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The Time Series Approach to Estimation for Repeated Surveys

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ABSTRACT

Papers by Scott and Smith (1974) and Scott, Smith, and Jones (1977) suggested the use of signal extraction results from time series analysis to improve estimates in repeated surveys, what we call the time series approach to estimation in repeated surveys. We review the underlying philosophy of this approach, pointing out that it stems from recognition of two sources of variation - time series variation and sampling variation - and that the approach can provide a unifying framework for other problems where the two sources of variation are present. We obtain some theoretical results for the time series approach regarding design consistency of the time series estimators, and uncorrelatedness of the signal and sampling error series. We observe that, from a design-based perspective, the time series approach trades some bias for a reduction in variance and a reduction in average mean squared error relative to classical survey estimators. We briefly discuss modeling to implement the time series approach, and then illustrate the approach by applying it to time series of retail sales of eating places and of drinking places from the U.S. Census Bureau's Retail Trade Survey.

KEY WORDS: Repeated surveys; Time series; Signal extraction; U.S. Retail Trade Survey.

1. INTRODUCTION

Papers by Scott and Smith (1974) and Scott, Smith, and Jones (1977), hereafter SSJ, suggested the use of signal extraction results from time series analysis to improve estimates in repeated surveys. If the covariance structure of the usual survey estimates (Y_t) and their sampling errors (e_t) for a set of time points is known, these results produce the linear functions of the available Y_t's that have minimum mean squared error as estimators of the population values being estimated (say θ_t) for θ_t a stochastic time series. To apply these results in practice one estimates a time series model for the observed series Y_t and estimates the covariance structure of e_t over time using knowledge of the survey design.

Section 2 of this paper gives a brief overview of the basic results and framework for the time series approach. Section 3 considers some theoretical issues and section 4 some application considerations for the approach. In section 5 we illustrate the approach with an example using two time series from the Census Bureau's Retail Trade Survey.

2. BASIC IDEAS AND GENERAL CONSIDERATION OF THE TIME SERIES APPROACH

The basic idea in using time series techniques in survey estimation that distinguishes it from the classical approach is the recognition of *two sources of variability*. Classical survey estimation deals with the variability due to sampling - having not observed all the units in the population. Time series analysis deals with variability arising from the fact that a time series is not perfectly predictable (often linearly) from past data. Consider the decomposition:

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$$Y_t = \theta_t + e_t, \tag{2.1}$$

where Y_t is a survey estimate at time t, θ_t is the population quantity of interest at time t, and e_t is the sampling error. The sampling variability of e_t is the focus of the classical survey sampling approach, which regards the θ_t's as fixed. From a time series perspective all three of Y_t, θ_t, and e_t can exhibit time series variation, as long as they are random and not perfectly predictable from past data. Standard time series analysis would treat Y_t directly and ignore the sampling error in the decomposition (2.1), not treating e_t explicity, but only indirectly in the aggregate Y_t. In fact, time series analysts typically behave as if the sampling variation is not present and the true values are actually observed. The most basic thing to keep in mind about the use of time series techniques in survey estimation is that there are two distinct sources of stochastic variation present that are conceptualized, modeled, and estimated differently.

2.1 Signal Extraction Results

Suppose that survey estimates Y_t are available at a set of time points labelled $t = 1, \ldots, T$. Let $\underset{\sim}{Y} = (Y_1, \ldots, Y_T)'$ and similarly define $\underset{\sim}{\theta}$ and $\underset{\sim}{e}$ so we have $\underset{\sim}{Y} = \underset{\sim}{\theta} + \underset{\sim}{e}$. Assuming the estimates Y_t are unbiased and θ_t and e_t are uncorrelated (see section 3.2)

$$E(\underset{\sim}{Y}) = E(\underset{\sim}{\theta}) \equiv \underset{\sim}{\mu} \equiv (\mu_1, \ldots, \mu_T)'$$

$$\Sigma_Y = \Sigma_\theta + \Sigma_e, \tag{2.2}$$

where E denotes expectation over both the sampling and time series model distributions, and Σ_Y is the covariance matrix of $\underset{\sim}{Y}$, *etc.* Here $\underset{\sim}{\mu}$ and Σ_θ refer to the time series structure of θ_t, which is not subject to sampling variation. If Y_t, θ_t, and e_t do not require differencing, it is well known that, since $\mathrm{Cov}(\underset{\sim}{\theta},\underset{\sim}{Y}) = \Sigma_\theta$, using (2.2) the minimum mean squared error linear predictor of $\underset{\sim}{\theta}$ can be written

$$\hat{\underset{\sim}{\theta}} = \underset{\sim}{\mu} + \Sigma_\theta \Sigma_Y^{-1}(\underset{\sim}{Y} - \underset{\sim}{\mu}) \tag{2.3}$$

$$= \underset{\sim}{\mu} + \left(I - \Sigma_e \Sigma_Y^{-1}\right)(\underset{\sim}{Y} - \underset{\sim}{\mu}) \tag{2.4}$$

$$= \underset{\sim}{\mu} + \left(I + \Sigma_e \Sigma_\theta^{-1}\right)^{-1}(\underset{\sim}{Y} - \underset{\sim}{\mu}). \tag{2.5}$$

Another standard result is that the variance of the error of this estimate is

$$\mathrm{Var}(\hat{\underset{\sim}{\theta}} - \underset{\sim}{\theta}) = \Sigma_\theta - \Sigma_\theta \Sigma_Y^{-1} \Sigma_\theta = \Sigma_e - \Sigma_e \Sigma_Y^{-1} \Sigma_e. \tag{2.6}$$

If normality of $(\underset{\sim}{\theta},\underset{\sim}{Y})$ is assumed (2.3) - (2.5) give $E(\underset{\sim}{\theta} \mid \underset{\sim}{Y})$, the conditional expectation of $\underset{\sim}{\theta}$ given $\underset{\sim}{Y}$, and (2.6) gives $\mathrm{Var}(\underset{\sim}{\theta} \mid \underset{\sim}{Y})$, the conditional variance.

If Y_t requires differencing the preceeding results need to be modified. Assume e_t does not require differencing, but θ_t and Y_t need to be differenced once (*i.e.* by applying $1 - B$ where $BY_t = Y_{t-1}$). Let the differenced data be $W_t = (1 - B)Y_t = (1 - B)\theta_t + (1 - B)e_t$ for $t = 2, \ldots, T$. Let $\Delta = [\Delta_{ij}]$ be the $(T - 1) \times T$ differencing matrix with $\Delta_{ii} = -1$, $\Delta_{i,i+1} = 1$, and all other elements zero, and write $\Delta \underset{\sim}{Y} \equiv \underset{\sim}{W} = \Delta \underset{\sim}{S} + \Delta \underset{\sim}{e}$. Then we use

$$\hat{\theta} = Y - \hat{e} = Y - \Sigma_e \Delta' \Sigma_W^{-1} \Delta (Y - \mu), \tag{2.7}$$

$$\mathrm{Var}(\hat{\theta} - \theta) = \Sigma_e - \Sigma_e \Delta' \Sigma_W^{-1} \Delta \Sigma_e. \tag{2.8}$$

The expressions (2.7) and (2.8) also apply when θ_t and Y_t require a more general differencing operator (*e.g.* seasonal differencing), with appropriate definition of the differencing matrix Δ, as long as e_t does not require differencing. These results are analogous to (2.4) and (2.6), but with $\Delta' \Sigma_W^{-1} \Delta$ playing the role of Σ_Y^{-1}. The results are given in Bell and Hillmer (1990), where their optimality properties are discussed. They were essentially given by Jones (1980), but without real justification.

Scott and Smith (1974) and SSJ used classical signal extraction results equivalent to (2.3) - (2.6) based on covariance generating functions rather than covariance matrices. Bell (1984) considers such results for models involving differencing. Another approach (Binder and Dick 1989, Bell and Hillmer 1989) involves putting time series models for θ_t and e_t in state space form and using the Kalman filter and smoother, which can be viewed as an efficient way to compute the matrix results given above. Also, see Tam (1987) for use of the Kalman filter in an explicitly model-based approach to analysis in repeated surveys. In subsequent discussions we generally refer to the results (2.3) - (2.6), though our remarks easily extend to cover the use of (2.7) - (2.8).

In many cases, for time series Y_t and θ_t that are always positive, we will want to take logarithms of Y_t to help induce stationarity of θ_t and the sampling errors. In such cases we rewrite (2.1) as

$$Y_t = \theta_t(1 + \tilde{u}_t) = \theta_t u_t, \tag{2.9}$$

where $\tilde{u}_t = e_t/\theta_t$ and $u_t = 1 + \tilde{u}_t$. Taking logs we get

$$\log(Y_t) = \log(\theta_t) + \log(1 + \tilde{u}_t) = \log(\theta_t) + \log(u_t). \tag{2.10}$$

Letting μ and Σ_θ now refer to $\log(\theta) \equiv (\log(\theta_1), \ldots, \log(\theta_T))'$, and $\Sigma_Y = \Sigma_\theta + \Sigma_u$ refer to $\log(Y)$, analogous to (2.4) our estimate is

$$\log(\hat{\theta}) = \mu + \left[I - \Sigma_u \Sigma_Y^{-1}\right](\log(Y) - \mu). \tag{2.11}$$

The analogues to (2.6) - (2.8) are obvious. To estimate $\hat{\theta}_t$ we use $\exp[\log(\hat{\theta}_t)]$; alternatively, one could use $\exp[\log(\hat{\theta}_t) + \mathrm{Var}(\log(\hat{\theta}_t) - \log(\theta_t))/2]$ for a more "unbiased" estimate of θ_t with minimum mean squared error (see Granger and Newbold 1976).

