Survey Methodology, December 1987
Vol. 13, No. 2, pp. 173–181
Statistics Canada

An Alternative Method of Controlling Current Population Survey Estimates to Population Counts

K.R. COPELAND, F.K. PEITZMEIER, and C.E. HOY[1]

ABSTRACT

The CPS uses raking ratio estimation in post-stratification estimation to adjust sample estimates of population to census-based estimates of the population. An alternative procedure, using generalized least squares, is compared to the current procedure.

KEY WORDS: Generalized least squares; Post-stratification; Raking ratio estimation.

1. INTRODUCTION

The Current Population Survey (CPS) produces labor force estimates for the total U.S. working-age civilian noninstitutional population, based on a monthly multi-stage probability sample of approximately 60,000 housing units in the U.S. Each month a rotating sample comprised of 8 panels (called rotation groups) of housing units is interviewed, with demographic and labor force data being collected for all civilian adult occupants of the sample housing units.

Monthly estimates are published, subaggregated by demographic characteristics. Estimates for other subaggregates of the population (states, families, veterans, wage and salary earners, persons not in the labor force, etc.) are also produced on a monthly, quarterly, and/or annual basis.

Sample person weights are derived through the application of probability of selection, adjustment for nonresponse, and ratio adjustment to reduce the contribution to the variance due to the sampling of primary sampling units. A post-stratification estimation procedure adjusts the sample person weights so as to control the survey estimates of population to independently derived estimates of the population. The resultant weights are used in a composite estimation procedure and then seasonally adjusted to produce national estimates (Hanson 1978).

Detailed estimates for certain population subdomains (families, wage and salary earners, persons not in the labor force, family earnings, and veterans) make use of sample weights derived from adjustment procedures built on top of the post-stratification estimation.

The use of a generalized least squares (GLS) approach could potentially be used in place of post-stratification estimation or to integrate the various CPS adjustment procedures. The use of GLS has been proposed and investigated for use in the Consumer Expenditure Survey (Zieschang 1986).

This article discusses and compares the current CPS post-stratification estimation (which uses raking ratio estimation) and the GLS procedure, based on two months' CPS data (July 1983 and July 1984). Both macro and micro level data were examined to evaluate differences, if any, in the two procedures in this application.

[1] K.R. Copeland, F.K. Peitzmeier, and C.E. Hoy, Division of Statistical Methods, Office of Employment and Unemployment Statistics, Bureau of Labor Statistics, Washington, D.C. 20212 U.S.A.

2. CURRENT CPS POST-STRATIFICATION ESTIMATION

The CPS post-stratification estimation uses raking ratio estimation (RRE) to adjust the sample weights within a rotation group so as to control the sample estimates for the population to independently derived estimates of the population in each of three categories (state, age/sex/ethnicity, age/sex/race).

The methodology for RRE was first proposed by Deming and Stephan (1940) as an iterative alternative to least squares adjustment of table data. The RRE procedure has been shown to produce best asymptotically normal (BAN) estimates under simple random sampling, and to minimize the adjustments made to the sample weights based on one measure of closeness, as discussed in subsection 4.2 (Ireland and Kullback 1968). In addition, RRE, although producing biased estimates, can sometimes be effective in reducing the mean square error of survey estimates. This is believed to be the case in the application of RRE for CPS (Hanson 1978).

For the CPS, the RRE procedure attempts to adjust the sample counts $\{n_{ijk}\}$ obtained from previous stages of weighting to adjusted sample counts $\{\tilde{n}_{ijk}\}$ under the condition that:

$$\text{(A)} \qquad \sum_{j,k} \tilde{n}_{ijk} = m_{i..}$$

$$\text{(B)} \qquad \sum_{i,k} \tilde{n}_{ijk} = m_{.j.}$$

$$\text{(C)} \qquad \sum_{i,j} \tilde{n}_{ijk} = m_{..k}$$

be satisfied simultaneously,

where i = state ($i = 1, \ldots, 51$),
j = age/sex/ethnicity ($j = 1, \ldots, 16$),
k = age/sex/race ($k = 1, \ldots, 70$),
$m_{i..}$ = independent state estimate,
$m_{.j.}$ = independent age/sex/ethnicity estimate,
$m_{..k}$ = independent age/sex/race estimate.

