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An Exact Test for the Presence of Stable Seasonality With Applications

BRAJENDRA C. SUTRADHAR, ESTELA BEE DAGUM and BINYAM SOLOMON[1]

ABSTRACT

The X-11-ARIMA seasonal adjustment method and the Census X-11 variant use a standard ANOVA-F-test to assess the presence of stable seasonality. This F-test is applied to a series consisting of estimated seasonals plus irregulars (residuals) which may be (and often are) autocorrelated, thus violating the basic assumption of the F-test. This limitation has long been known by producers of seasonally adjusted data and the nominal value of the F statistic has been rarely used as a criterion for seasonal adjustment. Instead, producers of seasonally adjusted data have used rules of thumb, such as, F equal to or greater than 7. This paper introduces an exact test which takes into account autocorrelated residuals following an SMA process of the $(0,q)\,(0,Q)_s$ type. Comparisons of this modified F-test and the standard ANOVA test of X-11-ARIMA are made for a large number of Canadian socio-economic series.

KEY WORDS: Standard Anova; Autocorrelated residuals; Seasonality.

1. INTRODUCTION

In the analysis of social and economic time series, it is traditional to decompose the observed series into four unobserved components, namely the trend, the cycle, the seasonal variations, and the irregulars.

Socio-economic time series are often presented in seasonally adjusted form so that the underlying short-term trend can be more easily analysed and current socio-economic conditions can be assessed. There are several seasonal adjustment methods available which estimate the seasonal component present in a time series, but the Census X-11 variant (Shiskin, Young and Musgrave 1967) and the X-11-ARIMA method (Dagum 1980) are the most widely applied. To identify the presence of stable seasonality in a time series, the X-11-ARIMA method as well as the Census X-11 variant use the results of the usual F-test in a one-way ANOVA between monthly seasonal variations and the residuals. However, the residuals in this ANOVA are often autocorrelated, so the nominal significance level of the F-test may not be valid. Aware of this limitation, producers of seasonally adjusted data, do not guide themselves by the nominal significance level of the F-test for presence of stable seasonality but by some rule of thumb based on empirical knowledge (see *e.g.* Shiskin and Plewes 1978). In fact, implicit in the X-11-ARIMA test for the presence of 'identifiable seasonality' is that the F-value for stable seasonality should be greater or equal to 7 if moving seasonality is not present.

The testing for stable seasonality (similarly for annual seasonal shifts) can be approached as a test for the significance of certain regression coefficients in a linear model with autocorrelated errors. The traditional Wald test, the likelihood ratio test, and the tests falling within a generalized least squares framework, all run into convergence problems in testing such a linear

[1] Brajendra C. Sutradhar, Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Newfoundland, A1C 5S7; Estela Bee Dagum, Time Series Research and Analysis Division, Statistics Canada, Ottawa, Ontario, K1A 0T6. Binyam Solomon, Directorate of Social and Economic Analysis, National Defence Headquarters, Ottawa, Ontario, K1A 0K2.

model with highly autocorrelated errors (*cf.* Sutradhar and Bartlett 1990). Pierce (1978) constructed an F-test based on transformed residuals which are approximately white noise. The transformation suggested in Pierce (1978) is equivalent to using the inversion of the error covariance matrix. But, the inverse of the error covariance matrix may not be obtained for highly autocorrelated errors. Recently Sutradhar, MacNeill and Dagum (1991) proposed a modified F-test, within a linear model framework for testing for the presence of stable seasonality. Their modified F-test is derived following Sutradhar, MacNeill and Sahrmann (1987), and the test accounts for the presence of autocorrelation in the residuals. The test does not require any transformation or any inversion of the error covariance matrix.

Exact tests for testing the null hypothesis that the seasonal pattern changes over time against the alternative that the seasonal pattern is constant have been developed by Franzini and Harvey (1983). Unlike Franzini and Harvey, the present approach assumes that the seasonal pattern is stable over time possibly at different levels (due to annual shifts) and then tests for the presence of significant stable seasonality.

In most empirical cases, a seasonal moving average (SMA) error model of the $(0,q)(0,Q)_s$ type is sufficient. In this investigation we simplify the exact test proposed by Sutradhar, MacNeill and Dagum (1991), for such error models. The test is applied to examine for the presence of stable seasonality as well as of annual seasonal shifts in a number of socio-economic series.