Notice that (2.3) - (2.6) require knowledge of μ and any two of Σ_Y, Σ_θ, and Σ_e (the third can be obtained from (2.2)). In practice these will not be known exactly and will need to be estimated. Thus, the true minimum mean squared error linear predictor $\hat{\theta}$ cannot be obtained exactly and (2.6) or (2.8) understates the mean squared error (MSE) since it does not account for modeling errors. (See Binder and Dick (1989) and Eltinge and Fuller (1989).) The basic assumption underlying the application of the preceeding results, which we shall call the time series approach to survey estimation, is that μ and Σ_Y can be well-estimated from the time series data on Y_t through a time series model, and Σ_e can be well-estimated using survey microdata and knowledge of the survey design (possibly also using a model). We discuss these issues further in section 4 and illustrate the approach with the example of section 5.

2.2 Some General Considerations of the Time Series Approach

Smith (1978), Jones (1980), and Binder and Dick (1986) review and discuss the approach known as Minimum Variance Linear Unbiased Estimation (MVLU). While both the MVLU and time series approaches can use data from time points other than t in estimating θ_t, they differ in that MVLU regards the θ_t's as fixed and still only treats one source of variation, that due to sampling. MVLU was developed for cases (such as many rotating panel surveys) where more than one direct estimate of θ_t is available for each t and the e_t's are correlated over time due to overlap in the survey design. The use of Y_j for $j \neq t$ in estimating θ_t then comes from generalized least squares results and the correlation of the e_t's. We can see the distinction in terms of our results for the simple case (2.1) where only one direct estimate, Y_t, of θ_t is available, by letting $\mathrm{Var}(\theta_t) \rightarrow \infty$ to get the MVLU. Then $\Sigma_\theta^{-1} \rightarrow 0$ and (2.5) becomes $\hat{\underset{\sim}{\theta}} = \underset{\sim}{Y}$, so without multiple estimates of θ_t the MVLU just uses Y_t to estimate θ_t. These remarks apply generally to composite estimation (Rao and Graham 1964, Wolter 1979), which is often used as an approximation to MVLU.

One question that may arise regarding the time series approach is why one should consider θ_t a stochastic time series? This issue has been discussed by SSJ and at length by Smith (1978). They observe that (1) users of data from repeated surveys treat the data Y_t as a stochastic time series in modeling and would do the same with θ_t if it were available (as it essentially is for surveys with very low levels of error), and (2) classical results (*e.g.* Patterson 1950) for estimation in repeated surveys (MVLU) assume a time series structure for the individual units in the population, while maintaining the anomalous position that θ_t, which is a function of these individual units (such as the total), is a sequence of fixed, unrelated quantities. In fact, if we assume θ_t is a sequence of fixed, unrelated quantities, then data through any time point are irrelevant to the future behavior of the true series θ_t. If this were the case, then there would be little point in doing the survey in the first place. The data would be out of date as soon as they were published. The real questions here are whether or not we can estimate the time series structure of θ_t and e_t well enough to make beneficial use of this in survey estimation, how worthwhile these benefits may be, and what risks are involved in doing so?

Along with opportunities for improving estimation in repeated surveys, the time series approach offers potential for improved results in other problems where typically only one of the two sources of variability is recognized. It also can potentially unify these as subproblems under one general approach. Such problems include preliminary estimation in repeated surveys (Rao, Srinath, and Quenneville 1989); seasonal adjustment (Wolter and Monsour 1981, Hausman and Watson 1985, Pfeffermann 1991); time series trend estimation and the related problem of detection of statistically significant change over time (Smith 1978); benchmarking, the reconciling of results from a repeated survey with the results from another survey or census estimating the same population characteristics (Hillmer and Trabelsi 1987, Trabelsi and Hillmer 1990); and inference about time series properties of the true series θ_t relevant to economic models (Bell and Wilcox 1990).

Finally, we note that the decomposition (2.1) or (2.10) does not allow for nonsampling errors, nor does the time series approach treat them explicitly. Whether nonsampling error is generally more or less of a problem for the time series approach than for the classical approach is unclear, but one may wish to consider the possible effects of known or suspected nonsampling errors on the time series estimators when applying them in particular situations.

3. THEORETICAL CONSIDERATIONS

We now obtain some theoretical results relevant to the time series approach, and some properties of the resulting estimators.

3.1 Consistency of Time Series Estimators

Following Fuller and Isaki (1981) we let Y_t^ℓ (from the ℓ^{th} sample at time t) be a sequence of estimators of the characteristic θ_t^ℓ of the ℓ^{th} population at time t where the populations and samples for $\ell = 1, 2, \ldots$ are nested. (See their paper for details.) Define $\underset{\sim}{Y}^\ell, \underset{\sim}{\theta}^\ell, \underset{\sim}{e}^\ell, \underset{\sim}{\mu}^\ell, \Sigma_Y^\ell, \Sigma_\theta^\ell, \Sigma_e^\ell, \hat{\underset{\sim}{\theta}}^\ell$, and $\hat{\theta}_t^\ell$ in the obvious fashion. We consider what happens to the time series estimators $\hat{\underset{\sim}{\theta}}^\ell$ when the estimators $\underset{\sim}{Y}^\ell$ are consistent, *i.e.* $Y_t^\ell \rightarrow \theta_t^\ell$ in some fashion as $\ell \rightarrow \infty$ for $t = 1, \ldots, T$, with T, the length of the series, remaining fixed. For now we assume $\underset{\sim}{\mu}^\ell$, Σ_θ^ℓ, and Σ_e^ℓ are known for each ℓ, which generally means the time series models (including their parameter values) for the components are known. Since $\underset{\sim}{\mu}^\ell$ and Σ_θ^ℓ are really superpopulation parameters for the time series, θ_t^ℓ, we wish to estimate, we shall assume these are the same for each population ℓ, that is, $\underset{\sim}{\mu}^\ell \equiv \underset{\sim}{\mu}$ and $\Sigma_\theta^\ell \equiv \Sigma_\theta$ (a positive definite matrix) for all ℓ. This is also partly for convenience since we could get the same results assuming $\underset{\sim}{\mu}^\ell \rightarrow \underset{\sim}{\mu}$ and $\Sigma_\theta^\ell \rightarrow \Sigma_\theta$ as $\ell \rightarrow \infty$.

From (2.5) it would appear that $\underset{\sim}{Y}^\ell \rightarrow \underset{\sim}{\theta}^\ell$ would imply $\hat{\underset{\sim}{\theta}}^\ell \rightarrow \underset{\sim}{\theta}^\ell$ as long as $\Sigma_e^\ell \rightarrow 0$. This condition suggests we need mean square convergence of Y_t^ℓ to θ_t^ℓ. We thus consider estimators Y_t^ℓ of θ_t^ℓ such that $E[(Y_t^\ell - \theta_t^\ell)^2] = E[(e_t^\ell)^2] \rightarrow 0$ as $\ell \rightarrow \infty$. Since $E[(e_t^\ell)^2] = \operatorname{Var}(e_t^\ell) + [E(e_t^\ell)]^2$ this implies both $\operatorname{Var}(e_t^\ell) \rightarrow 0$ and $E(e_t^\ell) \rightarrow 0$. Assuming $Y_t^\ell \rightarrow \theta_t^\ell$ in mean square for $t = 1, \ldots, T$ thus implies $\Sigma_e^\ell \rightarrow 0$. We can now establish

Result 3.1: Consider $\hat{\underset{\sim}{\theta}} = (\hat{\theta}_1, \ldots, \hat{\theta}_T)'$ given by (2.4). If $Y_t^\ell \rightarrow \theta_t^\ell$ in mean square as $\ell \rightarrow \infty$ for $t = 1, \ldots, T$, then $\hat{\theta}_t^\ell \rightarrow \theta_t^\ell$ in mean square as $\ell \rightarrow \infty$ for $t = 1, \ldots, T$.

Proof: From $\underset{\sim}{Y}^\ell = \underset{\sim}{\theta}^\ell + \underset{\sim}{e}^\ell$ with $\Sigma_e^\ell \rightarrow 0$ we have $\Sigma_Y^\ell \rightarrow \Sigma_\theta$ (even if $\underset{\sim}{\theta}^\ell$ and $\underset{\sim}{e}^\ell$ are correlated.) From (2.4) we have

$$\hat{\underset{\sim}{\theta}}^\ell - \underset{\sim}{\theta}^\ell = (\underset{\sim}{Y}^\ell - \underset{\sim}{\theta}^\ell) - \Sigma_e^\ell(\Sigma_Y^\ell)^{-1}(\underset{\sim}{Y}^\ell - \underset{\sim}{\mu}). \tag{3.1}$$

The first term on the right converges to $\underset{\sim}{0}$ in mean square; the second has mean $\underset{\sim}{0}$ and variance $\Sigma_e^\ell(\Sigma_Y^\ell)^{-1}\Sigma_e^\ell \rightarrow 0$ as $\ell \rightarrow \infty$. Since both terms converge to $\underset{\sim}{0}$ in mean square so does $\hat{\underset{\sim}{\theta}}^\ell - \underset{\sim}{\theta}^\ell$.

Convergence in probability is a more familiar concept in survey sampling. If $Y_t^\ell \rightarrow \theta_t^\ell$ as $\ell \rightarrow \infty$ in probability for $t = 1, \ldots, T$ this does not guarantee $\Sigma_e^\ell \rightarrow 0$, which is mean square convergence, a stronger condition. If we assume there are random variables ζ_t with finite variance such that $| e_t^\ell | \leq \zeta_t$ (almost surely) uniformly in ℓ, then $Y_t^\ell \rightarrow \theta_t^\ell$ in probability implies $Y_t^\ell \rightarrow \theta_t^\ell$ in mean square (Chung 1968, p. 64). Therefore, using Result 3.1, we have

Result 3.2: If $Y_t^\ell \rightarrow \theta_t^\ell$ in probability as $\ell \rightarrow \infty$ for $t = 1, \ldots, T$ and there exist random variables ζ_t with finite variance such that $| Y_t^\ell - \theta_t^\ell | \leq \zeta_t$ (almost surely) uniformly in ℓ, then $\hat{\theta}_t^\ell \rightarrow \theta_t^\ell$ in probability as $\ell \rightarrow \infty$ for $t = 1, \ldots, T$.

