Survey Methodology, June 1987
Vol. 13, No. 1, pp. 75-83
Statistics Canada

On Efficient Estimation of Unemployment Rates from Labour Force Survey Data

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ABSTRACT

The method of minimum $Q^{(T)}$ estimation for complex survey designs proposed by Singh (1985) provides asymptotically efficient estimates of model parameters analogous to Neyman's (1949) min X^2 estimation procedure for simple random samples. The $Q^{(T)}$ can be viewed as a X^2 type statistic for categorical survey data, and min $Q^{(T)}$ estimates provide a robust alternative to Weighted Least Squares estimates, which often display unstable behaviour for complex surveys. In this paper, the min $Q^{(T)}$ method is first described and then illustrated for the problem of estimating parameters of a logit model for survey estimates of unemployment rates which are obtained from the October 1980 Canadian LFS data cross-classified according to age-education covariate categories. It is seen that the trace efficiency of smoothed estimates obtained by Kumar and Rao (1986), who applied the method of pseudo maximum likelihood estimates (pseudo mle) to the same problem can be slightly improved by the min $Q^{(T)}$ method. Interestingly enough, pseudo mle for individual cells behave much the same way as the efficient min $Q^{(T)}$ estimates for the particular LFS example.

KEY WORDS: Pseudo mle; WLS estimator; Min $Q^{(T)}$ estimator; Asymptotic efficiency; Approximate likelihood; Generalized score statistic.

1. INTRODUCTION

Based on October 1980 Labour Force Survey (LFS) data, Kumar and Rao (1984, 1986) proposed and analysed a logistic regression (logit) model for unemployment rates. They used the theory developed by Roberts (1985) and Roberts, Rao and Kumar (1987) who generalized the Rao-Scott method (1981, 1984) of adjusting X^2 for impact of the underlying survey design to test the fit of the logit model. Kumar and Rao considered unemployment rates in various cells (or domains) that had been obtained by cross-classifying the population into a number of age and education categories. The logit model consisted of both linear and quadratic effects for the age variable, with only the linear effect for the education variable. The same LFS data were also analysed by Singh and Kumar (1986) using an alternative method, namely the $Q^{(T)}$ test proposed by Singh (1985). The test $Q^{(T)}$ is a X^2 type test based on a generalized score statistic of principal components. Results obtained by the $Q^{(T)}$ method were found to be in agreement with those arrived at by the adjusted X^2 method.

Whenever a suitable model is determined, it is of interest to find good estimates of model parameters. These, in turn, provide fairly good estimates of true rates for domains. Such estimates (often called "smoothed estimates") are especially useful for domains in which survey estimates lack precision because the number of observations is not sufficient. It may be noted that since smoothed estimates are obtained after a model is found to have a reasonable fit, the bias in the estimates is expected to be negligible. Kumar and Rao (1986) used the method of pseudo mle (pseudo maximum likelihood estimates) under the working form of the likelihood that corresponds to independent binomial samples for estimating parameters

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of a logit model after an adequate fit had been established for the October 1980 LFS data. They found a considerable gain in efficiency over survey estimates of unemployment rates in the particular LFS example.

Pseudo mle are known to be useful when the likelihood function is not available or when it is difficult to compute due to complexities of the survey design. Under suitable regularity conditions, the pseudo mle provide consistent and asymptotically normal estimates (Imrey, Koch and Stokes 1982). In this paper we consider the problem of finding asymptotically efficient (in a sense to be explained in Section 3) estimates of model parameters and therefore of domain estimates. We describe the min $Q^{(T)}$ estimator, proposed in Singh (1985), based on the generalized scores approach which can be viewed as analogous to Neyman's min X^2 estimator for simple random samples. It may be noted that the WLS (Weighted Least Squares) approach for complex survey designs (Koch, Freeman and Freeman 1975) also provides asymptotically efficient estimates. However, these estimates are usually unstable for moderate sample sizes due to near singularity of the estimated covariance matrix of survey cell estimates (see Imrey, Koch and Stokes 1982, Fay 1985). The min $Q^{(T)}$ estimates, on the other hand, are designed to guard against the instability problem mentioned above. It will be seen that the problem of instability can be overcome by the min $Q^{(T)}$ method by employing a modified version of the estimated covariance matrix in which the relatively very small eigenvalues from its spectral decomposition are trimmed.