The RRE procedure proportionately ratio adjusts the sample data each way (i.e., state, age/sex/ethnicity, and age/sex/race) of the table in successive steps, as follows.

(1) Ratio adjustment by state:

$$n_{ijk}^{(1,1)} = (m_{i..}/n_{i..})\, n_{ijk} = a_i^{(1)}\, n_{ijk}.$$

(2) Ratio adjustment by age/sex/etchnicity:

$$n_{ijk}^{(1,2)} = (m_{.j.}/n_{.j.}^{(1,1)})\, n_{ijk}^{(1,1)} = b_j^{(1)}\, n_{ijk}^{(1,1)}$$

$$= a_i^{(1)}\, b_j^{(1)}\, n_{ijk}.$$

(3) Ratio adjustment by age/sex/race:

$$n_{ijk}^{(1,3)} = (m_{..k}/n_{..k}^{(1,2)})\, n_{ijk}^{(1,2)} = d_k^{(1)}\, n_{ijk}^{(1,2)}$$

$$= a_i^{(1)}\, b_j^{(1)}\, d_k^{(1)}\, n_{ijk},$$

where $n_{i..}$ = sample row total
$n_{.j.}$ = sample column total
$n_{..k}$ = sample layer total.

The completion of the three adjustment steps constitutes one iteration of the raking process. The three steps are repeated substituting the current value of $n_{ijk}^{(h,3)}$ (adjusted sample count following the third way rake of the h-th iteration) for n_{ijk} in step (1) each time until 6 iterations are completed. (The number of iterations used in CPS was determined based on the convergence properties of the RRE for CPS and the relative gains achieved by number of iterations.) The final $\{n_{ijk}^{(6,3)}\}$ is taken as $\{\tilde{n}_{ijk}\}$.

In order to adjust the sample weights, the adjustment factor for sample records in cell $\{ijk\}$ is

$$F_{ijk} = n_{ijk}^{(6,3)} / n_{ijk}$$

$$= \prod_{h=1}^{6} a_i^{(h)} \, b_j^{(h)} \, d_k^{(h)}.$$

The sample weights prior to RRE are multiplied by the appropriate F_{ijk} to obtain the adjusted weights.

3. APPLICATION OF THE GLS IN THE CPS

The generalized least squares (GLS) procedure adjusts the sample weights from prior stages of weighting by minimizing the weighted squared adjustments, subject to a set of linear 'control' constraints the adjusted weights must satisfy. This is the problem which Deming and Stephan attempted to address in developing the RRE. The GLS procedure, like RRE, produces BAN estimates under certain conditions, in this case when all the cells are nonempty (Neyman 1949). GLS, by definition, minimizes the adjustments to the sample weights based on one measure of closeness (see subsection 4.2).

For the CPS, each dimension that defines a set of controls in the current post-stratification will define a set of linear constraints for the GLS procedure. The function to be minimized is

$$f(\underline{F}) = (\underline{F} - \underline{P})' \, P_0^{-1} \, (\underline{F} - \underline{P})$$

$$= \sum_i \, (W_{2i} - W_{1i})^2 / W_{1i},$$

subject to $X'\underline{F} = \underline{N}$,

where $\underline{F}$ = $(n \times 1)$ vector of derived final weights (W_{2i}) for each of the n sample persons,
$\underline{P}$ = $(n \times 1)$ vector of sample person weights prior to post-stratification (W_{1i}),
$\underline{P}_0$ = $(n \times n)$ diagonal matrix with the W_{1i} on the diagonal,
X = $(n \times k)$ design matrix whose rows correspond to sample persons, and whose columns correspond to control cells. The entries of the matrix (x_{ij}) are 0's or 1's, indicating the appropriate control categories for each of the n sample persons.
$\underline{N}$ = $(k \times 1)$ vector of independent population estimates, corresponding to the columns of X. These estimates are the same as those used in the CPS RRE.

