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Robust Small Domain Estimation Using Random Effects Modeling

PHILLIP S. KOTT[1]

ABSTRACT

This paper develops a design consistent small domain estimator using a random effects model. The mean squared error of this estimator is then evaluated *without* assuming the random effect component of the model is correct. Data from a complex sample survey shows how this approach to mean squared error estimation, while perhaps too instable to be used directly, can be employed to determine whether the design consistent small domain estimator proposed here is better than the conventional design-based estimator.

KEY WORDS: Finite population; Model; Mean squared error; Design consistent; Randomization.

1. INTRODUCTION

Suppose we were given a probability sample of unit values and were asked to estimate the mean of a small domain within the larger population covered by the sample. Scott and Smith (1969) introduced a Bayesian estimator for this purpose and showed that their estimator could also be developed using only unbiasedness and minimum variance (UMV) criteria. Their UMV approach, sometimes called random effects or components-of-variance modeling, will be adopted here.

Most attempts at small domain estimation paralleling Scott and Smith (*e.g.*, Fay and Herriot 1979, Battese and Fuller 1971, Ghosh and Meeden 1986, Prasad and Rao 1986, Fuller and Harter 1987, and Stroud 1987) assume that the sampling design is noninformative and so ignorable. The same assumption is made for synthetic estimators of small domain means, which will not be discussed at any depth here (for examples of these, see Gonzalez and Hora 1978).

Assuming a noninformative sampling design misses perhaps the most important contribution of randomization to inference. Since most statistical models in finite population inference are either wrong or (at best) incomplete, it is desirable for an estimation strategy to have the following property: if the sample were large enough, the estimator should approach what it is estimating almost certainly no matter what the "true" model. This desire receives formal expression in the criterion of design consistency introduced by Isaki and Fuller (1982).

Design consistency is an asymptotic property. As a result, it is often necessary to hypothesize a model (or models) when choosing among alternative design consistent estimation strategies. This is especially true in the case of small domain estimation, where the sample may be particularly small and the sampling design beyond one's control. Nevertheless, limiting attention to design consistent estimators does offer some, albeit small, protection against model failure. Using this reasoning, Särndal (1984) focused his attention on design consistent small domain estimators. We will follow that practice here.

Section 2 develops a design consistent random effects estimator for a small domain population mean. Section 3 introduces a robust (but unstable) estimator for the model and design mean squared errors of the small domain estimator. It is robust in the sense of not depending

[1] Phillip S. Kott, Senior Mathematical Statistician, Survey Research Branch, National Agricultural Statistics Service, USDA, S-4801, Washington, D.C., 20250, USA.

on the necessary, but heroic, model that links the small domains together. Section 4 contains an empirical example and Section 5 a discussion.

2. THE ESTIMATOR

We begin with the *basic* (or fixed effects) model:

$$y_{gi} = \theta_g + \epsilon_{gi}, \tag{1}$$

where the ϵ_{gi} are uncorrelated random variables with means of zero, and $\text{var}(\epsilon_{gi}) = \delta_g^2$. The subscript gi denotes a unit in domain g. There are N_g units in the population from domain g and m domains.

Let us focus on a particular domain j. The problem is to estimate the domain mean:

$$\bar{y}_{jP} = \sum_{i=1}^{N_j} y_{ji}/N_j.$$

Let p_{ji} be the probability of selecting unit ji for the sample and n_j be the number of units selected from domain j. It is well known that a design unbiased and model efficent linear estimation strategy for $\bar{y}_{jP}$ would set the p_{ji} equal to n_j/N_j and the estimator equal to $\sum_{i=1}^{n_j} y_{ji}/n_j$, where the units are relabeled so that $j1, \ldots, jn_j$ are in the sample.

Unfortunately, one is often required in practice to estimate a domain mean using a sample that has not been selected primarily for that purpose. Consequently, the selection probabilities within domain j may not all equal n_j/N_j. A popular estimator in this circumstance is

$$d_j = \sum_{i=1}^{n_j} w_{ji} y_{ji}, \tag{2}$$

where

$$w_{ji} = p_{ji}^{-1} / \sum_{k=1}^{n_j} p_{jk}^{-1},$$

denotes the *sampling weight* of unit ji. This estimator was suggested by Brewer (1963) and Hajek (1971).