These consistency results show that if the errors in the original estimates Y_t of θ_t are small (Σ_e is small) then the errors $\hat{\theta}_t - \theta_t$ will be small as well. From (3.1) we see this is because $\hat{\underset{\sim}{\theta}} - \underset{\sim}{Y}$ becomes small as Σ_e becomes small, thus when there is little error in the original estimates Y_t the time series approach will not change them much. Binder and Dick (1986) have noted this phenomenon, and also pointed out that in this case it does not matter what time series model is used. That is, the convergence to $\underset{\sim}{0}$ of (3.1) depends only on $\Sigma_e^\ell \rightarrow 0$ and not on $\underset{\sim}{\mu}$ or Σ_θ. Thus, the consistency results extend to allowing $\underset{\sim}{\mu}$, Σ_θ, and also Σ_e^ℓ to be replaced by estimates $\hat{\underset{\sim}{\mu}}^\ell$, $\hat{\Sigma}_\theta^\ell$, and $\hat{\Sigma}_e^\ell$ (which will generally come from estimated models – see sections 4 and 5), as long as $\hat{\underset{\sim}{\mu}}^\ell$ and $\hat{\Sigma}_\theta^\ell$ converge to something as $\ell \rightarrow \infty$ (it doesn't matter what as long as the limit of $\hat{\Sigma}_{\theta}^{\ell}$ is positive definite) and $\hat{\Sigma}_{e}^{\ell} \to 0$, which should generally hold when $\Sigma_{e}^{\ell} \to 0$. It is also obvious that these results extend to the nonstationary case where $\hat{\underset{\sim}{\theta}}$ is given by (2.7) instead of (2.4). While the results show that the time series estimates behave sensibly in the situation of small error in the original estimates Y_t, the gains from the time series approach will come in the opposite case – when $\mathrm{Var}(e_t)$ is large.

We can extend the consistency results to the case where we take logarithms and estimate $\log(\theta_t)$ using (2.11). In this case let $\Sigma_{u}^{\ell} = \mathrm{Var}(\log(\underset{\sim}{u}^{\ell}))$ where $\log(\underset{\sim}{u}^{\ell}) \equiv (\log(u_1^{\ell}), \ldots, \log(u_T^{\ell}))'$. If we are taking logarithms it is reasonable to assume Y_t^{ℓ} and θ_t^{ℓ} remain bounded away from 0, say $| Y_t^{\ell} | \geq \kappa$ and $| \theta_t^{\ell} | \geq \kappa$ (almost surely) for all t and ℓ for some constant $\kappa > 0$.

Result 3.3: If $Y_t^{\ell} \to \theta_t^{\ell}$ in mean square as $\ell \to \infty$ for $t = 1, \ldots, T$, then $\log(Y_t^{\ell}) \to \log(\theta_t^{\ell})$ and $\log(\hat{\theta}_t^{\ell}) \to \log(\theta_t^{\ell})$ in mean square as $\ell \to \infty$ for $t = 1, \ldots, T$.

Proof: The analogue to (3.1) is

$$\log(\hat{\underset{\sim}{\theta}}^{\ell}) - \log(\underset{\sim}{\theta}^{\ell}) = (\log(\underset{\sim}{Y}^{\ell}) - \log(\underset{\sim}{\theta}^{\ell})) - \Sigma_{u}^{\ell}(\Sigma_{Y}^{\ell})^{-1}(\log(\underset{\sim}{Y}^{\ell}) - \underset{\sim}{\mu}).$$

If we can show $\Sigma_{u}^{\ell} \to 0$ we will have the result since this implies $\log(\underset{\sim}{Y}^{\ell}) \to \log(\underset{\sim}{\theta}^{\ell})$ in mean square, and the second term on the right behaves exactly as that in (3.1). Notice

$$E\left[(\tilde{u}_t^{\ell})^2\right] = E\left[(e_t^{\ell})^2/(\theta_t^{\ell})^2\right] \leq \left(E(e_t^{\ell})^2\right)/\kappa^2 \to 0 \quad \text{as} \quad \ell \to \infty,$$

thus $E[(\tilde{u}_t^{\ell})^2] = E[(u_t^{\ell} - 1)^2] \to 0$. This implies $\mathrm{Var}(u_t^{\ell}) \to 0$ and $E(u_t^{\ell}) \to 1$. By Jensen's inequality (Chung 1968, p. 45), since $\exp(\cdot)$ is a convex function,

$$1 \leq \exp\left(E\left[\log(u_t^{\ell})^2\right]\right) \leq E\left(\exp\left[\log(u_t^{\ell})^2\right]\right) = E\left[(u_t^{\ell})^2\right].$$

But $E[(u_t^{\ell})^2] = \mathrm{Var}(u_t^{\ell}) + [E(u_t^{\ell})]^2 \to 1$ so $\exp(E[\log(u_t^{\ell})^2]) \to 1$ implying $E[\log(u_t^{\ell})^2] \to 0$. This yields $\mathrm{Var}(\log(u_t^{\ell})) \to 0$ as desired.

As before we could get a convergence in probability result by imposing a boundedness condition on the $\log(u_t^{\ell})$. Having $\log(\hat{\theta}_t)$ as an estimate of $\log(\theta_t)$, we have the following Corollary to Result 3.3 for using $\exp[\log(\hat{\theta}_t)]$ as an estimate of θ_t.

Corollary 3.4: If $Y_t^{\ell} \to \theta_t^{\ell}$ in mean square as $\ell \to \infty$ for $t = 1, \ldots, T$, then (see (2.11)) $\exp[\log(\hat{\theta}_t^{\ell})] \to \theta_t^{\ell}$ in probability as $\ell \to \infty$ for $t = 1, \ldots, T$.

Proof: Since $\log(\hat{\theta}_t^{\ell}) \to \log(\theta_t^{\ell})$ in mean square implies convergence in probability, the result follows since $\exp(\cdot)$ is a continuous function (Chung 1968, p. 66).

An analogous result obviously holds for using $\exp[\log(\hat{\theta}_t^{\ell}) + \mathrm{Var}(\log(\hat{\theta}_t^{\ell}) - \log(\theta_t^{\ell}))/2]$ to estimate θ_t, since then $\mathrm{Var}(\log(\hat{\theta}_t^{\ell}) - \log(\theta_t^{\ell})) \to 0$ as $\ell \to \infty$.

3.2 Uncorrelatedness of θ and e

Standard time series signal extraction results corresponding to (2.3) – (2.8) typically assume and θ_t and e_t are uncorrelated with each other at all leads and lags (equivalent to independence under normality). Previous papers on the time series approach to repeated survey estimation have merely assumed this, but since θ_t and e_t depend on the same population units it is not obvious that this assumption is valid. Fortunately, we can establish that it is valid under fairly general conditions. (Tam (1987) discusses how this fails under an explicitly model-based approach.)

We let y_{it} be the value of the characteristic of interest for the i^{th} unit in the population at time t, and let $\Omega_t = \{y_{it}: i = 1, \ldots, N_t\}$ be the collection of all N_t of these units. We consider time points $t = 1, \ldots, T$ and let $\underset{\sim}{\Omega} = (\Omega_1, \ldots, \Omega_T)'$. The y_{it} are random variables, as is $\theta_t = \theta_t(\Omega_t)$, which is a function of the y_{it}. The sample at time t, s_t (denoting the indices, not the values, of the units selected), has probability of selection $p(s_t \mid \underset{\sim}{\Omega})$. The estimator Y_t of θ_t is a function of the values y_{it} for the units sampled, thus a function of both Ω_t and s_t, *i.e.* $Y_t = Y_t(\Omega_t, s_t)$. We could let Y_t depend on the sample at times other than t, but we ignore that here for simplicity.

We consider estimators Y_t of θ_t that are *design unbiased*, which we shall define as $E(Y_t \mid \underset{\sim}{\Omega}) \equiv \Sigma_{s_t} Y_t p(s_t \mid \underset{\sim}{\Omega}) = \theta_t$. We could alternatively define design unbiasedness as $E(Y_t \mid \Omega_t) \equiv \Sigma_{s_t} Y_t p(s_t \mid \Omega_t) = \theta_t$, and then would need to assume the sample selection process is such that $p(s_t \mid \underset{\sim}{\Omega}) = p(s_t \mid \Omega_t)$, so $E(Y_t \mid \underset{\sim}{\Omega}) = E(Y_t \mid \Omega_t)$. If the sample design is noninformative then s_t and $\underset{\sim}{\Omega}$ are independent, implying $p(s_t \mid \underset{\sim}{\Omega}) = p(s_t \mid \Omega_t) = p(s_t)$, and either definition of design unbiasedness reduces to $\Sigma_{s_t} Y_t p(s_t) = \theta_t$. This is the usual definition, which generally assumes the y_{it}, and so Ω_t and θ_t, are fixed. (The assumption $p(s_t \mid \underset{\sim}{\Omega}) = p(s_t \mid \Omega_t)$ allows the sample selection process at time t ($p(s_t \mid \underset{\sim}{\Omega})$) to depend on the population values at time t (Ω_t), but assumes the population values at time points other than t (Ω_j for $j \neq t$) offer no additional information on s_t beyond that in Ω_t. This might occur if sampling was with probability proportional to the size of an auxiliary variable at time t that was correlated with the y_{it} only at time t.) The assumptions we make here might even be generalized.

Result 3.5: If Y_t is design unbiased for all t then θ_t and e_t are uncorrelated time series.

