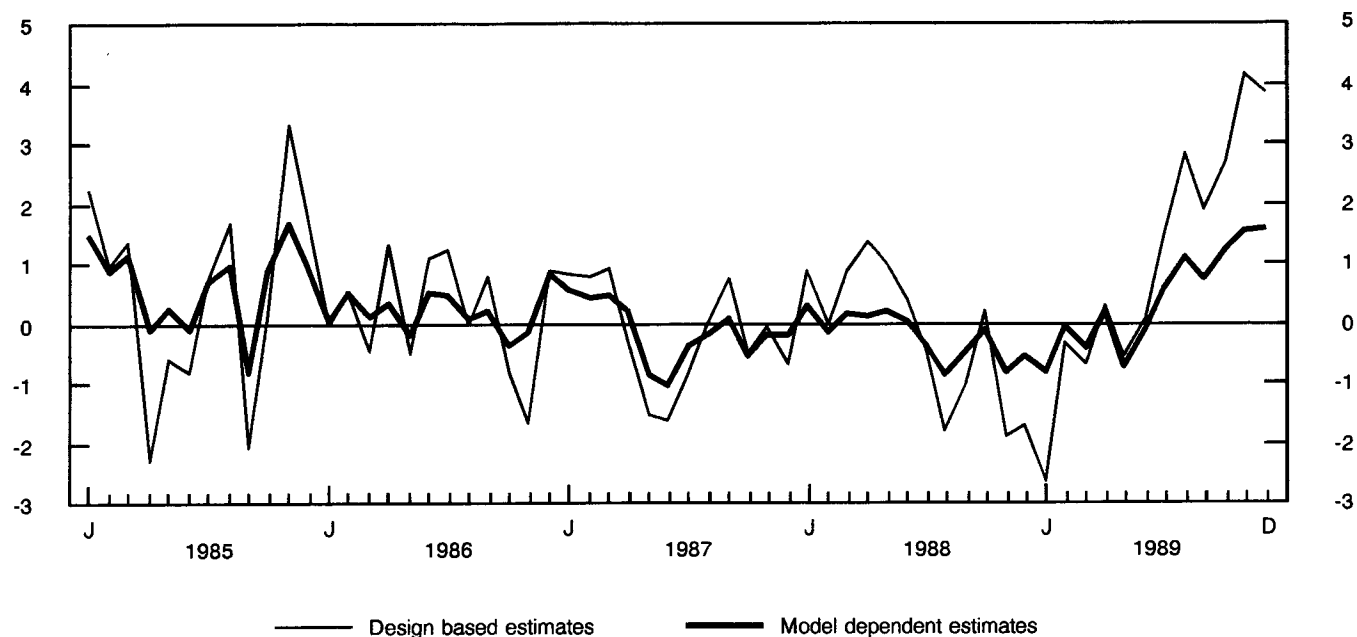
Figure 6. Year to Year Changes in Design Based and Model Dependent Estimates of P.E.I. Unemployment Rates ($\times$ 100)
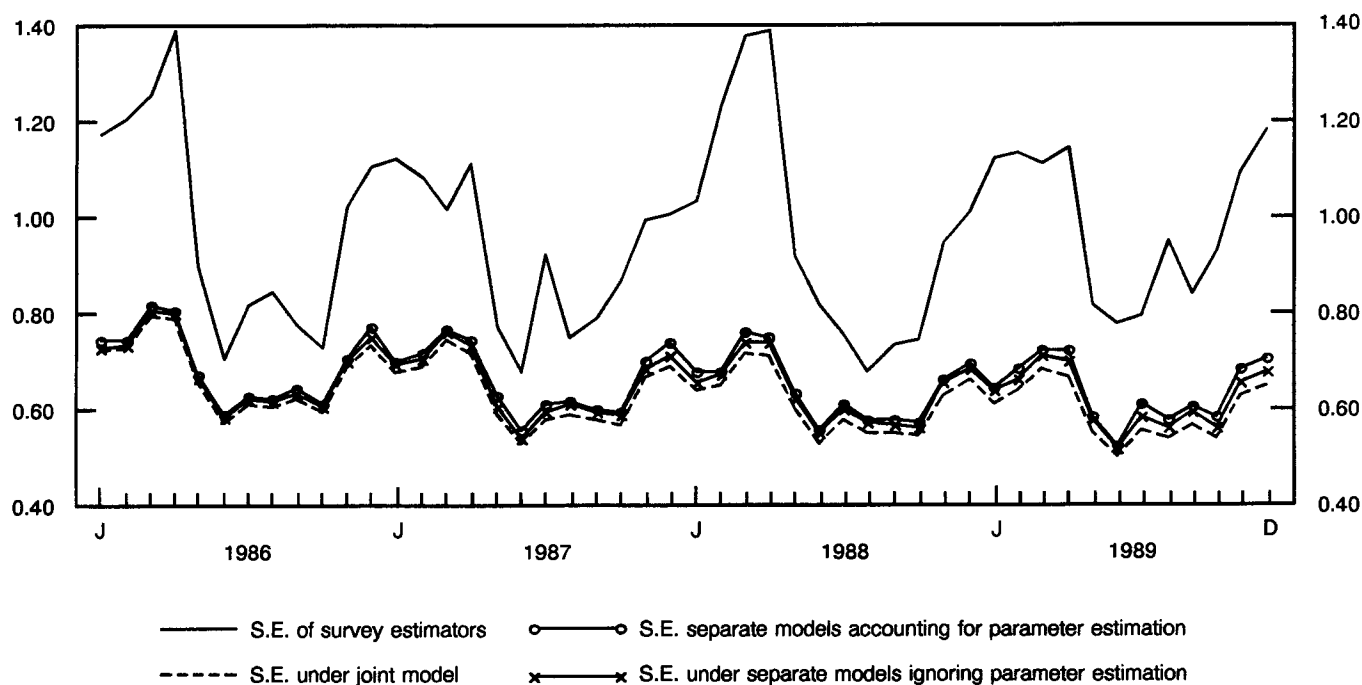

Figure 7. S.E. of Survey Estimators and of Model Dependent Estimators With and Without Accounting for Parameter Estimation ($\times$ 100) for P.E.I. Province

The model used in this article can be extended in various directions. Foremost, the model should be applied simultaneously to more provinces or other small areas to ensure that the aggregate sample estimators $\sum_{a=1}^{A} w_{ta} \bar{y}_{ta}$ are sufficiently close to the corresponding population values. See the discussion in section 2.7. Incorporating in the model an outlier detection mechanism to further assess the performance and suitability of the model is another valuable addition.

Two other extensions are to relax the assumption of constant variance for the error term ϵ_t in the census model and to let the rotation group biases to change over time. The first extension is suggested by the observation made in section 3.1 that the variances of the survey errors are subject to seasonal effects, with a seasonal pattern that is similar to the seasonal pattern of the raw estimates. Fitting the equations (2.4) in the four provinces indicates also the existence of a mild trend in the variances which again behaves similar to the trend of the raw survey estimates. Thus, the variances of the survey errors seem to depend on the magnitude of the survey estimators which suggests that the variances $\sigma_t^2 = V(\epsilon_t)$ change with the level of the population values. As a first approximation one could assume that σ_t^2 is proportional to the corresponding variance of the survey error.

Letting the rotation group biases change over time is a natural extension of the model, considering that the population values means are time dependent. Modelling the evolution of the group biases can however be problematic because of possible identifiability problems with the models holding for the trend and the seasonal effects. See the discussion in Pfeffermann (1991).

The last two extensions are important and should be explored but based on our experience with the unemployment data, we expect that they will affect the model estimators very mildly.

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