The estimator d_j is clearly model unbiased under (1), in the sense that $E_\epsilon(d_j - \bar{y}_{jP}) = 0$. Under many sampling designs, d_j is also *design consistent*; *i.e.*,

$$\underset{n_j \to \infty}{plim_\pi}(d_j - \bar{y}_{jP}) = 0,$$

where π denote the probability space generated by the random selection process rather than the model in (1).

Isaki and Fuller (1982) give sufficient conditions for d_j to be design consistent, and it is under most sampling designs in common practice. Notable exceptions involve systematic sampling from a predetermined list (see Kott 1986). A popular alternative to design consistency is Brewer's (1979) *asymptotic design unbiasedness* (ADU) property. The estimator d_j is always ADU.

The trouble with d_j is that it may not be very efficient for small n_j. One solution is to "draw strength" from the other domains by treating the fixed parameter θ_j as if it was a realization of a random variable satisfying this *linking* model:

$$\theta_j = \mu + \tau_j, \tag{3}$$

where $E(\tau_j) = 0$, and $E(\tau_j \tau_g) = \sigma^2$ when $j = g$ and 0 otherwise. This is sometimes called "random effects modeling," because the heretofore fixed effect of being a unit in domain j, θ_j, is now being treated as a random variable.

Combining equations (1) and (3) results in the reduced form components-of-variance model:

$$y_{ji} = \mu + \tau_j + \epsilon_{ji}. \tag{4}$$

Many analysts start with equation (4). We have separated the basic and linking models to underscore the greater level of confidence one often has in the validity of the basic model (especially when it is assumed as part of the linking model that all $\delta_g^2 = \delta^2$, as it soon will be).

Any estimator of the form:

$$f_j(\alpha, \boldsymbol{c}) = (1 - \alpha)d_j + \alpha\,\hat{\mu},$$

where

$$\boldsymbol{c} = (c_1, \ldots, c_{j-1}, 0, c_{j+1}, \ldots, c_m),$$

$$\hat{\mu} = \sum_{g=1}^{m} c_g \bar{y}_{gS},$$

$$\bar{y}_{gS} = \sum_{i=1}^{n_g} y_{gi}/n_g,$$

and

$$\sum^{m} c_g = 1$$

is unbiased under the model in (4). (Note: although the variables $\boldsymbol{c}$ and $\hat{\mu}$ depend on domain j, additional denotation has been suppressed for simplicity.)

If all the δ_g^2 are assumed equal to δ^2, then using a Lagrangian multiplier technique it is not difficult to show that the choices for α and the c_g that minimize the model variance of $f_j(\alpha, c) - \bar{y}_{jP}$ are

$$\alpha^* = \frac{\sum_{i=1}^{n_j} w_{ji}^2 - 1/N_j}{\sum_i w_{ji}^2 + \sum_g c_g^{*2}/n_g + (1 + \sum_g c_g^{*2})\,(\sigma^2/\delta^2)}, \tag{5}$$

and

$$c_g^* = \frac{[(\sigma^2/\delta^2) + n_g^{-1}]^{-1}}{\sum_h [(\sigma^2/\delta^2) + n_h^{-1}]^{-1}}, \quad \text{for } g \neq j. \tag{6}$$

In practice, σ^2 and δ^2 are rarely known. Ghosh and Meeden (1986) have proposed estimating the ratio σ^2/δ^2 from the sample in a *model* consistent manner (as $m \to \infty$) by

$$L = \max\left\{0, \left[\frac{\sum_g n_g(\bar{y}_{gS} - \bar{y}_S)^2/(m-1)}{\sum_g \sum_i (y_{gi} - \bar{y}_{gS})^2/(n-m)} - 1\right](m-1)/(n - \sum_g n_g^2/n)\right\}, \tag{7}$$

where

$$\bar{y}_S = \sum n_g \bar{y}_{gS}/n$$

and

$$n = \sum n_g.$$

Let $\alpha'(L)$ and $\mathbf{c}'(L)$ be the right hand sides of equations (5) and (6) respectively with L replacing σ^2/δ^2. Now call

$$e_j = f_j[\alpha'(L), \mathbf{c}'(L)]$$

the *random effects estimator*, where $\hat{\mu}$ in $e_j = f_j(.,.)$ is set equal to $\mu'(L) = \sum c_g'(L)\bar{y}_{gS}$. As m grows large, e_j become indistinguishable from $f_j(\alpha^*, \mathbf{c}^*)$.