The columns of X are required to be linearly independent so that an inverse of the matrix $(X' \ P_0 \ X)$ is achievable. In setting up matrices X and $\underline{N}$ for CPS, the 137 control cells used in the RRE (state, age/sex/ethnicity, age/sex/race) were reduced to a set of $k = 132$ linearly independent cells.

The unique solution to $X'\underline{F} = \underline{N}$ that minimizes $f(\underline{F})$ is, as shown in Luery (1986)

$$\underline{F} = \underline{P} + P_0 \, X \, (X' \, P_0 \, X)^{-1} \, (\underline{N} - X' \, \underline{P})$$

Although the elements of $\underline{F}$ are not constrained to be positive, in this application of GLS for CPS, the elements of $\underline{F}$ were all positive without the need for additional constraints. Methodology for providing non-negative weights in this context is discussed in Huang and Fuller (1978) and Zieschang (1986), among others.

4. RESULTS

4.1 Macro-Level

a. Estimates

Labor force estimates were tabulated for several demographic groups for July 1983 and July 1984, using the final weights derived from RRE and GLS. Standard errors for both RRE and GLS were calculated using a random group estimator of the form Wolter (1985)

$$\sum_{k=1}^{8} (8Y_k - \hat{Y})^2 / 56,$$

where Y_k = sum of the weights for sample records from the k-th rotation group with the characteristic Y,
$\hat{Y}$ = sum of the Y_k.

This variance estimator, while not accounting for the multi-stage design of the CPS, was used due to the unavailability of design information on the CPS public use microdata file.

Relative differences were calculated for both estimates of level and estimates of standard error. The relative difference was defined as:

$$(Y_{GLS} - Y_{RRE}) / Y_{RRE},$$

where Y_{RRE} = estimate of Y based on the weights derived through the use of RRE,
Y_{GLS} = estimate of Y based on the weights derived through the use of GLS.

As the data in Table 1 indicate, neither weighted labor force estimates nor estimates of standard error based on the current CPS RRE procedure and the GLS procedure showed any noticeable differences or trends when subaggregated to the sex by race/ethnicity level.

For labor force estimates by sex by race/ethnicity the estimated absolute relative differences between the CPS RRE and GLS estimates were all less than 0.3% (well below the estimated CVs of each estimate). For the majority of these estimates, in particular for total and whites, the absolute relative difference was less than 0.1%.

For many of the characteristics the sign of the relative difference changed from 1983 to 1984; thus there does not appear to be a pattern to the differences in the estimates obtained from the two procedures.