The plan of this paper is as follows. Section 2 presents the exact test. Section 3 analyses the results from the application of the modified F-test to a set of socio-economic time series and compares them with the values given by the X-11-ARIMA method. Section 4 gives the conclusions.

2. MODIFIED F-TEST

2.1 Selected Model

Consider a stationary seasonal time series $\{Z_t\}$, given by

$$Z_t = S_t + U_t, \tag{2.1}$$

where Z_t is the observed series at time t, S_t is the seasonal component, and U_t the irregulars. If the time series contains a trend, which is most likely, it is assumed that a suitable detrending technique will yield the model (2.1). In the latter case, the detrended series may be obtained from the original series by taking appropriate differences as in ARIMA modelling (Box and Jenkins 1970) or as is traditionally done by statistical agencies which use the X-11-ARIMA method or Census X-11 variant.

Next, suppose there are k seasons in a year and there are kn observations in a time series of n years. Let $Z\{(i-1)n+j\}$ be the jth ($j = 1, \ldots, n$) observation under the ith season ($i = 1, \ldots, k$) which corresponds to Z_t in (2.1). We shall denote in similar manner the (i,j)th components of S_t and U_t, for all $t = 1, \ldots, kn$. Then, the model assumed for S_t is (*cf.* Sutradhar and MacNeill 1989):

$$S((i-1)n+j) = \mu + \alpha_i + \beta_j, \tag{2.2}$$

with $\sum_{i=1}^{k}\alpha_i = 0$, $\sum_{j=1}^{n}\beta_j = 0$.

The α's and β's in (2.2) represent, respectively, the stable seasonality and annual seasonal shifts in the seasonal time series. Thus, when testing for the presence of stable seasonality, we test the hypotheses

$$H_0\colon \alpha_i = 0 \quad \text{vs.} \quad H_1\colon \alpha_i \neq 0 \text{ for at least one } i; \tag{2.3}$$

and when testing for the presence of annual seasonal shifts, we test the hypotheses

$$H_0\colon \beta_j = 0 \quad \text{vs.} \quad H_1\colon \beta_j \neq 0 \text{ for at least one } j. \tag{2.4}$$

Consequently, the rejection of H_0 in (2.3) and (2.4) would indicate that the series contains significant stable seasonality as well as annual seasonal shifts.

Taking into account model (2.2), the model (2.1) can be written as

$$Z^* = X\gamma + U^*, \tag{2.5}$$

where

$$Z^* = [Z(1), \ldots, Z(n), Z(n+1), \ldots, Z(kn)]',$$

$$U^* = [U(1), \ldots, U(n), U(n+1), \ldots, U(kn)]',$$

$$\gamma = [\mu, \alpha_1, \ldots, \alpha_{k-1}, \alpha_k, \beta_1, \ldots, \beta_{n-1}, \beta_n]'$$

and X is the appropriate $kn \times (k + n + 1)$ design matrix.

2.2 Test Statistics

U^* in (2.5) can be represented by seasonal autoregressive moving average (SARMA) stationary process $(p,q)(P,Q)_s$. In most empirical cases we found, however, that a $(0,q)(0,Q)_s$ model is sufficient. Let Σ^* denote the $kn \times kn$ covariance matrix of U^*. Naturally, Σ^* will contain $\theta \equiv (\theta_1, \ldots, \theta_q)$ and $\Theta \equiv (\Theta_1, \ldots, \Theta_Q)$, where θ and Θ's are the parameters associated with the SARMA $(0,q)(0,Q)_s$ process.