If the model in (4) is correct and all the $\delta_j^2 = \delta^2 > 0$, then for sufficiently large m, L must be positive. Even if the model fails, as long as L is bounded from below by a positive number, $|\mu'(L)|$ is bounded, and $n_j \sum_{i=1}^{n_j} w_{ji}^2$ is bounded as n_j (but not m) grows arbitrarily large, then e_j is design consistent whenever d_j is. This is because

$$\underset{n_j \to \infty}{plim_\pi} [\alpha'(L)] = 0,$$

so that e_j converges to the design consistent d_j.

3. MODEL AND DESIGN MEAN SQUARED ERROR

Under some sampling designs there exists an estimator of the design variance of d_j that is also a model unbiased estimator of the variance of d_j as an estimator for $\bar{y}_{jP}$ under the basic model (henceforth I will omit the clarifying phrase "as an estimator for $\bar{y}_{jP}$" to simplify the exposition). Often, however, one must settle for a design consistent estimator of the design mean squared error of d_j (assuming, as we will, one exists). This is particularly true when $\sum_{k=1}^{n_j} p_{jk}^{-1} \neq N_j$. Kott (1987) shows how (when necessary) this estimator of the design mean squared error of d_j can be adjusted to be simultaneously a design consistent estimator of the design mean squared error of d_j and a model unbiased estimator of the variance of d_j under the *basic* model. Call this adjusted "variance estimator" $\mathrm{v}(d_j)$.

We are now ready to address the model and design mean squared errors of the random effects estimator, e_j. Although we needed to assume that the δ_j^2 were all equal to determine e_j, we need not make that assumption in assessing the accuracy of e_j. In fact, we need not even assume that the linking model in equation (3) holds! Instead, we assume only that m is large enough so that L may be viewed as (virtually) independent of the units in domain j. Alternatively, L can be redefined by excluding units from domain j in the summations on the right hand side of (7).

Either way, $E_\epsilon[(d_j - \bar{y}_{jP})(\bar{y}_{jP} - \mu'(L))] = 0$. As a result,

$$E_\epsilon[\{d_j - \mu'(L)\}^2] = \mathrm{var}_\epsilon(d_j - \bar{y}_{jP}) + E_\epsilon[\{\bar{y}_{jP} - \mu'(L)\}^2].$$

It is now a simple matter to show that under the basic model in (1),

$$\mathrm{v}(e_j) = [1 - 2\alpha'(L)]\,\mathrm{v}(d_j) + [\alpha'(L)]^2\,[d_j - \mu'(L)]^2$$

is an unbiased estimator of the model mean squared error of e_j *given* L and $\mu'(L)$. Since $\alpha'(L)$ is asymptotically zero as n_j approaches infinity, $\mathrm{v}(e_j)$ is also a design consistent estimator of the design mean squared error of e_j whenever $\mathrm{v}(d_j)$ is a design consistent estimator of the design mean squared error of d_j.

It is not necessary for L to converge to σ^2/δ^2 or $\mu'(L)$ to converge to μ for $\mathrm{v}(e_j)$ to have the properties described above. In fact, it is not necessary for the limits of L and $\mu'(L)$ to have any interpretations at all, since these properties have been defined independently of the model in equation (3).

Statisticians often have much more confidence in the basic model in equation (1) than the linking model in equation (3), especially when the latter is coupled with the assumption of constant unit variances (δ_g) across domains. It is therefore reassuring that the accuracy of the e_j can be estimated without invoking (3) or requiring that the δ_g be equal.