The necessary notation along with a brief review of the test $Q^{(T)}$ are presented in Section 2. Next the min $Q^{(T)}$ estimator and its asymptotic behaviour are described in Section 3. The example using LFS data is given in Section 4 as an illustration. For this numerical example, an interesting finding was that over individual cells, the pseudo mle perform almost at par with efficient min $Q^{(T)}$ estimates. In terms of an overall measure as given by trace efficiency, pseudo mle are found to be only slightly inferior to min $Q^{(T)}$ estimates. Finally, Section 5 contains some concluding remarks.

2. THE TEST $Q^{(T)}$: A BRIEF REVIEW

We shall briefly describe the test $Q^{(T)}$ in order to motivate the min $Q^{(T)}$ method of estimation (for more details, see Singh 1985, Singh and Kumar 1986). Let I denote the number of disjoint domains and v_i denote the parameter of interest for the i-th domain. Consider a model for $v = (v_1, v_2, \ldots, v_I)'$ as

$$H_0: h(v) = X\theta \tag{2.1}$$

where X is a known $I \times r$ matrix of full rank r, θ is an r-vector of unknown parameters, and h is a continuously differentiable one-to-one function, for instance, log or logit.

Let $\hat{v}$ denote the I-vector of survey estimates. Assume that under a suitable central limit theorem

$$\hat{v} \sim MVN(v, \Gamma/n) \tag{2.2}$$

where " $\sim$ " means "asymptotically distributed as", n is the total sample size, and Γ is the asymptotic covariance matrix of $\sqrt{n}(\hat{v} - v)$.

Now, choose a small level $\epsilon\,(>0)$ of dimensionality reduction (eg., .01 or .005 can be taken as working values of ϵ). Find a number T such that with the eigenvalues $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \ldots \geq \hat{\lambda}_I$ of the estimated covariance matrix $\hat{\Gamma}$, we have

$$T = \max \left\{ t: t > r \text{ and } \sum_{i=t}^{I} \hat{\lambda}_i / \sum_{i=1}^{I} \hat{\lambda}_i \geq \epsilon \right\}. \tag{2.3}$$

The variable T, although random, can be regarded as fixed for our asymptotics. It may be noted that if there are no relatively very small eigenvalues (i.e. if $\hat{\Gamma}$ is not ill-conditioned), then there will usually be no effect of dimensionality reduction for small ϵ and T will coincide with I in those situations.

Consider the problem of testing H_0 against alternatives K_0: $h(v) \neq X\theta$ in the class of tests based on the first T principal components W of $\hat{v}$. Let the normalized eigenvector corresponding to $\hat{\lambda}_i$ be P_i (it need not be unique) and let M_T denote the $I \times T$ matrix of eigenvectors P_i's corresponding to the first T largest eigenvalues. Then

$$W = M_T'\hat{v} \stackrel{.}{\sim} MVN(\mu, D_T/n), \tag{2.4}$$

where

$$\mu = M_T'v,\; D_T = diag(\lambda_1, \ldots, \lambda_T).$$

Based on W, the original testing problem concerning an I-dimensional v is reduced to testing a hypothesis about the T-dimensional parameter μ given by

$$H_0': \mu = M_T'h^{-1}(X\theta) \text{ vs } K_0': \mu \neq M_T'h^{-1}(X\theta). \tag{2.5}$$

The test statistic $Q^{(T)}$ can be obtained as a score statistic of principal components by employing the approximate likelihood of θ given by the limiting distribution (2.4) of W for computing the efficient scores (see Cox and Hinkley 1974, p. 321-324). We shall refer to $Q^{(T)}$ as a generalized score test that would reject H_0 for large values of the quadratic form

$$\begin{aligned} Q^{(T)}(\theta^o) &= Y(\theta^o)'\Delta_T Y(\theta^o) - Z_T(\theta^o)'\Lambda_T Z_T(\theta^o) \\ &\stackrel{.}{\sim} \chi^2_{T-r} \end{aligned} \tag{2.6}$$

where

$$Y(\theta^o) = \hat{v} - v(\theta^o),\; \Delta_T = n \sum_{i=1}^{T} (P_i P_i' / \hat{\lambda}_i),$$

$$Z_T(\theta^o) = B'\Delta_T Y(\theta^o),\; B = (\partial v / \partial \theta),\; \Lambda_T = (B'\Delta_T B)^{-1},$$

and θ^o is some fixed point in the null parameter space. In computing $Q^{(T)}$, any root n-consistent estimate of θ under H_0 can be substituted for θ^o, such as pseudo mle of θ. Notice that $Q^{(T)}$ of (2.6) is in fact a quadratic form in W but is expressed in $\hat{v}$ for the sake of convenience.