Table 1

Labor Force Estimates by Sex/Race or Ethnicity

		1983 GLS		1983 (GLS-RRE)/RRE		1984 GLS		1984 (GLS-RRE)/RRE	
		Total (000)	S.E. (000)	Total (%)	S.E. (%)	Total (000)	S.E. (000)	Total (%)	S.E. (%)
Total									
Total	Emp	103516	403	0.00	−0.14	107535	352	−0.01	1.12
	UE	10669	221	−0.04	−0.75	8765	118	−0.06	−0.21
	Rate	*9.34%*	*0.19%*	*−0.04*	*−0.56*	*7.54%*	*0.09%*	*−0.05*	*0.27*
	NILF	59938	373	0.01	−0.68	60080	419	0.02	0.41
White	Emp	91338	344	0.00	−0.33	94417	274	0.00	0.70
	UE	7928	236	0.00	−0.27	6282	120	0.00	−0.14
	Rate	*7.99%*	*0.23%*	*0.00*	*−0.26*	*6.24%*	*0.10%*	*0.00*	*−0.16*
	NILF	51915	340	0.00	−0.36	51700	358	0.00	0.39
Black	Emp	9871	69	0.06	−3.44	10371	98	0.02	0.17
	UE	2434	68	−0.12	−1.07	2202	60	−0.03	1.41
	Rate	*19.78%*	*0.55%*	*−0.14*	*−1.60*	*17.51%*	*0.42%*	*−0.04*	*1.49*
	NILF	6628	26	−0.04	−1.47	6765	109	−0.02	0.09
Hispanic	Emp	6132	73	−0.03	−0.59	6607	102	−0.03	1.90
	UE	920	79	−0.05	−0.29	786	70	−0.08	−0.03
	Rate	*13.04%*	*1.10%*	*−0.02*	*−0.33*	*10.63%*	*0.96%*	*−0.05*	*0.35*
	NILF	3760	31	0.05	−0.39	3786	73	0.04	1.02
Male									
Total	Emp	58985	147	0.00	−1.58	61045	188	0.00	1.74
	UE	5980	134	−0.05	−0.88	4682	79	−0.02	0.77
	Rate	*9.20%*	*0.19%*	*−0.05*	*−0.79*	*7.12%*	*0.11%*	*−0.02*	*1.30*
	NILF	17495	178	0.01	−1.81	17840	214	0.02	0.64
White	Emp	52674	482	0.00	0.42	54261	111	0.00	0.34
	UE	4484	131	0.01	−0.49	3394	93	0.01	−0.12
	Rate	*7.84%*	*0.21%*	*0.00*	*−0.47*	*5.89%*	*0.15%*	*0.01*	*−0.13*
	NILF	14985	160	−0.02	−0.40	15077	150	0.00	0.16
Black	Emp	5047	56	0.07	−1.70	5263	84	0.01	−0.50
	UE	1300	45	−0.20	−1.87	1137	33	0.08	1.12
	Rate	*20.49%*	*0.71%*	*−0.21*	*−2.02*	*17.76%*	*0.51%*	*0.05*	*0.94*
	NILF	2097	40	−0.04	−0.13	2236	88	−0.07	−0.48
Hispanic	Emp	3781	48	0.01	−0.86	4064	79	−0.02	1.29
	UE	534	45	−0.16	−0.83	451	41	−0.05	0.51
	Rate	*12.38%*	*0.99%*	*−0.15*	*−0.89*	*9.99%*	*0.95%*	*−0.03*	*0.66*
	NILF	981	42	0.00	−0.42	964	57	0.07	1.40
Female									
Total	Emp	44531	320	−0.01	−0.01	46490	194	−0.01	1.48
	UE	4689	107	−0.04	−0.19	4083	88	−0.10	−1.22
	Rate	*9.53%*	*0.23%*	*−0.03*	*−0.02*	*8.07%*	*0.16%*	*−0.09*	*−0.80*
	NILF	42443	287	0.01	−0.26	42240	217	0.02	0.34
White	Emp	38664	315	0.00	−0.29	40156	191	0.00	0.66
	UE	3444	115	−0.01	0.16	2888	68	0.00	−0.32
	Rate	*8.18%*	*0.28%*	*−0.01*	*0.11*	*6.71%*	*0.15%*	*0.00*	*−0.34*
	NILF	36929	283	0.01	−0.32	36623	214	0.00	0.53
Black	Emp	4824	57	0.05	0.56	5108	50	0.02	1.69
	UE	1134	46	−0.02	0.07	1065	46	−0.14	−0.62
	Rate	*19.03%*	*0.80%*	*−0.06*	*0.08*	*17.25%*	*0.67%*	*−0.13*	*−0.63*
	NILF	4531	24	−0.04	2.99	4529	59	0.01	1.49
Hispanic	Emp	2350	44	−0.08	−0.46	2543	38	−0.05	3.04
	UE	385	41	0.10	0.51	335	34	−0.13	−0.62
	Rate	*14.08%*	*1.46%*	*0.16*	*0.57*	*11.64%*	*1.18%*	*−0.07*	*−0.11*
	NILF	2778	33	0.07	−0.87	2822	27	0.03	0.13

The absolute relative differences between the CPS RRE and GLS estimates of standard errors for national labor force estimates were all less than: 1.9% for total population; 0.7% for whites; 3.5% for blacks; and 3.1% for Hispanics.

b. Month-in-Sample Indexes

It is a well-documented fact that the estimates produced from the CPS final weights have certain patterns of relative bias based upon the time the rotation group has been in sample (Bailar 1975). Month-in-sample indexes

$$I_k = (8Y_k / \hat{Y}) \times 100,$$

were calculated for both July 1983 and July 1984 based upon both the RRE estimates and the GLS estimates.