Unfortunately, $\mathrm{v}(e_j)$ is unstable and can even be negative when $\alpha'(L)$ exceeds 0.5. Nevertheless, a simple comparison of the relative sizes of $\mathrm{v}(d_j)$ and $\mathrm{v}(e_j)$ over the m domains $(j = 1, \ldots, m)$ provides a robust method for choosing between the two estimators, d_j and e_j.

4. AN EMPIRICAL EXAMPLE

The Human Nutrition Information Service (HNIS) conducted a stratified, multistage survey of one day food intake by women aged 19-50 in 1985 as part of its Continuing Survey of Food Intakes by Individuals (CSFII). Responses were converted into measured intakes from among 60 food groups and 27 nutrients. See Human Nutrition Information Service (1985) for more details on the survey and its sample design.

We will restrict our attention here to the estimation of mean intake of milk and milk products (one of the 60 food groups) by women 19-34 and 35-50 within 12 mutually exclusive domains. These domains are defined by two cross classifications: region (northeast, midwest, south, and west) and level of urbanization (central city, suburban, non-metropolitan). HNIS published mean food group intakes separately for these two age groups on the national level only. Mean nutrition intakes were published for each age group by region and level of urbanization but were not cross-classified.

The CSFII sample design employed an independent stratified multistage sample with each of these domains. First primary sampling units (cities or town) were chosen using probability proportional to size sampling *with* replacement, then a random subsample of area segments was selected from which a smaller random subsample of households were chosen. I added another level of subsampling. When more than one woman per household from an age group was in the CSFII sample, I randomly chose one.

For each group, d_j in equation (2) defines the conventional design-based estimated of the domain mean. The SESUDAAN program (Shah 1980) provided design consistent estimators of all the d_j and their design root mean squared errors $(\sqrt{\mathrm{MSE}(d_j)})$. These estimators, when squared, are not necessarily model unbiased estimators of the model variance of d_j under equation (1) however.

To see this, we confine our attention not only to an age group but to a domain as well and suppress the subcript j. Let $h = 1, \ldots, H$ denote strata, $k = 1, \ldots, K_h$ denote primary sampling units (PSU's) in h, and $i = 1, \ldots, n_{hk}$ denote sampled women in hk. The estimate for the mean intake estimate is

$$d = \sum_{h=1}^{H} \sum_{k=1}^{K_h} \sum_{i=1}^{n_{hk}} w_{hki}\, y_{hki}.$$

We need more notation before we proceed. Let

$$x_{hk} = \sum_{i=1}^{n_{hk}} w_{hki},$$

$$z_{hk} = \sum_{i=1}^{n_{hk}} w_{hki}^2,$$

$$f_{hk} = \sum_{i=1}^{n_{hk}} w_{hki}\,(y_{hki} - d),$$

and

$$f_h = \sum_{k=1}^{K_h} f_{hk}/K_h.$$

If we assume the population size of the domain is large enough to be ignored (this also virtually assures that no individual had been sampled twice), the model variance of d is

$$\mathrm{var}_\epsilon(d) = \delta^2 \sum_h \sum_k \sum_i w_{hki}^2$$
$$= \delta^2 \sum_h \sum_k z_{hk}.$$

The SESUDAAN (linearization) estimator for the design mean squared error of d is

$$\mathrm{v}^*(d) = \sum_{h=1}^{H} (K_h/[K_h - 1]) \sum_{k=1}^{K_h} (f_{hk} - f_h)^2 .$$

After much manipulation the model expectation of this can be shown to be

$$E_\epsilon[v^*(d)] = \delta^2 \Bigg[\sum_h \sum_k z_{hk}$$
$$- 2 \sum_h (K_h/[K_h - 1]) \left(\sum_k^{K_h} z_{hk} x_{hk} - \sum_k^{K_h} z_{hk} \sum_k^{K_h} x_h/K_h \right)$$
$$+ \left(\sum_h \sum_k z_{hk} \right) \sum_h (K_h/[K_h - 1]) \left(\sum_k^{K_h} x_{hk}^2 - \left\{ \sum_k^{K_h} x_{hk} \right\}^2 / K_h \right) \Bigg] .$$

Following Kott (1987),

$$\mathrm{v}(d) = \mathrm{v}^*(d)\ \mathrm{var}_\epsilon(d)/\mathrm{E}_\epsilon[\mathrm{v}^*(d)]$$

is both a design consistent estimator for the mean squared error of d (under certain conditions) and a model unbiased estimator of the model variance of d.