For testing H_0 vs K_0 in the class of tests based on W, the asymptotic optimality of the test $Q^{(T)}$ follows from that of the score statistic. For small $\epsilon > 0$, $\hat{v}$ and W will be close in the sense that principal components provide the best possible way of dimensionality reduction with a minimum loss of information. Thus $Q^{(T)}$ (for small ϵ) is expected to be robust with respect to the test Q corresponding to no dimensionality reduction. However, Q may be unstable (in the sense of inflated Type I error rate) for finite samples due to possible near singularity of $\hat{\Gamma}$. The test $Q^{(T)}$ is expected to control this problem of instability at the cost of sacrificing some information in the data that gives rise to possibly unreliable components in Q in the directions of eigenvectors that correspond to relatively very small eigenvalues. The loss of information implies that the test $Q^{(T)}$ will lack power for alternatives in directions of (near) singularities. However, this loss of power is offset by the gain in control of Type I error rate. The instability control is further ensured by the fact that, since H_0 is a subset of H_0', $Q^{(T)}$ will be a conservative test for H_0.

A special asymptotically equivalent version of $Q^{(T)}$ (θ°) which has a simpler expression similar to that of the standard Pearson-Fisher's X^2, is obtained by replacing θ° with an estimator $\tilde{\theta}$ that minimizes the expression $(\hat{v} - v(\theta))'\Delta_T(\hat{v} - v(\theta))$. We then have

$$\begin{aligned} Q^{(T)}(\tilde{\theta}) &= Y(\tilde{\theta})'\Delta_T Y(\tilde{\theta}) \\ &= \sum_{i=1}^{T} [P_i'(\hat{v} - v(\tilde{\theta}))]^2/\hat{\lambda}_i \\ &\stackrel{\cdot}{\sim} \chi^2_{T-r} \end{aligned} \tag{2.7}$$

Henceforth we assume that, for a given data vector $\hat{v}$, a model H_0 has been deemed appropriate based on the test $Q^{(T)}$ or some other test such as the adjusted X^2 test. In the next section, we give an asymptotically efficient method of estimating parameters θ under H_0, using the statistic $Q^{(T)}$. The θ estimates in turn provide a set of smoothed estimates of v corresponding to survey estimates $\hat{v}$.

3. THE MIN $Q^{(T)}$ ESTIMATOR

Consider the approximate likelihood for the mean μ of the first T principal components W of $\hat{v}$, given earlier by (2.4). Suppose the model H_0: $h(v) = X\theta$ is accepted. Then, the kernel function $K(\theta)$ of the approximate likelihood for $\mu(\theta)$ is given by

$$\begin{aligned} K(\theta) &= (W - \mu(\theta))'D_T^{-1}(W - \mu(\theta)) \\ &= (\hat{v} - v(\theta))'\Delta_T(\hat{v} - v(\theta)) \end{aligned} \tag{3.1}$$

The value $\tilde{\theta}$ that minimizes $K(\theta)$ corresponds to the mle of θ for the approximate likelihood of μ under H_0. The estimator $\tilde{\theta}$ will be asymptotically efficient (or best asymptotically normal (BAN) in the sense of Neyman, 1949), in a restricted class, namely in the class of estimates based on W. Following the min X^2 estimator of Neyman (1949), the estimator $\tilde{\theta}$ was termed min $Q^{(T)}$ estimator in Singh (1985). Notice that the estimator $\tilde{\theta}$ depends on the level ϵ of dimensionality reduction via Δ_T. Thus $\tilde{\theta}$ varies if ϵ does.