Proof: Consider $\text{Cov}(\theta_t, e_j)$ for any two time points t and j. Since Y_j is design unbiased $E(e_j \mid \underset{\sim}{\Omega}) = E(Y_j - \theta_j \mid \underset{\sim}{\Omega}) = 0$, implying $E[E(e_j \mid \underset{\sim}{\Omega})] = E(e_j) = 0$. Also $E(\theta_t \cdot e_j \mid \underset{\sim}{\Omega}) = \theta_t \cdot E(e_j \mid \underset{\sim}{\Omega}) = 0$ implying $E(\theta_t \cdot e_j) = 0$. Thus $\text{Cov}(\theta_t, e_j) = E(\theta_t \cdot e_j) - E(\theta_t)E(e_j) = 0$.

Comment: If $E(e_j \mid \underset{\sim}{\Omega})$ does not depend on $\underset{\sim}{\Omega}$ then e_j is said to be "mean independent" of $\underset{\sim}{\Omega}$, which is known to be a stronger condition than e_j and $\underset{\sim}{\Omega}$ uncorrelated, though not as strong as stochastic independence (unless we have normality). This shows that actually we only need $E(e_t \mid \underset{\sim}{\Omega}) = E(Y_t \mid \underset{\sim}{\Omega}) - \theta_t$ to not depend on $\underset{\sim}{\Omega}$ for θ_t and e_t to be uncorrelated time series. This would cover cases where Y_t has a constant additive bias (not dependent on Ω_t) as an estimate of θ_t, or, using approximate Result 3.6 which follows, a constant percentage (multiplicative) bias.

We now consider the logarithmic decomposition (2.10) when the Y_t are design unbiased. We assume that $\tilde{u}_j$ is $O_p(r_\ell)$ where $r_\ell \rightarrow 0$ as $\ell \rightarrow \infty$ in the superpopulation framework of the previous section, omitting the superscript ℓ from random variables here for convenience. (See Wolter (1985, p. 222) for definition of the order in probability notation $O_p(r_\ell)$. For example, when estimating a population mean we would often have $\text{Var}(\tilde{u}_j) \leq K/n_{j\ell}$ where K is some constant and $n_{j\ell}$ is the sample size at time j in the ℓ^{th} population. Then $\tilde{u}_j = O_p(n_{j\ell}^{-.5})$ from Wolter (1985, theorem 6.2.1).) From a Taylor series linearization of $\log(u_j) = \log(1 + \tilde{u}_j)$ we have from Wolter (1985, theorem 6.2.2)

$$\log(u_j) = \tilde{u}_j + O_p(r_\ell^2). \tag{3.2}$$

Using this we obtain the following.

Result 3.6: If Y_t is design unbiased for all t and $\tilde{u}_j$ is $O_p(r_\ell)$, then to terms that are $O_p(r_\ell^3)$, $\log(\theta_t)$ and $\log(u_t)$ are uncorrelated time series.

Proof: From theorem 6.2.5 of Wolter (1985) $\mathrm{Cov}(\log(\theta_t), \log(u_j)) = \mathrm{Cov}(\log(\theta_t), \tilde{u}_j) + O_p(r_t^3)$. Notice $E(\tilde{u}_j \mid \Omega) = E(e_j \mid \Omega)/\theta_j = 0$ implies $E(\tilde{u}_j) = 0$, and $E(\log(\theta_t)\tilde{u}_j \mid \Omega) = \log(\theta_t)E(\tilde{u}_j \mid \Omega) = 0$ implies $E(\log(\theta_t)\tilde{u}_j) = 0$, so $\mathrm{Cov}(\log(\theta_t), \tilde{u}_j) = 0$, establishing the result.

3.3 Design-Based Properties of Signal Extraction Estimates

Unconditionally, $\hat{\theta}$ in (2.3) is unbiased ($E(\hat{\theta}) = E(\theta) = \mu$) and has minimum MSE given by (2.6). It is easy to see that this is not the case when viewed from a design-based perspective. Suppose we begin with design-unbiased estimators Y, *i.e.* $E(Y \mid \Omega) = \theta$. From (2.2) and (2.4) we have $\hat{\theta} - \theta = (I - \Sigma_e \Sigma_Y^{-1})\, e - \Sigma_e \Sigma_Y^{-1}(\theta - \mu)$. With some algebra, we can show the design bias, variance, and MSE of $\hat{\theta}$ are given by

$$\begin{aligned}
E(\hat{\theta} \mid \Omega) - \theta &= -\Sigma_e \Sigma_Y^{-1}(\theta - \mu), \\
\mathrm{Var}(\hat{\theta} - \theta \mid \Omega) &= \Sigma_e - \Sigma_e \Sigma_Y^{-1}\Sigma_e - \Sigma_e \Sigma_Y^{-1}\Sigma_\theta \Sigma_Y^{-1}\Sigma_e, \\
E[(\hat{\theta} - \theta)(\hat{\theta} - \theta)' \mid \Omega] &= \Sigma_e - \Sigma_e \Sigma_Y^{-1}\Sigma_e \\
&\quad - \Sigma_e \Sigma_Y^{-1}\left[\Sigma_\theta - (\theta - \mu)(\theta - \mu)'\right]\Sigma_Y^{-1}\Sigma_e. \qquad (3.3)
\end{aligned}$$

From a design-based perspective we see use of $\hat{\theta}$ trades bias for a reduction in variance, since $\Sigma_e - \mathrm{Var}(\hat{\theta} - \theta \mid \Omega)$ is a positive semidefinite matrix. Whether this reduces the conditional MSE (3.3) below Σ_e, the MSE of Y, depends on the last two terms in (3.3), and in turn on θ. There can be particular realizations of θ for which the conditional MSE of $\hat{\theta}$ exceeds Σ_e, though on average signal extraction reduces the MSE by $\Sigma_e \Sigma_Y^{-1} \Sigma_e$, since the unconditional expectation of the bracketed term in (3.3) is zero. (Of course, (3.3) is unusable in practice since it depends on θ.) Also, as noted earlier, modeling error will contribute additional MSE to $\hat{\theta}$, so another fundamental question, more difficult to answer (see Eltinge and Fuller 1989), is how the real unconditional MSE of $\hat{\theta}$ compares to Σ_e?

4. APPLICATION CONSIDERATIONS

Application of the time series approach to survey estimation requires estimation of the autocovariance structure of the sampling errors, estimation of the mean and autocovariance structure of the signal, and computation of the estimates $\hat{\theta}_t$ and $\mathrm{Var}(\hat{\theta}_t - \theta_t)$ as discussed in section 2. The first two generally involve use of time series models, and are discussed in some detail in Bell and Hillmer (1989). Here we make some general remarks. We assume the Y_t are design unbiased estimators of the θ_t. We illustrate application of the methods in the next section with two time series from the Census Bureau's Retail Trade Survey.

Sampling error autocovariances, $\mathrm{Cov}(e_t, e_{t+k})$, can be estimated in an analogous fashion to sampling variances, $\mathrm{Var}(e_t)$, which is done routinely and for which many methods are available. (See Wolter 1985.) In practice, there may be difficulties in linking survey microdata over time to directly estimate sampling error covariances. Nevertheless, in what follows we assume we have available such estimates $\widehat{\mathrm{Cov}}(e_t, e_{t+k})$ for some set of time points t and lags k. Unfortunately, if there is a substantial amount of sampling error present (the situation where time series methods can make a difference), such autocovariance estimates are likely to have high variances themselves. This suggests some sort of averaging to improve the autocovariance estimates.

First, if we assume e_t is covariance stationary, so $\mathrm{Cov}(e_t,e_{t+k}) \equiv \gamma_e(k)$ depends on k but not t, then each $\widehat{\mathrm{Cov}}(e_t,e_{t+k})$ is estimating $\gamma_e(k)$ and we can simply average them, *i.e.* take $\hat{\gamma}_e(k) = (T - k)^{-1}\sum_t \widehat{\mathrm{Cov}}(e_t,e_{t+k})$ if we have $\widehat{\mathrm{Cov}}(e_t,e_{t+k})$ for $t = 1, \ldots, T - k$. Alternatively, $\widehat{\mathrm{Corr}}(e_t,e_{t+k}) = \widehat{\mathrm{Cov}}(e_t,e_{t+k})/[\widehat{\mathrm{Var}}(e_t)\widehat{\mathrm{Var}}(e_{t+k})]^{.5}$ can be averaged over t to estimate $\mathrm{Corr}(e_t,e_{t+k})$, which also depends on k but not t, and the variance estimates can be averaged as before.

Now suppose we are assuming e_t is relative covariance stationary, so $\mathrm{Cov}(e_t/\theta_t, e_{t+k}/\theta_{t+k}) = \mathrm{Cov}(\tilde{u}_t,\tilde{u}_{t+k}) \equiv \gamma_u(k)$ depends on k but not t. If $\tilde{u}_t$ is $O_p(r_t)$ for all t, as in section 3.2, then from (3.2) and theorem 6.2.5 of Wolter (1985), $\mathrm{Cov}(\log(u_t),\log(u_{t+k})) = \mathrm{Cov}(\tilde{u}_t,\tilde{u}_{t+k}) + O_p(r_t^3) \approx \gamma_u(k)$. Taking $\widehat{\mathrm{Cov}}(e_t,e_{t+k})/(Y_t Y_{t+k})$ as estimates of $\mathrm{Cov}(\tilde{u}_t,\tilde{u}_{t+k})$, these can be averaged over t to estimate $\gamma_u(k)$. Alternatively, using corollary 5.1.5 of Fuller (1976) we can show that $\mathrm{Corr}(\log(u_t),\log(u_{t+k})) = \mathrm{Corr}(\tilde{u}_t,\tilde{u}_{t+k}) + O_p(r_t^3)$, and taking as estimates of $\rho_u(k) \equiv \mathrm{Corr}(\tilde{u}_t,\tilde{u}_{t+k}), \{\widehat{\mathrm{Cov}}(e_t,e_{t+k})/Y_t Y_{t+k}\}/\{[\widehat{\mathrm{Var}}(e_t)\widehat{\mathrm{Var}}(e_{t+k})]^{.5}/Y_t Y_{t+k}\} = \widehat{\mathrm{Corr}}(e_t,e_{t+k})$, we can average the estimated autocorrelations of e_t over t to estimate $\rho_u(k)$, which are approximately the autocorrelations of $\log(u_t)$. Relative variance estimates can be averaged as before.