Calculations for n_j, d_j, $\alpha'(L)$, e_j, $\mathrm{v}(d_j)$ and $\mathrm{v}(e_j)$ for the 12 domains in each of the two groups are displayed in Table 1 (the domain subscript j has been returned to d_j and e_j). Using equation (5), L was calculated to be 0.055 for women 19-34 and 0.037 for women 35-50. This suggests that women in the same domain had little in common over and above their membership in the same age group. Nevertheless, $\alpha'(L)$ exceeded 0.5 only for five (out of 24) cells all with samples of under 25 women.

The estimate $\mathrm{v}(e_j)$ was negative twice and less than $\mathrm{v}(d_j)$ 18 out of 24 times, nine times for each age group. These latter group of numbers suggest to me that the e_j are indeed better estimates than the d_j. Formally, if we treat each of the 24 differences, $\mathrm{v}(e_j) - \mathrm{v}(d_j)$, as if they were independent across domains (they aren't quite), the hypothesis that the true model (or design) mean squared errors of e_j and d_j are equal and the random variable $\mathrm{v}(e_j) - \mathrm{v}(d_j)$ as likely positive as negative is soundly rejected.

The reduction in mean squared error from using e_j in place of d_j is estimated (by $\sum\{\mathrm{v}(e_j) - \mathrm{v}(d_j)\}/\sum\mathrm{v}(d_j)$) to be 40.6%. This translates into a standard error reduction of 22.9%. Note that because we are summing 24 near independent random variates, we have much more confidence in this estimate than any particular $\mathrm{v}(e_j)$ (or $\mathrm{v}(d_j)$ for that matter).

Table 1

Estimated Values for the Domains by Age Group

Domain	Women 19-34					
	Sample Size	d_j	e_j	$v(d_j)$	$v(e_j)$	$\alpha'(L)$
N - C	68	220.6	222.1	683.0	367.5	.233
N - S	95	195.7	203.1	568.8	367.8	.225
N - R	12	219.1	223.8	5266.7	-1349.5	.630
M - C	55	270.7	258.6	2021.5	1152.5	.251
M - S	107	277.2	267.8	625.8	509.6	.164
M - R	73	301.1	285.9	4027.1	2754.3	.187
So - C	66	212.4	215.7	3011.6	1700.1	.220
So - S	112	156.8	167.9	472.8	457.3	.146
So - R	81	117.0	139.3	592.0	868.9	.184
W - C	39	403.0	333.2	2064.2	5438.4	.364
W - S	74	205.0	209.6	1704.0	1018.3	.207
W - R	13	120.0	190.7	3533.5	3924.3	.652
	Women 35-50					
N - C	44	205.3	197.4	1716.1	318.4	.425
N - S	67	135.0	153.1	1068.8	698.0	.326
N - R	21	206.1	195.4	579.2	56.6	.550
M - C	28	89.0	139.5	470.3	2559.9	.482
M - S	87	200.3	196.1	2128.5	1049.2	.258
M - R	38	304.9	250.7	6065.3	3973.9	.415
So - C	47	136.1	159.6	266.7	592.6	.421
So - S	93	161.0	167.7	1492.5	809.1	.244
So - R	77	128.8	146.3	1023.4	790.9	.263
W - C	23	205.5	193.9	7497.1	-1067.6	.580
W - S	88	245.1	229.1	2484.7	1432.2	.263
W - R	11	132.1	173.3	743.3	1344.1	.734

Domain Codes
N - Northeast; M - Midwest; So - South; W - West; C - Central City; S - Suburban; R - Non-metropolitan.