The smoothed estimates of v under H_0 based on W can be obtained as follows. Find $\tilde{\theta}$ which minimizes $K(\theta)$, i.e. $\tilde{\theta}$ is the solution of r equations

$$B'\Delta_T(\hat{v} - v(\theta)) = 0 \tag{3.2}$$

where both $B(=\partial v/\partial\theta)$ and v involve θ. An iterative procedure such as Newton-Raphson can be used to solve (3.2). Weighted least squares (WLS) estimates or pseudo mle can be used as possible initial choices for θ. We can then compute the min $Q^{(T)}$ estimator of v as

$$\tilde{v} = h^{-1}(X\tilde{\theta}). \tag{3.3}$$

The asymptotic behaviours of $\tilde{\theta}$ and $\tilde{v}$ are given by the following proposition.

Proposition 3.1 As before, let $\wedge_T$ denote $(B'\Delta_T B)^{-1}$. We have

$$\begin{aligned} &\text{(a)}\quad \tilde{\theta} - \theta \approx \wedge_T B'\Delta_T(\hat{v} - v(\theta)) \stackrel{\sim}{\sim} MVN(0,\wedge_T) \\ &\text{(b)}\quad \tilde{v} - v \approx B\wedge_T B'\Delta_T(\hat{v} - v(\theta)) \stackrel{\sim}{\sim} MVN(0,B\wedge_T B') \end{aligned} \tag{3.4}$$

where " $\approx$ " indicates that the difference between the two sides is negligible in probability.

The proof follows from the application of the δ-method to the functions $B'\Delta_T(\hat{v}-v(\theta))$ and $\tilde{v}-v(\theta)$, which gives

$$B'\Delta_T(\hat{v}-v(\theta)) - (B'\Delta_T B)(\tilde{\theta}-\theta) = o_p(1),$$

$$\tilde{v} - v(\theta) - B(\tilde{\theta}-\theta) = o_p(1).$$

From the above proposition it follows that the asymptotic covariance matrix of the min $Q^{(T)}$ estimator $\tilde{\theta}$ is the inverse of the information matrix $B'\Delta_T B$ for θ, which was obtained from the approximate likelihood of θ as given by (2.4). It can then be seen that in the absence of dimensionality reduction, the estimator $\tilde{\theta}$ will be asymptotically equivalent to the WLS estimator of Koch, Freeman and Freeman (1975). As mentioned in the Introduction, the WLS estimator generally shows unstable finite sample behaviour because of the inefficient estimation of Γ. In contrast, the estimator $\tilde{\theta}$ for a given $\epsilon > 0$ is expected to show stable finite sample behaviour in the sense that it can be approximated well by its asymptotic behaviour. This is achieved at the cost of compromising the asymptotic optimality of $\tilde{\theta}$ by restricting it to a smaller class, namely the class of estimates based on the first T principal components W. The WLS estimator, on the other hand, is asymptotically optimal in a wider class, namely the class of estimates based on the full data vector $\hat{v}$. If, for a small ϵ, the $Q^{(T)}$ test statistic indicates insignificance for H_0, then the corresponding min $Q^{(T)}$ estimator $\hat{v}$ will likely provide a robust alternative to the WLS estimator.

4. MIN $Q^{(T)}$ ESTIMATES OF UNEMPLOYMENT RATES

The Canadian labour force survey (LFS) data for October 1980 was analysed by Kumar and Rao (1984, 1986) and Roberts, Rao and Kumar (1987). Both sets of authors applied the extension of the Rao-Scott adjusted X^2 method to the case of logistic regression. They showed that the logit model given below provided an adequate fit to the survey estimates of employment rates ($v_{j\ell}$) for the table of 60 cells cross-classified by age (10 categories) and education (6 categories). The model is

$$log\,\frac{v_{j\ell}}{1-v_{j\ell}} = \beta_0 + \beta_1 A_j + \beta_2 A_j^2 + \beta_3 E_\ell \tag{4.1}$$

where A_j represents the midpoint $12 + 5j$ for j-th age group ($j = 1, \ldots, 10$), and $E_\ell(\ell = 1, \ldots, 6)$ represents the median years of schooling with values 7, 10, 12, 13, 14 and 16.

The model (4.1) can be expressed in the notation of Section 2 by numbering the sixty cells lexicographically. Thus, (4.1) can be rewritten as $h(v) = X\theta$, where v is the vector of employment rates, h is the logit function, X is a 60×4 matrix whose i-th row is $(1, A_i, A_i^2, E_i)$, and θ is $(\beta_0, \beta_1, \beta_2, \beta_3)'$. We also have

$$H = (\partial h / \partial v) = D_v^{-1} D_{1-v}^{-1}, \; B = H^{-1} X, \tag{4.2}$$

where D_v and D_{1-v} are diagonal matrices with diagonal elements given by the subscripts.