Actually, the usual survey estimates of variances and autocovariances will be estimating $\mathrm{Var}(e_t \mid \underset{\sim}{\Omega})$ and $\mathrm{Cov}(e_t,e_{t+k} \mid \underset{\sim}{\Omega})$. These estimates may also be suitable as estimates of $\mathrm{Var}(e_t)$ and $\mathrm{Cov}(e_t,e_{t+k})$, *e.g.* if they make sense from a model-based perspective. If not, and if Y_t is design unbiased so $E(e_t \mid \underset{\sim}{\Omega}) = 0$, then averaging autocovariance estimates over time still makes sense. First, if e_t is assumed stationary, then $\gamma_e(k) \equiv \mathrm{Cov}(e_t,e_{t+k}) = E[\mathrm{Cov}(e_t,e_{t+k} \mid \underset{\sim}{\Omega})]$, so we can average estimates of $\mathrm{Cov}(e_t,e_{t+k} \mid \underset{\sim}{\Omega})$ to estimate $\gamma_e(k)$. Or if e_t is relative covariance stationary, then since $E(\tilde{u}_t \mid \underset{\sim}{\Omega}) = E(e_t \mid \underset{\sim}{\Omega})/\theta_t = 0, \gamma_u(k) \equiv \mathrm{Cov}(\tilde{u}_t,\tilde{u}_{t+k}) = E[\mathrm{Cov}(\tilde{u}_t,\tilde{u}_{t+k} \mid \underset{\sim}{\Omega})] = \mathrm{Cov}(\log(u_t),\log(u_{t+k})) + O_p(r_t^3)$, and estimates of $\mathrm{Cov}(\tilde{u}_t,\tilde{u}_{t+k} \mid \underset{\sim}{\Omega})$ can be averaged to estimate $\gamma_u(k)$. It is less clear how to justify averaging estimates of conditional (on $\underset{\sim}{\Omega}$) correlations, since $E[\mathrm{Corr}(e_t,e_{t+k} \mid \underset{\sim}{\Omega})] \neq \mathrm{Corr}(e_t,e_{t+k})$, though this may be true to a sufficient approximation. In general, approaches to estimation of sampling error autocovariance structures bear more investigation.

Given an estimate of the sampling error covariance structure, and using any relevant information about the design of the survey, we can attempt to determine a time series model and its parameters to closely reproduce this structure. This is illustrated in the example of section 5.

We now turn to developing a model for the signal, θ_t. Since the behavior of most published time series Y_t is dominated by their signals (otherwise, they would not be published), in developing models for signals θ_t we can draw on experience modeling time series Y_t without allowing for sampling error. Such experience suggests use of nonlinear transformations, differencing, and regression mean functions in the model for θ_t will be important. The logarithm is the most common nonlinear transformation used in time series, and taking $\log(Y_t)$ lets us model $\log(\theta_t)$ through (2.10), with consequences for the sampling error discussed above. The following remarks are given in terms of use of (2.1), but apply equally well to use of (2.10). While other transformations could be considered, they would not generally yield a convenient decomposition of transformed Y_t in terms of transformed θ_t and some sampling error. Choosing between taking logarithms or not transforming seems sufficient for modeling many series.

Assuming e_t has mean zero (implied by design unbiasedness) and does not require differencing, θ_t and Y_t will require the same differencing and have the same mean function. The mean function can often be modeled with a linear regression function, $\mu_t = \underset{\sim}{X}_t'\underset{\sim}{\beta}$, for some vector of regression variables $\underset{\sim}{X}_t$ and parameters $\underset{\sim}{\beta}$. We often use ARIMA (autoregressive-integrated-moving average) models to account for the needed differencing and to explain the autocovariance structure of the differenced θ_t. A convenient approach to developing the θ_t model is to first model Y_t ignoring the sampling error, and then use a model with the same regression terms and ARIMA order for θ_t. The parameters of the θ_t model can then be estimated using the time series data for Y_t and the previously developed model for e_t, holding the parameters in the model for e_t fixed. Diagnostic checking may suggest modifications to the θ_t model. The final fitted model can then be used in the signal extraction estimation of θ_t. The model fitting and signal extraction computations are not trivial; Kalman filter/smoother algorithms are discussed in Bell and Hillmer (1989). These have been implemented in some software recently developed in cooperation with members of the time series staff of the Statistical Research Division of the Census Bureau. This software was used in the analysis of the next section.

5. EXAMPLE: U.S. RETAIL TRADE SURVEY - SALES OF EATING AND DRINKING PLACES

As an illustrative example we analyze time series of sales (in millions of dollars) of Eating Places and of Drinking Places, which are estimated in the monthly U.S. Retail Trade Survey. The Retail Trade Survey has a list panel of large businesses that are selected into the sample with certainty and report sales every month, and 3 rotating list panels of smaller businesses that are selected into the sample by stratified simple random sampling. There is also a rotating panel area sample covering companies not in the list universe. Quarterly, a sample of new firm births is introduced, and firm deaths as determined from activity checks are removed from the sample. The rotating panels report current month and previous month sales at intervals of 3 months for the list sample and 6 or 12 months for the area sample. Horvitz-Thompson (HT) estimates of current and previous months' sales are constructed; the resulting time series shall be denoted Y_t' and Y_{t-1}''. From these composite estimators are constructed as described in Wolter (1979). The final composite estimates will make up our time series Y_t. (While it might be interesting to instead analyze Y_t' and Y_{t-1}'' directly, these estimates are not saved for a long enough period of time for seasonal time series modeling.) Sampling variances are estimated using the random group method (Wolter 1985, chapter 2) for the list sample with 16 random groups, and the collapsed stratum method for the area sample. Further information on the survey is given in Isaki *et al.* (1976), Wolter *et al.* (1976), Wolter (1979), Garrett, Detlefsen and Veum (1987), Bell and Wilcox (1990).

There are several complicating factors in the survey. The sample is redesigned and independently redrawn about every five years, with new samples having been introduced in September of 1977, and January of 1982 and 1987. This produces a break in the covariance structure of e_t every five years, which can be handled by the Kalman filter/smoother as discussed in Bell and Hillmer (1989). We shall use data from September, 1977 through December, 1986, so there is one redrawing of the sample near the middle of our series. When a new sample is introduced approximate MVLU estimates are used for the first three months before switching to the composite estimates (Wolter 1979). This introduces a transient effect into the sampling error autocorrelations that we shall ignore. Finally, the monthly estimates are benchmarked to annual totals estimated from an annual survey and from the economic census taken every five years. To avoid this complication we use data that are not benchmarked. The reader should be aware, however, that for this reason the data used here do not agree with published estimates.

Table 1
Sampling Error Correlations for Horvitz-Thompson Estimates

	Lag					
	4	8	12	16	20	24
Eating Places						
Averaged[1]	.72	.71	.79	.63	.65	.77
From (5.1)[2]	.75	.69	.81	.60	.53	.61
Drinking Places						
Averaged[1]	.70	.67	.78	.60	.60	.61
From (5.1)[2]	.72	.66	.80	.56	.50	.59
Number of Correlations Averaged	23	19	15	11	7	3
Weights Used in Determining $\hat{\phi}$'s	1	1	1	.5	0	0

[1] Raw estimates of $\mathrm{Corr}(e_i', e_j')$ and $\mathrm{Corr}(e_{i-1}'', e_{j-1}'')$ were available for all pairs of months from January, 1973 through March, 1975. Averages of the correlations for the lags shown were taken after applying Fisher's transformation, and the results then transformed back.

[2] Correlations are shown from model (5.1) for $m = 4$ with parameters $\hat{\phi}^4 = .604$, $\hat{\phi}_{12} = .723$ (Eating Places) and $\hat{\phi}^4 = .580$, $\hat{\phi}_{12} = .714$ (Drinking Places). These parameter values were determined to minimize the weighted sum of squared deviations of the correlations from model (5.1) and the averaged correlations using the weights shown. Lags 20 and 24 were not used (given zero weight) because of the small number of correlation estimates available at these lags.

5.1 Development of Sampling Error Models

Our first step will be to develop a model for the correlation structure of the sampling errors. Let us write $Y_t' = \theta_t + e_t'$ for the current month (t) HT estimate, and $Y_{t-1}'' = \theta_{t-1} + e_{t-1}''$ for the previous month $(t - 1)$ HT estimate. We shall use the same models for e_t' and e_{t-1}''. Estimates of $\mathrm{Corr}(e_t', e_{t-1}'')$ are extremely high - typically .98 or higher. While this is partly artificial (due to businesses reporting the same figure for current and previous month sales, and possibly due to the way missing values are imputed), in the absence of other information it is difficult to distinguish characteristics of e_t' from those of e_{t-1}''.

Since the three rotating panels in the survey are drawn (approximately) independently (Wolter 1979), auto- and cross-correlations for (e_t', e_{t-1}'') should be nonzero only for lags that are multiples of 3. Estimates of such lag correlations can be averaged over time assuming correlation stationarity. While estimates of lag correlations are not regularly produced for the Retail Trade Survey, this was done as part of a special study using micro-data (random group totals) from the Retail Trade Survey sample for January, 1973 through March, 1975, albeit at a time when the survey had four rotating list panels. Lacking more recent data, we "averaged" the correlations at lags 4, 8, 12, 16, 20, and 24 for e_t' and e_{t-1}''. (This was actually done after applying Fisher's transformation $.5 \log((1 + r)/(1 - r))$, to make the distribution of the transformed correlations more symmetric, and then transforming the results back.) The results are shown in Table 1. They show fairly strong positive correlation in the sampling errors, and evidence of seasonality from the correlations at lag 12. A possible model given such data is

$$(1 - \phi^m B^m)(1 - \Phi B^{12}) e_t' = v_{1t}, \tag{5.1}$$

where $m = 4$ for the 4-panel survey, with the same model assumed for e_{t-1}'' with $v_{2,t-1}$ replacing v_{1t}. (v_{1t} and $v_{2,t-1}$ are white noise with variance σ_v^2.)