5. DISCUSSION

Let $n_j^* = 1/\sum_{i=1}^{n_j} w_{ji}^2$ define the *effective sample size* within domain j. Observe that $n_j^* \leq n_j$ where equality holds if and only if all the sampling weights within j are all equal to $1/n_j$. For a known σ^2/δ^2, the only difference between the optimal estimator developed here, $f_j(a^*, c^*)$, and the best linear unbiased predictor in Scott and Smith (1969) is that $1/n_j^*$ has replaced $1/n_j$ in the formula for α^* (equation (5)). The effect of this when the w_{ji} within j are not all equal is to increase α^*; that is, to increase the dependence on sample information from outside domain j. This happens because forcing the estimator to be design consistent results in the domain j sample not being used as efficiently as possible. We could penalize the sample from outside the domain in a conformal manner by using sample weights in determining $\mu'(L)$, but that would only decrease the model efficiency of the estimator without improving any design-based characteristic.

Equation (7) assures that L can be no less than zero. This means that $\alpha'(L)$ can be no greater than $\sum_{g\neq j} n_g / (\sum_{g\neq j} n_g + n_j^*)$. If $\alpha'(L)$ were equal to its upper bound and $n_j^* = n_j$, then e_j would collapse into the simple mean of the y_{gi} across the entire sample. This makes sense because when the full model in equation (4) is correct and $\sigma^2 = 0$, the most efficient estimator of $\mu + \tau_j = \mu$ is the full sample mean.

If $n_j^* < n_j$ and $L = 0$, however, then e_j will be calculated with more weight given to units outside of domain j than to units inside the domain, which makes little sense. One *ad hoc* way to get around this phenomenon is to set an upper bound of $1 - (n_j / \sum n_g)$ (or smaller) on $\alpha'(L)$. Another approach would be to abandon small domain estimation entirely when $\alpha'(L)$ as calculated in the text exceeds $1 - (n_j / \sum n_g)$. Note that L, the estimated value for σ^2/δ^2, would have to be very small for this to happen. In the empirical study discussed in the previous section, L was in the 0.03 to 0.06 range, yet $\alpha'(L)$ was always well below $1 - (n_j / \sum n_g)$.

There are two ways the full model in equation (4) may fail. The fixed effects model within each domain (equation (1)) can fail or the linking model in (3) can fail. In the real world, both models are likely to be wrong. Equation (1) for its part ignores stratification and clustering effects as well as any subtle effect of membership in a household with more than one woman in the same age group. None of these effects are likely to be great. Moreover, by incorporating sampling weights into the estimate d_j and forcing the mean squared error estimators to be design consistent, we have done as much as we can do to protect ourselves against the potential for model failure in equation (1).

On the other hand, we should have little faith in the viability of the linking model. It is hardly more than a statistical convenience that, among other things, fails to allow for any correlation in the intakes of women from the same region but from different levels of urbanization or *vice versa*.

As noted, simply counting the number of times $\mathrm{v}(e_j) - \mathrm{v}(d_j)$ is negative provides a means for choosing between the estimators d_j and e_j that is independent of the linking model. The estimator $\mathrm{v}(e_j)$ is unstable, however, and should not be used by itself as an estimate of mean squared error in practice.

Not only are the estimates of the mean squared error of e_j unstable, the $\mathrm{v}(d_j)$ are only slightly better. At best $\mathrm{v}(d_j)$ has "degrees of freedom" equal to the number of PSU's minus the number strata in j. For the CSFII sample, these range from 2 to 7.

Since it is becoming increasingly necessary for statisticians to provide estimated standard errors along with the estimated means they publish, it is imperative that more stable estimators than $\mathrm{v}(d_j)$ and $\mathrm{v}(e_j)$ be found. One idea might be to fit the $\mathrm{v}(d_j)$ and the $\mathrm{v}(e_j)$, either together or separately, with a variance estimating function. This approach is *ad hoc*, however, and may do little more than return values close to fully model-dependent estimates of the mean squared errors of the d_j and e_j (see Prasad and Rao 1986, for a good discussion of these) by "averaging out" the effects of model failure.

One intriguing idea is to combine the stable, but biased, model-dependent mean squared error estimates with the design consistent estimates developed here, much like e_j does for means. How this should be done is a topic that deserves future attention.

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