The pseudo mle of θ for the model (4.1) were obtained by Kumar and Rao (1984) under the pseudo product-binomial likelihood as

$$\bar{\theta} = (-3.10, 0.211, -0.00218, 0.1509)'. \tag{4.3}$$

They also computed Rao-Scott's first order adjusted X^2 (G_c^2 in their notation) as 55.3, which shows acceptance of the model (2.1) when referred to the χ_{56}^2 distribution.

The $Q^{(T)}$ method was applied for testing (4.1) (see Singh 1985, and Singh and Kumar 1986) also resulting in the acceptance of the model (4.1). For $\epsilon = .01$, T turns out to be 51 using the estimated covariance matrix $\hat{\Gamma}$ as obtained by Kumar and Rao (1984). Now using the pseudo mle $\bar{\theta}$, we have

$$Q^{(51)}(\bar{\theta}) = 58.665 - 4.454 = 54.211 \tag{4.4}$$

When $\epsilon = .005$, T is found to be 54, and

$$Q^{(54)}(\bar{\theta}) = 67.774 - 2.343 = 65.431 \tag{4.5}$$

When $\epsilon = 0$, $T = 58$ because two cells had zero observed unemployment rates. In this case,

$$Q^{(58)}(\bar{\theta}) = 87.302 - 0.812 = 86.49 \tag{4.6}$$

By referring $Q^{(51)}$ to the χ_{47}^2 distribution, $Q^{(54)}$ to a χ_{50}^2 and $Q^{(58)}$ to a χ_{54}^2 distribution, it is clear that both $Q^{(51)}$ and $Q^{(54)}$ accept (4.1) while $Q^{(58)}$ does not. An instability check can be performed by considering the difference $Q^{(58)} - Q^{(T)}$ for $T = 51, 54$, which can be seen to be highly significant when referred to the χ_{58-T}^2 distribution. These indicate presence of the instability problem in the Q-test statistic that corresponds to no dimensionality reduction. It is clear that WLS test would also have an instability problem due to the difficulty involved in inverting the matrix $\hat{\Gamma}$ which is singular. Thus, min $Q^{(T)}$ method would be preferable to min Q or WLS methods. In the interests of reducing loss of information, the method with the largest value of T is recommended, providing of course that the corresponding $Q^{(T)}$ shows insignificance for the model.

We shall now compute asymptotically efficient estimates. Neither min Q nor WLS estimates were computed because $\hat{\Gamma}$ was singular. The min $Q^{(T)}$ estimates $\tilde{\theta}$ were computed for $\epsilon = .005$ and $\epsilon = .01$ by using the Newton-Raphson iterative procedure and $\bar{\theta}$ as the initial estimate of θ for solving (3.2). The values of $\tilde{\theta}_T$ and $Q^{(T)}(\tilde{\theta})$ (in this case the negative term in (2.6) drops out) for $\epsilon = .005$, $T = 54$ were obtained as

$$\tilde{\theta}_{54} = (-2.7112, 0.1944, -0.00196, 0.1432)', \text{ and}$$

$$Q^{(54)}(\tilde{\theta}_{54}) = 63.4737 \tag{4.7}$$

For $\epsilon = .01$, $T = 51$, we have

$$\tilde{\theta}_{51} = (-2.6739,\ 0.19702,\ -0.00202,\ 0.1364)', \text{ and}$$

$$Q^{(51)}(\tilde{\theta}_{51}) = 55.2518. \tag{4.8}$$

Conclusions based on the statistic $Q^{(T)}(\tilde{\theta})$ for both $T = 54$ and 51 agree with those obtained from $Q^{(T)}(\bar{\theta})$.