Month-in-sample indexes for labor force by race, labor force by sex, and labor force by ethnicity were virtually identical for estimates based upon the CPS RRE and GLS procedures.

4.2 Micro-Level

a. Adjustments to Sample Weights

Both RRE and GLS minimize some measure of closeness between the pre- and post- adjustment sample weights. For RRE the measure is (Ireland and Kullback 1968)

$$M_A = \sum_i W_{2i} \ln (W_{2i} / W_{1i}).$$

For GLS, the measure is (Luery 1986)

$$M_B = \sum_i (W_{2i} - W_{1i})^2 / W_{1i},$$

where W_{1i} = weight for sample record i prior to adjustment,
W_{2i} = weight for sample record i following adjustment.

Tabulation of the measures of closeness (summarized in Table 2) provided some interesting and, in some cases, puzzling results. The CPS RRE yielded smaller values for both measures. The GLS procedure did tend to produce smaller values for the measures for certain subgroups, most notably for blacks and Hispanics. It should be noted that the differences between the values for the measures for RRE and GLS were almost always less than 1%.

Although M_B should be minimized through the use of the GLS procedure, the value of M_B based upon the GLS weights for the total sample was greater than the value of M_B for the CPS RRE weights for 11 of the 16 rotation groups.

In seeking a reason for this apparent contradiction, it was noted that the CPS RRE had yet to converge to the age/sex/ethnicity controls after six iterations. The extent of this non/convergence is *very small;* less than 1.0% for all control categories. However, given the difference in M_B between the RRE and GLS, a change in the RRE sample weights of only 0.1%-0.2% could reverse the results. Rerunning RRE using 15 iterations, although still not achieving convergence did provide indications that the slight lack of convergence of the RRE is the reason for the results for M_B. (It should be noted that the GLS procedure minimizes M_B among the class of adjustment procedures yielding estimates that meet the population controls. Since the CPS RRE did not converge to the population controls, it is not a member of this class.)

Table 2

Comparison of measures of closeness based on 8 RGs for each year (# of RGs with RRE < GLS)

	M_A		M_B	
	1983	1984	1983	1984
Total	8	8	4	7
White	7	7	3	4
Black	3	3	1	1
Hispanic	0	0	0	0
Male	2	7	1	5
Female	8	8	8	8

Although an adjustment procedure such as RRE or GLS may minimize some measure of closeness for the total sample, it does not necessarily minimize that measure of closeness for subaggregates of the sample which were controlled for (e.g., blacks, Hispanics, males). Given the use of controls, and the fact that the overall measure of closeness is being minimized, it would seem desirable to have an adjustment procedure produce small measures of closeness at the subaggregate level also. The GLS procedure yielded smaller measures in almost every rotation group for Hispanics, in many rotation groups for blacks, and in several rotation groups for whites and males.

b. Comparison of Adjustments

Both RRE and GLS determine adjustment factors within cells defined by the intersection of the marginal constraints. Each sample record within a cell receives the same factor. To compare the adjustments made by the two procedures, the factors determined for each sample record by each procedure were compared using the following ratio

$$RRE/GLS = [(W_{2i}/W_{1i})_{RRE}]/[(W_{2i}/W_{1i})_{GLS}].$$

This ratio indicates the relationship between the adjustments made to a sample person weight by the RRE and GLS procedures. For comparison purposes, values of *RRE/GLS* less than 0.95 or greater than 1.05 were used to denote differences in the adjustments made by RRE and GLS.

For each set of independent population controls, ratios E/C (i.e., coverage rates), where E is the sample estimate based on the sample person weights prior to post-stratification and C is the independent control, were derived.

Within each set of controls (state, age/sex/ethnicity, age/sex/race) sample records were categorized by their coverage rates. Table 3 provides the sample distribution by coverage rate categories and by the *RRE/GLS* values, as well as the proportion of records within each coverage rate category that have the *RRE/GLS* values.

The data in Table 3 indicate that, for each set of controls, sample records from population groups which were over- or under-covered to some extent by the survey (i.e., for which the coverage rate is not near 1) were more likely to be adjusted differently by RRE and GLS than were sample records in population groups adequately covered by the survey.