A particularly convenient property of (5.1) is that if the sampling error in each panel would follow (5.1) with $m = 1$ if it were observed every month, then for any number m (that is a divisor of 12) of independent panels reporting successively, e_t' follows (5.1). This allows us to use the 4-panel survey results in Table 1 to estimate ϕ^4 and Φ, and (assuming $\phi > 0$) convert these to estimates of ϕ^3 and Φ, the parameters of the model for the current 3-panel survey. This was done by finding ϕ^4 and Φ to minimize the sum of squared deviations of the correlations from (5.1) with those of Table 1. (Lags 20 and 24 were dropped, and lag 16 given a weight of .5, due to the smaller number of correlation estimates that were averaged together at these higher lags.) This resulted in $\hat{\phi}^3 = .685$, $\hat{\Phi} = .723$ for Eating Places, and $\hat{\phi}^3 = .664$, $\hat{\Phi} = .714$ for Drinking Places. The resulting correlations for $m = 4$ from (5.1) are shown in Table 1, and may be compared to the averaged correlations. More formal statistical estimation procedures for ϕ^3 and Φ, as well as a possible test of model fit, could be considered. (We may pursue this later if sampling error autocorrelation estimates can be produced from more recent micro-data from the 3-panel survey.)

We make the further assumption that $\mathrm{Corr}(e_t', e_{t-1-k}'') = \rho\,\mathrm{Corr}(e_t', e_{t-k}')$ for all k. To justify this, note the population regression of e_{t-1-k}'' on e_{t-k}' is $\rho e_{t-k}' + \epsilon$, where if ϵ is not uncorrelated with e_t', at least it is certainly small since $\mathrm{Var}(\epsilon) = (1 - \rho^2)\mathrm{Var}(e_t')$ and ρ is very near 1. With this assumption (5.1) leads to the following bivariate model for (e_t', e_{t-1}''):

$$\left(1 - \phi^3 B^3\right)\left(1 - \Phi B^{12}\right)\begin{bmatrix} e_t' \\ e_{t-1}'' \end{bmatrix} = \begin{bmatrix} v_{1t} \\ v_{2,t-1} \end{bmatrix} \quad \mathrm{Var}\begin{bmatrix} v_{1t} \\ v_{2,t-1} \end{bmatrix} = \sigma_v^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \tag{5.2}$$

with $\rho = \mathrm{Corr}(v_{1t}, v_{2,t-1}) = \mathrm{Corr}(e_t', e_{t-1}'')$. Estimates of $\mathrm{Corr}(e_t', e_{t-1}'')$ are regularly produced and were available for 1982 through 1986. Averaging these (with Fisher's transformation) produced $\hat{\rho} = .985$ for Eating Places and $\hat{\rho} = .986$ for Drinking Places.

We can now use (5.2) to derive a model for the sampling error of the linear form of the composite estimator (Wolter 1979), which is given by

$$\begin{aligned} Y_t''' &= (1 - \beta)Y_t' + \beta(Y_{t-1}''' + Y_t' - Y_{t-1}'') && \text{(preliminary estimator)}, \\ Y_{t-1} &= (1 - \alpha)Y_{t-1}'' + \alpha Y_{t-1}''' && \text{(final estimator)}. \end{aligned} \tag{5.3}$$

In the (3-panel) retail trade survey, values of $\alpha = .8$, $\beta = .75$ are used. It is easily seen that (5.3) also holds for the sampling errors, *i.e.* with Y replaced by e. We can use the resulting relations to derive the following equation for e_t in terms of e_t' and e_{t-1}'':

$$(1 - .75B)e_t = .2\,e_t'' - .75\,e_{t-1}'' + .8\,e_t'. \tag{5.4}$$

Using (5.2) and (5.4) we then get

$$(1 - .75B)(1 - \phi^3 B^3)(1 - \Phi B^{12})\,e_t = .2\,v_{2t} - .75\,v_{2,t-1} + .8\,v_{1t}. \tag{5.5}$$

The right hand side is a first order moving average process (Box and Jenkins 1976, p. 121) whose parameters can be determined given estimates of σ_v^2 and ρ. Thus, (5.5) would yield an ARMA model for e_t.

Rather than pursue this further, we shall instead make the rather strong assumption that a model of the same form holds for $\log(u_t)$ in $\log(Y_t) = \log(\theta_t) + \log(u_t)$, thus

$$(1 - .75B)(1 - \phi^3 B^3)(1 - \Phi B^{12})\log(u_t) = (1 - \eta B)\,c_t. \tag{5.6}$$

Table 2

Coefficients of Variation (CV)[1] for Retail Sales Estimates

	Horvitz-Thompson	Final Composite[2]	Signal Extraction[3]	
	CV	CV	Low	High
Eating Places	.042	.025	.017	.023
Drinking Places	.088	.052	.032	.038

[1] CV = (Relative Variance)$^{.5}$.
[2] The values for the final composite estimator are obtained using models (5.7a,b).
[3] The values for signal extraction actually vary over time, being highest at the end of the series and lowest near the middle. We show the lowest and highest values, which are attained for both series in January 1982 (low) and December 1986 (high). The signal extraction variances are not symmetric in time because the sample redraw in January 1982 is not exactly at the center of the series.

We do this because estimates of sampling variance for these series are highly dependent on the level of the series; estimates of relative variance are much more stable over time. We also assume we can use estimates of relative variance and of ρ in determining η and σ_c^2. Estimates Y_t', Y_{t-1}'', $\widehat{\mathrm{Var}}(e_t')$ and $\widehat{\mathrm{Var}}(e_{t-1}'')$ were available for 1982 through 1986. The resulting relative variance estimates were used in the spirit of maximum likelihood estimation for the lognormal distribution - taking the average of the logs of the relative variance estimates, adding one half of the sample variance of the logged estimates to this, and exponentiating the results. (Merely averaging the relative variance estimates produced similar results.) This was done separately for Rel Var(Y_t') and Rel Var(Y_{t-1}''), and these two results were then averaged, producing a common relative variance estimate that is constant over time. The results are shown in Table 2 under the heading "Horvitz-Thompson". Using these and the $\hat{\rho}$'s given earlier, one can solve for η and σ_c^2 for the right side of (5.6). The resulting sampling error models are

$$(1 - .75B)(1 - .685B^3)(1 - .723B^{12}) \log(u_t) = (1 + .130B)c_t \qquad (5.7a)$$

$$\text{(Eating Places)} \qquad \hat{\sigma}_c^2 = 1.948 \times 10^{-5}$$

$$(1 - .75B)(1 - .664B^3)(1 - .714B^{12})\log(u_t) = (1 + .134B)c_t \qquad (5.7b)$$

$$\text{(Drinking Places)} \qquad \hat{\sigma}_c^2 = 9.301 \times 10^{-5}.$$

One can use the method of McLeod (1975,1977) to solve for Var(log(u_t)) in these models, which is an estimate of the relative variance of the final composite estimator. The results are shown in Table 2. The corresponding coefficients of variation, .025 for Eating Places and .052 for Drinking Places, are quite close to estimates published in the Census Bureau's Monthly Retail Trade Reports that are obtained more directly.

5.2 Time Series Modeling and Signal Extraction

Figures 1a,b show plots of the time series of final composite estimates Y_t for Eating Places and for Drinking Places, respectively. To develop models for θ_t we shall begin by modeling the Y_t series directly. Both series show trends and strong seasonality, with the magnitude of the seasonal fluctuations larger the higher the level of the series. This suggests taking logarithms and the need for differencing; both are typical for economic time series. Examination

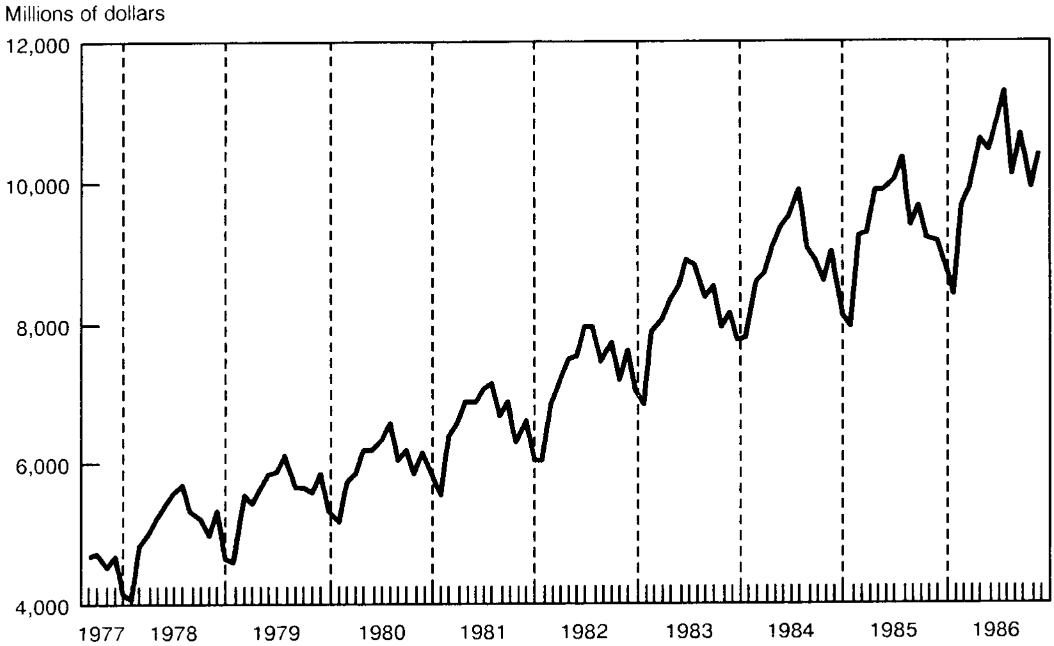

Figure 1.a Retail Sales of Eating Places – Composite Estimates (not benchmarked)