For the usual ANOVA model, *viz.*, when the components of U^* are i.i.d. $N(0,\sigma^2)$, one tests the null hypotheses $\beta_j = 0$, and $\alpha_i = 0$ by using the classical F-statistics F_{A1} and F_{A2} respectively, given by

$$F_{A1} = (k-1)Q_1/Q_3, \quad \text{and} \quad F_{A2} = (n-1)Q_2/Q_3,$$

where

$$Q_1 = k \sum_{j=1}^{n} (\bar{Z}_{.j} - \bar{Z}_{..})^2, \quad Q_2 = n \sum_{i=1}^{k} (\bar{Z}_{i.} - \bar{Z}_{..})^2,$$

and

$$Q_3 = \sum_{i=1}^{k} \sum_{j=1}^{n} (Z_{ij} - \bar{Z}_{i.} - \bar{Z}_{.j} + \bar{Z}_{..})^2$$

with

$$\bar{Z}_{i.} = \sum_{j=1}^{n} Z_{ij}/n, \quad \bar{Z}_{.j} = \sum_{i=1}^{k} Z_{ij}/k, \quad \text{and} \quad \bar{Z}_{..} = \sum_{i=1}^{k} \sum_{j=1}^{n} Z_{ij}/kn,$$

Z_{ij} being the jth observation under the ith season. In the present set-up, however, these statistics are inappropriate for testing the above hypotheses. This is because, the expected values of the sums of squares are affected by the dependence among observations. Also, sums of squares are not mutually independent. For the case when U^* in (2.5) follow a SARMA $(0,q)(0,Q)_s$ process, it can be shown that

$$E(Q_1) = k \sum_{j=1}^{n} \beta_j^2 + \sigma^2(n-1)C_1(\theta,\Theta),$$

$$E(Q_2) = n \sum_{i=1}^{k} \alpha_i^2 + \sigma^2(k-1)C_2(\theta,\Theta),$$

and

$$E(Q_3) = \sigma^2(k-1)(n-1)C_3(\theta,\Theta),$$

where, for example, for the SARMA $(0,1)(0,1)_{12}$ process,

$$C_1(\theta,\Theta) = (1+\theta_1^2)(1+\Theta_1^2) - (\theta_1/6)(1+\Theta_1^2)(11-1/n) + (2\Theta_1/n)(1+\theta_1^2)$$
$$+(\theta_1\Theta_1/6)\{1 - 22/n - (n-2)/n(n-1)\},$$

$$C_2(\theta,\Theta) = (1+\theta_1^2)(1+\Theta_1^2) - 2(1-1/n)\Theta_1(1+\theta_1^2)$$
$$+1/6\{1 + (1-1/n)/11\}\theta_1(1+\Theta_1^2) - (4/11)(1-1/n)\theta_1\Theta_1,$$

$$C_3(\theta,\Theta) = (1+\theta_1^2)(1+\Theta_1^2) + (2\Theta_1/n)(1+\theta_1^2) + (\theta_1/6)(1+\Theta_1^2)(1-1/11n)$$
$$-(\theta_1\Theta_1/6n)[n/11 - 2(n-2)/11(n-1) - 2].$$

Consequently, the null hypotheses $\beta_j = 0$, and $\alpha_i = 0$ may be tested by using the modified F-statistics F_{M1} and F_{M2} respectively, given by

$$F_{M1} = d_1(\hat{\theta},\hat{\Theta})F_{A1}, \tag{2.6}$$

$$F_{M2} = d_2(\hat{\theta},\hat{\Theta})F_{A2}, \tag{2.7}$$

(see also Sutradhar, MacNeill and Sahrmann 1987, Sutradhar, MacNeill and Dagum 1991), where $d_1(\theta,\Theta) = C_3(\theta,\Theta)/C_1(\theta,\Theta)$, $d_2(\theta,\Theta) = C_3(\theta,\Theta)/C_2(\theta,\Theta)$. The modified F-statistics F_{M1} and F_{M2} account for autocorrelation of the residuals.

Notice that in the independence case when $\theta = 0$, $\Theta = 0$, $C_1(\cdot) = C_2 = (\cdot) = C_3(\cdot) = 1$, which is obvious. In that case the problem reduces to testing the hypotheses by using standard ANOVA F-statistics.

2.3 Computation of p-value

A simulation study (*cf.* Sutradhar and Bartlett 1989, Table IV, p. 1587) indicates that for the cases when k groups are independent, the distribution of the modified F-statistics for the SMA/ $(0,q)(0,Q)_s$ process, may be approximated by the usual F-distribution. In general, the F approximation to the modified F-statistic would be inappropriate, in particular when k groups are correlated and n is small.