Table 3
Comparison of RRE and GLS adjustments, 1984

Control Marginal	Coverage Rate Category	Proportion of Total Sample	Proportion of Sample with RRE/GLS <0.95 or >1.05	Proportion of Category with RRE/GLS <0.95 or >1.05
Age/Sex/ Race	<0.7	0.007	0.057	0.219
	0.7-0.8	0.022	0.116	0.136
	0.8-0.9	0.241	0.147	0.019
	0.9-1.1	0.699	0.504	0.019
	1.1-1.2	0.021	0.069	0.084
	>1.2	0.010	0.106	0.275
Age/Sex/ Ethnicity	<0.7	0.010	0.078	0.198
	0.7-0.8	0.014	0.032	0.058
	0.8-0.9	0.106	0.135	0.033
	0.9-1.1	0.869	0.741	0.022
	1.1-1.2	0.001	0.007	0.202
	>1.2	0.001	0.007	0.373
State	<0.7	0.056	0.068	0.031
	0.7-0.8	0.111	0.180	0.042
	0.8-0.9	0.278	0.325	0.030
	0.9-1.1	0.479	0.342	0.018
	1.1-1.2	0.026	0.009	0.009
	<1.2	0.049	0.077	0.040

4.3 Computer Resources

The CPS RRE and GLS procedures were run on an IBM System 370 at the National Institutes of Health using PROC MATRIX in the SAS System. The CPU time to prepare the files and perform the weighting was approximately three times as much for the GLS procedure than it was for the RRE procedure. There was also more storage of files involved with the GLS procedure. (The size of the matrices involved for CPS are quite large, with the number of rows for $\underline{P}$, P_0, X, and $\underline{N}$ being around 14,000 for each rotation group.)

5. SUMMARY AND CONCLUSIONS

This investigation was intended to provide a comparison of RRE and GLS as applied to the CPS, at both the macro and micro level.

The results obtained at the macro level do not indicate any difference in the estimates obtained from the RRE and GLS procedures.

The measures of closeness indicated that the CPS RRE made slightly smaller changes overall to the sample weights to meet the control constraints than did the GLS. The CPS RRE tended to produce slightly larger measures of closeness for subaggregates of minority populations. The two procedures differ most notably in the adjustments made to portions of the population which are either over- or under-covered.

Based on the work done in this investigation, it does appear that the RRE takes less computer time to run for the CPS second-stage adjustment than the GLS.

ACKNOWLEDGEMENT

The authors are grateful to Fritz Scheuren for his review of the original version of this paper, and to the referees and the Associate Editor for their very useful comments, incorporation of which resulted in the improvement of the paper.

REFERENCES

BAILAR, B. (1975). The Effects of rotation group bias on estimates from panel surveys. *Journal of the American Statistical Association,* 70, 23-30.

DEMING, W.E., and STEPHAN, F.F. (1940). On a least squares adjustment of a sampled frequency table when the expected marginal totals are known. *Annals of Mathematical Statistics,* 11, 427-444.

HANSON, R.H. (1978). The Current Population Survey design and methodology. Technical Paper 40, U.S. Bureau of the Census.

HUANG, E.T., and FULLER, W.A. (1978). Nonnegative regression estimation for sample survey data. *Proceedings of the Section on Social Statistics, American Statistical Association,* 300-305.

IRELAND, C.T., and KULLBACK, S. (1968). Contingency tables with given marginals. *Biometrika,* 55, 179-188.

LUERY, D. (1986). Weighting sample survey data under linear constraints on the weights. *Proceedings of the Section on Social Statistics, American Statistical Association,* 325-350.

NEYMAN, J. (1949). Contribution to the Theory of the X^2 Test. In *Proceedings of the First Berkeley Symposium on Mathematical Statistics and Probability,* (Ed. J. Neyman), Berkeley: University of California Press, 239-273.

WOLTER, K.M. (1985). *Introduction to Variance Estimation.* New York: Springer-Verlag.

ZIESCHANG, K.D. (1986). A Generalized least squares weighting system for the Consumer Expenditure Survey. *Proceedings of the Section on Survey Research Methods, American Statistical Association,* 64-71.