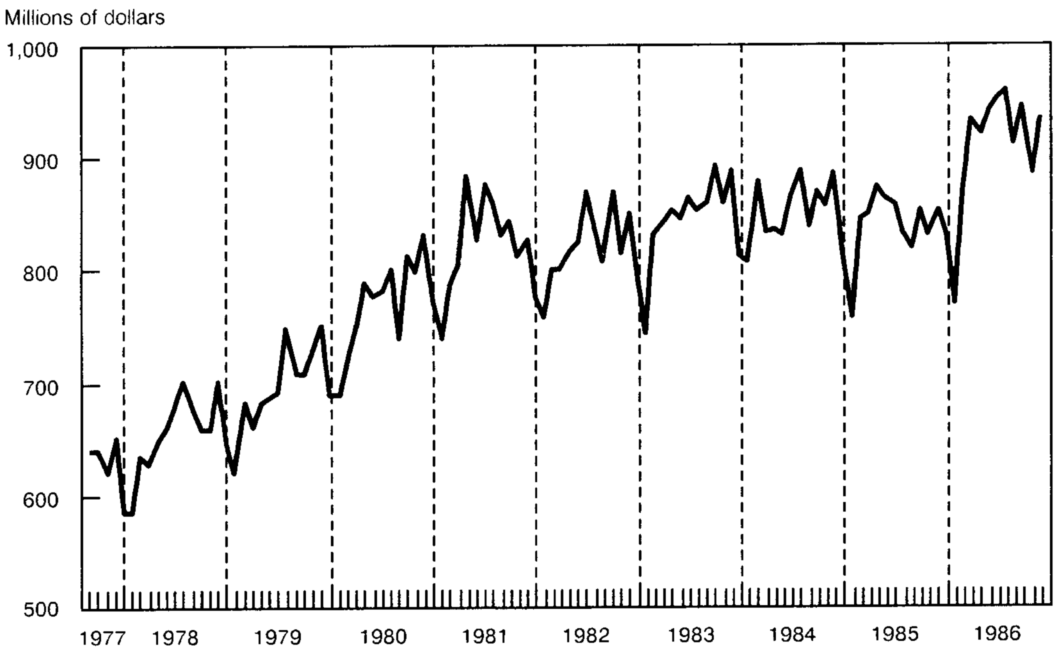

Figure 1.b Retail Sales of Drinking Places – Composite Estimates (not benchmarked)

of sample autocorrelations for $\log(Y_t)$ and its differences suggested the difference operator $(1 - B)(1 - B^{12})$ for both series. Retail trade series are known to contain trading-day variation, which can be modeled by including seven regression variables in the model: X_{1t} = number of Mondays in month t, ..., X_{7t} = number of Sundays in month t. Following Bell and Hillmer (1983), a more convenient parameterization is obtained by using instead the variables $T_{1t} = X_{1t} - X_{7t}$ (number of Mondays − number of Sundays), ..., $T_{6t} = X_{6t} - X_{7t}$ (number of Saturdays − number of Sundays), $T_{7t} = \sum_1^7 X_{it}$ (length of month t). To identify the ARMA structures, the autocorrelations and partial autocorrelations of the residuals from regressions of $(1 - B)(1 - B^{12}) \log(Y_t)$ on $(1 - B)(1 - B^{12})T_{it}$, $i = 1, \ldots, 7$, were examined. This suggested an ARIMA $(0,1,2)(0,1,1)_{12}$ model for Eating Places, and an ARIMA $(0,1,3)(0,1,1)_{12}$ model for Drinking Places. The resulting estimated models were

$$(1 - B)(1 - B^{12})\left[\log(Y_t) - \sum_i \beta_i T_{it}\right] = (1 - .25B - .22B^2)(1 - .79B^{12})\, a_t$$

$$\text{(Eating Places)} \qquad \hat{\sigma}_a^2 = .000230 \qquad (5.8\text{a})$$

$$(1 - B)(1 - B^{12})\left[\log(Y_t) - \sum_i \beta_i T_{it}\right] = (1 - .21B - .15B^2 + .03B^3)(1 - .56B^{12})\, a_t$$

$$\text{(Drinking Places)} \qquad \hat{\sigma}_a^2 = .000587. \qquad (5.8\text{b})$$

For brevity, we omit the estimates of the trading-day parameters. While the lag 2 and lag 3 moving average parameters in (5.8b) are small, we shall retain them since we shall only use (5.8a,b) as starting points for modeling $\log(\theta_t)$ for both series.

Taking models of the form of (5.8a,b) for $\log(\theta_t)$ with models (5.7a,b) for $\log(u_t)$, the parameters of the models for $\log(\theta_t)$ were estimated. For both series the seasonal moving average parameters were estimated to be very near 1(.985 for Eating Places and .992 for Drinking Places), implying nearly deterministic seasonality that can be modeled by cancelling a $(1 - B^{12})$ from both sides of the θ_t model and instead including a trend constant and a seasonal regression function of the form $\sum_1^{11} \gamma_i M_{it}$, where M_{1t} is 1 in January, −1 in December, and 0 otherwise, ..., M_{11t} is 1 in November, −1 in December, and 0 otherwise (Bell 1987). Estimation of the resulting models produced the following:

$$(1 - B)\left[\log(\theta_t) - \sum_i \hat{\beta}_i T_{it} - \sum_i \hat{\gamma}_i M_{it}\right] = .00762 + (1 - .20B - .29B^2)b_t$$

$$\text{(Eating Places)} \qquad \hat{\sigma}_b^2 = .000139 \qquad (5.9\text{a})$$

$$(1 - B)\left[\log(\theta_t) - \sum_i \hat{\beta}_i T_{it} - \sum_i \hat{\gamma}_i M_{it}\right] = .00352 + (1 - .18B - .09B^2 - .42B^3)b_t$$

$$\text{(Drinking Places)} \qquad \hat{\sigma}_b^2 = .000244. \qquad (5.9\text{b})$$

We again omit the estimates of the regression parameters. We do not provide standard errors for the ARMA parameters; doing so for models of the sort used here is a topic for further research, made particularly difficult here by the unrealistic assumption that the sampling error

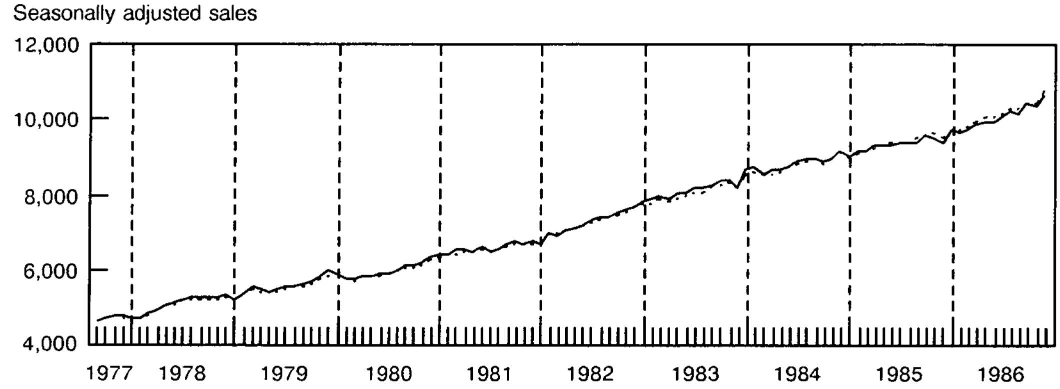

Figure 2.a Eating Places: Composite (solid) and Signal Extraction (dotted) Estimates

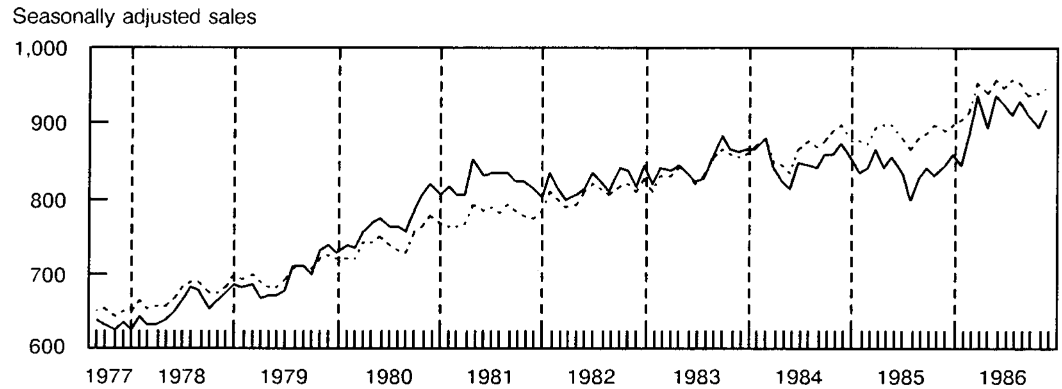

Figure 2.b Drinking Places: Composite (solid) and Signal Extraction (dotted) Estimates

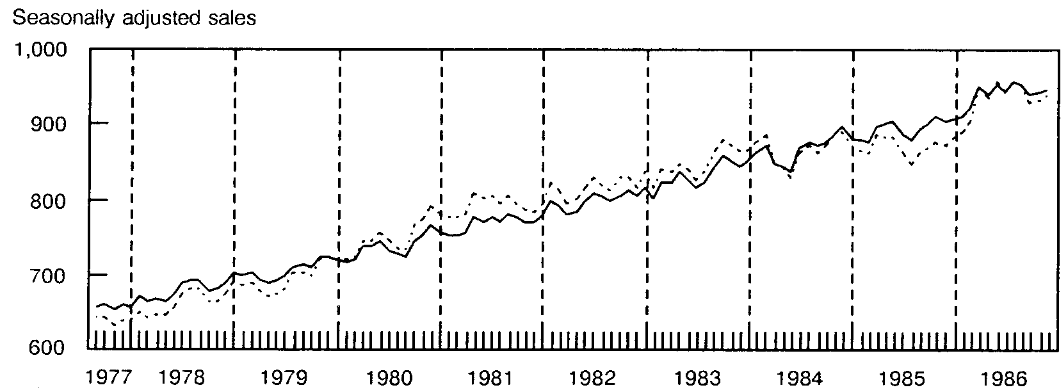

Figure 2.c Drinking Places: Alternative Signal Extraction Estimates

model is known. Examination of standardized residuals produced by the Kalman filter, and of their autocorrelations, suggested no major inadequacies with the fitted models for either series.