In this paper we use the well known Satterthwaite (1946) approximation (*cf.* Sutradhar, MacNeill and Dagum 1991) to calculate the p-value, namely, $P_r(F_{M1} \geq f_{M1})$, where f_{M1} is the data based value of F_{M1}. In order to do it, we first compute the eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_r > 0 = \lambda_{r+1} = \ldots = \lambda_s > \lambda_{s+1} \geq \ldots \geq \lambda_n$ of

$$\Sigma^{*1/2}[d_1(\theta,\Theta)D_1 - f_{M1}(I_{kn} - D_2)]\,\Sigma^{*1/2}, \tag{2.8}$$

where $d_1(\cdot)$ is given in equation (2.6), $D_1 = R(RR')^{-1}R'$, with $R = C(X'X)^{-1'}$, $D_2 = X(X'X)^{-1}X'$, C being a suitable matrix obtained by expressing the: $H_0 : \beta_j = 0$ in the form $C\gamma = 0$, where γ is given in model (2.5). In equation (2.8) I_{kn} is the $kn \times kn$ identity matrix. Then the Satterthwaite approximation yields

$$P_r(F_{M1} \geq f_{M1}) = P_r[F_{a,b} \geq bd/ac], \tag{2.9}$$

where $F_{a,b}$ denotes the usual F-ratio with degrees of freedom a and b, with

$$a = \left(\sum_{j=1}^{r} \lambda_j\right)^2 \Big/ \sum_{j=1}^{r} \lambda_j^2, \quad b = \left(\sum_{j=s+1}^{n} \lambda_j\right)^2 \Big/ \sum_{j=s+1}^{n} \lambda_j^2.$$

In equation (2.9),

$$c = \sum_{j=1}^{r} \lambda_j^2 \Big/ \sum_{j=1}^{r} \lambda_j, \quad d = \sum_{j=s+1}^{n} \lambda_j^2 \Big/ \sum_{j=s+1}^{n} |\lambda_j|.$$

Similarly, $P_r(F_{M2} \geq f_{M2})$ may be calculated by using $d_2(\cdot)$ and f_{M2} in place of $d_1(\cdot)$ and f_{M1} respectively in equation (2.9). The construction of D_1 will now depend on a different C matrix which will be obtained by expressing the $H_0 : \alpha_i = 0$ in the form $C\gamma = 0$.

3. APPLICATIONS

3.1 Monthly Series

The modified F statistics F_{M1} and F_{M2} of equations (2.6) and (2.7) were calculated for a set of 26 monthly series obtained from various economic sectors, namely, Imports, Exports, Consumer Prices and Labour. All series cover the period January 1979 till December 1988 inclusive.

Since the modified F-test is not valid when moving seasonality is present (except for annual seasonal shifts), none of the series selected are affected by moving seasonality according to certain preliminary tests available in X-11-ARIMA. (We also looked at the plots of the seasonal-irregular ratios.)

The X-11-ARIMA method was applied to obtain the detrended series $\{Z_t: t = 1, \ldots, 120\}$. Diagnostic checks show that the errors of the detrended series, U_t (see equation 2.1) follow a $(0,1)(0,1)_{12}$ SMA model for each of the monthly series. The estimates $\hat{\theta}_1$ and $\hat{\Theta}_1$ are used to compute the modified F-statistics F_{M1} and F_{M2}.

In testing for the presence of annual seasonal shifts, the p-values for the modified F-test based on the Satterthwaite approximation and on the standard ANOVA F-test generally were found to be different. For both cases, however, the p-values were very large for each of the series indicating that there is no moving seasonality in the form of annual shifts.