The estimated models, (5.7a,b) with (5.9a,b), were used to produce signal extraction estimates of $\log(\theta_t)$, which were then exponentiated to produce estimates of θ_t. The results are shown in Figures 2a,b for the series with the estimated seasonal and trading-day effects removed. Notice that signal extraction makes only slight differences in the estimates for Eating Places,which contained little sampling error (low relative variance), but it makes a considerable difference in the estimates for Drinking Places, which contained much more sampling error (higher relative variance). Signal extraction variances for $\log(\theta_t)$ were also produced; these are relative variances for the estimates of θ_t. Table 2 shows that, depending on the location in the series, signal extraction produces about an 8%-32% improvement in CV over the final composite estimates for Eating Places (though the composite estimate CV is small), and about a 27%-38% improvement in CV for Drinking Places. As noted previously, these results are optimistic, since they assume the true component models are those that were estimated. To partly address concerns about this, we next examine the sensitivity of the results for Drinking Places to variation in the model parameters.

5.3 Sensitivity Analysis for Drinking Places

Here we focus on sensitivity of results to variation in the sampling error model, since this was determined with less information than the signal model. Our approach is to vary parameters of the sampling error model, then reestimate the signal model and redo the signal extraction. While it would be preferrable to have more formal statistical measures of the signal extraction error due to model error (which the present state of theory and computer software does not allow), this approach should at least help indicate in what respects the signal extraction results are sensitive to parameter variation and in what respects they are not.

Comparing models (5.8b) and (5.9b) gives some indication of the sensitivity of the signal model to changes in σ_c^2, the innovation variance of the sampling error model, since (5.8b) corresponds to $\sigma_c^2 = 0$ and (5.9b) to $\sigma_c^2 = 9.3 \times 10^{-5}$. The most noticeable differences are in the estimate of σ_b^2, which is to be expected, and in the estimate of the seasonal moving average parameter, η_{12} say, which was found to be essentially 1 in obtaining (5.9b). Reestimation of the signal model for other values of σ_c^2 yielded $\hat{\eta}_{12} \geq .99$ as long as $\sigma_c^2 \geq 3.0 \times 10^{-5}$. In light of this, and to simplify presentation of results, we assume $\eta_{12} = 1$ and use a signal model with seasonal indicator variables as in (5.9b).

Figure 2.c. shows (seasonally and trading-day adjusted) signal extraction estimates $\hat{\theta}_t$ corresponding to sampling error models with $(\phi^3,\Phi) = (.564,.614)$ and $(.764,.814)$, and with $\rho = .986$ and $\mathrm{Var}(\log(u_t)) = .00776$ (the relative variance of the Horvitz-Thompson estimates) held fixed. These cover the extremes of $\hat{\theta}_t$ for the sensitivity analysis. The nature of the different estimates $\hat{\theta}_t$ we have generated seems to roughly correspond to the value of $\mathrm{CV}_{56} = [\mathrm{Var}(\log(\hat{\theta}_{56}) - \log(\theta_{56})]^{1/2}$, the signal extraction coefficient of variation achieved at the middle of the series. (CV_{56} is very close to the lowest value, which is achieved at $t = 53$ - see Table 2.) The lower CV_{56} is, the smoother $\hat{\theta}_t$ is. CV_{56} is 2.78%, 3.28%, and 3.70% for (ϕ^3,Φ) equal to $(.564,.614)$, $(.664,.714)$, and $(.764,.814)$ respectively. Other estimates $\hat{\theta}_t$ we generated lie closest to the signal extraction estimate in Figure 2.b. or 2.c. with the closest CV_{56}.

We now consider the sensitivity of CV_{56} to variations in the sampling error model parameters, beginning with ρ. The only parameter in (5.7b) affected by a change in ρ is η. Table 3 reports the values of η and corresponding values of ρ considered, and the resulting CV_{56}'s. We see CV_{56} is somewhat sensitive to changes in ρ, especially increases: CV_{56} for $\rho = 1$ (3.49) is 6% larger than for $\rho = .985$ (3.28), the value used for (5.7b).

Table 3

Sensitivity of CV_{56}[1] for Drinking Places to Changes in η (Changes in ρ)

η	.00	−.05	−.10	−.15	−.20	−.25
ρ	.9375	.9642	.9792	.9888	.9953	1.000
CV_{56}	3.03	3.12	3.21	3.31	3.40	3.49

[1] CV_{56} is the signal extraction coefficient of variation for $t = 56$ (the middle of the series), expressed as a percentage, *i.e.* the square root of $\mathrm{Var}(\log(\hat{\theta}_t) - \log(\theta_t))$ multiplied by 100.

Table 4

Sensitivity of CV_{56} for Drinking Places to Changes in $\mathrm{Var}(\log(u_t))$[1] (Changes in σ_c^2)

$\mathrm{Var}(\log(u_t))$	.00676	.00726	.00776	.00826	.00876
CV(HT)[2]	8.22	8.52	8.81	9.09	9.36
$\sigma_c^2 \times 10^5$	8.16	8.76	9.30	9.97	10.57
CV_{56}	3.16	3.23	3.28	3.35	3.40

[1] $\mathrm{Var}(\log(u_t))$ is the relative variance of the Horvitz-Thompson estimators.
[2] CV(HT) is the coefficient of variation of the Horvitz-Thompson estimators, expressed as a percentage, *i.e.* the square root of $\mathrm{Var}(\log(u_t))$ multiplied by 100.

Table 5

Sensitivity of Results for Drinking Places to Changes in (ϕ^3, Φ)

		(i) Values of $\sigma_c^2 \times 10^5$ for given (ϕ^3,Φ)				
		ϕ^3				
		.564	.614	.664	.714	.764
	.614	16.90	14.70	12.36	9.98	7.64
	.664	15.03	13.00	10.87	8.72	6.62
Φ	.714	13.04	11.23	9.30	7.44	5.60
	.764	10.96	9.40	7.78	6.15	4.58
	.814	8.79	7.51	6.17	4.85	3.58
		(ii) Values of CV_{56} for given (ϕ^3,Φ)				
		ϕ^3				
		.564	.614	.664	.714	.764
	.614	2.78	2.88	2.99	3.12	3.27
	.664	2.95	3.04	3.14	3.26	3.38
Φ	.714	3.10	3.19	3.28	3.39	3.50
	.764	3.24	3.33	3.42	3.51	3.60
	.814	3.36	3.45	3.54	3.62	3.70

We next consider the sensitivity of CV_{56} to changes in $\mathrm{Var}(\log(u_t))$. The only sampling error model parameter this affects is σ_c^2. Table 4 reports the values of $\mathrm{Var}(\log(u_t))$, its square root CV(HT), the corresponding σ_c^2, and the resulting CV_{56}. We see less sensitivity of CV_{56} here than in Table 3.

Finally, we examine the sensitivity of CV_{56} to ϕ^3 and Φ. Holding $\mathrm{Var}(\log(u_t))$ fixed at .00776 and changing (ϕ^3,Φ) also changes σ_c^2. Table 5 reports the grid of values used for (ϕ^3,Φ), and resulting values of σ_c^2 and CV_{56}. Notice σ_c^2 varies more here than in Table 4. We see CV_{56} increases substantially as ϕ^3 and Φ are increased.

We conclude from this analysis that moderate changes in the sampling error model parameters have relatively small impacts on $\hat{\theta}_t$. The largest changes we observed in $\hat{\theta}_t$ were around 2 percent. The same moderate changes in the sampling error model parameters have relatively larger impacts on the signal extraction variances, with CV_{56}'s changing by as much as 17 percent. This suggests that for this example the greatest concern in not knowing the sampling error model parameters may be in the effect on signal extraction variances, and the resulting measures of improvement over the composite estimates. However, in all the cases considered in the sensitivity analysis the signal extraction estimates showed a significant improvement in variance.

5.4 Conclusions

The Drinking Places example illustrates the potential gains that may be achieved with the time series approach to survey estimation. Both examples also illustrate the complex and delicate nature of the time series modeling that may be required. We view the results as preliminary for several reasons. First,the optimistic nature of the signal extraction variances that do not reflect parameter estimation error has been mentioned. Second, we have no clear explanation of why the signal extraction estimates lie above or below the composite estimates for long stretches of time. (This is obvious in Figure 2.b., and actually the case in Figure 2.a. as well.) For the Drinking Places example this behavior was evident throughout the sensitivity analysis, and so does not appear to be due to uncertainty in the parameters of the sampling error model. We are in the process of exploring whether this may be due to the forms of the sampling error model or signal model being incorrect. In fact, Bell and Wilcox (1990) report that the correlations of e_t' and e_{t-1}' at lags not multiples of three are not necessarily zero, as was assumed by the model.

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