Table 1

Diagnostics of Stable Seasonality in Monthly Series

Series	Parameter Estimates θ_1	Parameter Estimates Θ_1	X-11-ARIMA F-Test[a]	Modified F F_{M2} (p-value in %)	Final Diagnostic[c]
IMPORTS					
1. Fodder and feed	−0.09*	−0.01	3.68	3.43(0.06)	Y
2. Coal related materials	0.02	−0.01	64.40	58.76(0.00)	Y
3. Crude vegetable products	0.02	−0.07*	3.48	2.94(0.27)	Y
4. Wool & man made materials	0.02	0.29*	10.98	20.63(0.00)	Y
5. Precious metals	0.27*	0.01	1.25	1.20(31.10)	N
6. Oils & fats	0.41*	0.01	8.59	8.22(0.00)	Y
7. Non-metal minerals	0.04	0.02	16.50	16.68(0.00)	Y
8. Aircraft engines	0.32*	0.00	2.53[b]	2.36(1.79)	N
9. Other trans. equipments	0.19*	−0.18*	3.48[b]	2.43(1.31)	N
EXPORTS					
10. Wheat	0.04	−0.03	1.89	1.71(8.71)	N
11. Asbestos	0.13*	−0.03	6.83	6.15(0.00)	Y
12. Wood pulp	−0.27	0.20*	6.45	9.61(0.00)	Y
13. Textile fabrics	0.52*	0.13*	12.05	15.06(0.00)	Y
14. Other fabrics	0.04	0.11*	5.03	6.19(0.00)	Y
15. Television & telecommunication	0.12*	0.01	9.26	8.99(0.00)	Y
16. Domestic export pass.	−0.30*	−0.14*	24.50	18.52(0.00)	Y
CPI					
17. Eggs	−0.04	−0.01	6.90	6.50(0.00)	Y
18. Pasta	−0.05*	−0.04	3.69	3.24(0.10)	Y
19. Onions	−0.42*	−0.03	26.90	23.49(0.00)	Y
20. Housing	0.11*	−0.34*	19.02	9.28(0.00)	Y
21. Clothing	0.03	−0.42*	47.42	24.30(0.00)	Y
22. Transport	−0.09*	−0.02	4.21	3.74(0.02)	Y
LABOUR					
23. Sask. employment (25-34)	−0.19*	−0.11*	67.40	52.35(0.00)	Y
24. Sask. not in labour force	0.12*	−0.36*	22.98	12.69(0.00)	Y
25. Ontario unemployment (25-44)	−0.21*	0.07*	31.4	34.23(0.00)	Y
26. Ontario unemployment male & female (20-24)	−0.02	0.19*	24.27	34.78(0.00)	Y

a Critical value is $F(11,99;\ 0.01) = 2.47$.
b X-11-ARIMA and Modified F give conflicting inference.
c Y (Yes) – stable seasonality is significant
N (No) – stable seasonality is not present.
* Significant values at 5% level.

To test for the presence of stable seasonality, we computed the p-values of the modified F-statistic F_{M2} (2.7) by using the Satterthwaite approximation and compared them to those given by the X-11-ARIMA F-test (which is equivalent to the standard ANOVA F_{A2}) for the 26 monthly series. The results are shown in Table 1.

The p-values of the modified F-statistic in Table 1 show that among the nine import series, three series do not have significant stable seasonality at the 1% significance level (the critical value of F (11,99; 0.01) = 2.47). Among the seven exports series, only one series, namely Wheat, appears to have no seasonality. All six CPI series have significant stable seasonality and similarly the four Labour series.

The X-11-ARIMA F-test values give same results (either rejection or acceptance of the null hypothesis) as the modified F-test for a large number of series. It seems that for most of the monthly series, under the SMA $(0,1)(0,1)_s$ error structure, the X-11-ARIMA F-test (or equivalently standard ANOVA F-test) is more affected by large negative values of Θ_1, *i.e.* when there is seasonal autocorrelation in the residuals. This can be generalized by looking at the values of $C_3(\theta,\Theta)/C_2(\theta,\Theta)$. By examining when this fraction is greater or less than 1, it may be seen that the direction of the inequality is affected by the signs of θ_1 and the size by the value of Θ_1. Only two series, namely, Imports Aircraft Engines and Imports other transportation Equipments, have standard F-test values which lead to contradictory conclusions with respect to the modified F-test. On the other hand, if we would follow the rule of thumb of $F \geq 7$ to justify seasonal adjustment, then the modified F-test would be in contradiction for eight out of twelve series. We then seasonally adjusted these eight series with the X-11-ARIMA method and found that the quality of the adjustment was acceptable for six out of the eight cases. All series passed the extrapolation ARIMA model automatically chosen for the program, six out of the eight series passed the X-11-ARIMA guidelines criteria for acceptance; and the four series for which the F_{M2} values were relatively small, that is, falling between 3.24 and 3.74 were really strongly affected by trading-day variations. Only Imports Fodder and Feed and Imports Crude Vegetable products gave a seasonally adjusted output that could not be considered reliable.

3.2 Quarterly Series

The X-11-ARIMA method was applied to four quarterly series of the System of National Accounts to obtain the detrended values (z_t, $t = 1, \ldots, 40$). It was found that for all four series U_t follow a $(0,1)(0,1)_4$ model. The computation for the modified F-test is quite similar to the case for monthly series but since the covariance matrix Σ^* is different, the formulas for $C_1(\cdot)$, $C_2(\cdot)$, and $C_3(\cdot)$ in equations (2.6) and (2.7) were adjusted accordingly.

Similar to the monthly series, the p-values for testing the presence of annual shifts based on the F_{M1} test were found very large and thus rejecting this pattern of moving seasonality.

The results of the modified F_{M2} test and the X-11-ARIMA F-test for testing for the presence of stable seasonality in each of the four series, are given in Table 2. The p-value for two series namely, Deposits in other Institutions and Small Mortgages are not significant and in agreement with those obtained from X-11-ARIMA. Thus we conclude that these two series contain significant stable seasonality. For the remaining two quarterly series, the modified F-test and the X-11-ARIMA F-test give conflicting inferences. Contrary to the X-11-ARIMA F-test, the modified F-test yields significant p-values for these two series. Thus we conclude that these two quarterly series, namely, Net Financial Investments and Corporate claims should not be seasonally adjusted.

Table 2

Diagnostics of Stable Seasonality in Quarterly Series

Series	Parameter Estimates θ_1	Parameter Estimates Θ_1	X-11-ARIMA F-Test[a]	Modified F F_{M2} (p-value in %)	Final Diagnostic[c]
1. Deposits in other institutions	0.53*	0.11*	9.03	9.67(0.04)	Y
2. Net financial investments	0.77*	−0.37*	4.86[b]	2.56(8.16)	N
3. Small mortgages	0.17*	−0.01	6.65	4.88(1.02)	Y
4. Corporate claims	0.77*	−0.31*	7.88[b]	3.58(3.20)	N

a Critical value is $F(3,27; 0.01) = 4.51$.
b X-11-ARIMA and Modified F give conflicting inference.
c Y (Yes) – Stable seasonality is significant.
N (No) – Stable seasonality is not present.
* Significant values at 5% level.

4. CONCLUSIONS

This paper has introduced an exact test for the presence of stable seasonality and annual seasonal shifts based on the modified F-test by Sutradhar, MacNeill and Sahrmann (1987). The new test takes into account the possibility of autocorrelated residuals in the seasonal-irregular ratios of the X-11-ARIMA method. The residuals are assumed to follow a simple Seasonal Moving Average (SMA) model $(0,q)\,(0,Q)_s$. This test is applied to a set of quarterly and monthly series from the system of National Accounts, Imports, Exports, Consumer Prices and Labour. The residuals from the X-11-ARIMA method are found to follow seasonal moving average models (SMA) where either $\hat{\theta}$ and/or $\hat{\Theta}$ were significant. The exact F-test gives values very different from those of the F-test in X-11-ARIMA (also in the Census X-11 variant) when the autocorrelation of the residuals is of a seasonal character, *i.e.*, whenever $\hat{\Theta}$ is significantly different from zero.

Among the 26 monthly series analysed, only in two cases, the standard F-test values gave conflicting conclusions with respect to the modified F-test. On the other hand, if we would follow the common rule of thumb of $F \geq 7$ to justify seasonal adjustment, then the modified F-test gave contradictory results for eight out of twelve series.

By looking at the seasonal adjustment output of these eight series we found that six can be soundly seasonally adjusted by the X-11-ARIMA method.

Concerning the quarterly series, the modified F-test indicates that there is no stable seasonality in two out of the four series analysed. Furthermore, in one case, the F-test of X-11-ARIMA gives an F value greater than 7 whereas the modified F accepts the null hypothesis.

It has been assumed throughout the paper that moving seasonality may be present in the series only in the form of annual shifts. The present test is not suitable to detect other types of moving seasonal patterns in the series. This raises the necessity of further investigations in this direction.

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