Table 1 gives efficiencies relative to survey estimates of unemployment rates $1 - v$ for all cells (excepts two with zero observed unemployment rates) corresponding to the three smoothed estimates. The three smoothed estimates are the pseudo mle, min $Q^{(51)}$, and min $Q^{(54)}$. The pseudo mle variances are taken from Kumar and Rao (1986), while those for min $Q^{(T)}$ estimates are obtained from the diagonal elements of $B \wedge_T B'$ of (3.4). As noted by Kumar and Rao (1986) for pseudo mle, smoothed estimates based on min $Q^{(T)}$ also lead to considerable efficiency gains over survey estimates. The relative trace efficiency of smoothed estimates over survey estimates is 17.9 for pseudo mle, 18.95 for min $Q^{(51)}$ and 19.88 for min $Q^{(54)}$ estimates. Thus the min $Q^{(T)}$ estimators provide a slight improvement in the

Table 1

Efficiencies of Smoothed Estimates of Unemployment rates relative to Survey Estimates[a]

Cell Number	Min $Q^{(51)}$	Min $Q^{(54)}$	Pseudo mle	Cell Number	Min $Q^{(51)}$	Min $Q^{(54)}$	Pseudo mle
1	5.87	5.74	5.44	31	9.01	9.32	8.65
2	3.62	3.62	3.28	32	8.76	9.46	10.68
3	3.45	3.55	3.12	33	36.93	42.93	51.59
4	52.45	51.65	43.46	34	51.55	60.23	81.12
5	104.77	114.30	96.21	35	69.76	79.93	98.37
7	5.33	5.14	4.38	36	9.17	11.01	15.07
8	9.36	9.53	8.09	37	3.48	3.01	3.45
9	6.85	7.16	6.70	38	13.74	15.91	18.00
10	25.65	28.40	26.31	39	66.87	80.98	97.30
11	13.34	14.13	17.73	40	154.81	187.73	221.50
12	27.74	30.85	30.85	41	49.14	67.56	80.61
13	8.64	8.84	7.15	42	17.32	21.73	24.98
14	13.84	13.84	12.37	43	8.57	9.28	8.49
15	8.20	8.49	9.47	44	27.42	31.65	30.74
16	23.14	24.09	27.75	45	58.55	70.67	75.72
17	18.20	18.20	21.49	46	94.11	114.13	121.49
18	9.87	11.14	12.51	47	82.12	112.65	108.52
19	15.87	16.03	13.66	48	26.54	39.41	41.22
20	11.44	11.98	12.56	49	4.95	5.37	4.41
21	12.39	12.39	15.53	50	12.11	14.10	11.17
22	24.83	24.83	32.02	51	6.75	8.61	7.50
23	16.43	18.16	21.55	52	8.83	11.45	9.90
24	6.98	7.83	10.06	53	52.64	71.49	61.14
25	7.49	7.74	6.99	55	3.59	3.93	3.03
26	10.33	11.33	12.32	56	7.33	8.96	8.23
27	6.47	7.18	8.69	57	23.50	29.83	22.11
28	125.81	140.57	172.91	58	221.23	294.59	208.77
29	33.88	38.13	52.00	59	6.45	8.82	6.62
30	14.89	15.24	20.43	60	38.90	52.84	41.96

[a] Cells 6 and 54 are omitted due to zero observed unemployment rates.

efficiency of smoothed estimates compared to pseudo mle. With regard to performance over individual cells Table 1 indicates that the pseudo mle behave very well as compared to efficient min $Q^{(T)}$ estimates for the example under consideration.

5. CONCLUDING REMARKS

For computing pseudo mle, the working form of the likelihood function corresponds to simple random samples (i.e. multinomial or product-multinomial sampling). The pseudo mle do provide consistent estimates of model parameters without requiring an estimate of the covariance matrix Γ. However, the pseudo mle are not asymptotically efficient for complex survey data. By contrast, the min $Q^{(T)}$ estimates are asymptotically efficient with respect to the class of estimates based on W (the first T principal components of the vector $\hat{v}$ of survey estimates). For investigating the relative performance of pseudo mle and min $Q^{(T)}$, it would be desirable to perform a simulation study for efficiency comparisons. The min $Q^{(T)}$ estimates do take into account of the underlying complex design by employing an appropriate $\hat{\Gamma}$. If $\hat{\Gamma}$ is not ill-conditioned, i.e. it has no relatively very small eigenvalues, then there is no instability problem with the well known WLS estimates which are of course asymptotically efficient. In this case, it will usually turn out that there is no dimensionality reduction for small ϵ, that T will coincide with I and that there will be no loss in efficiency of min $Q^{(T)}$ estimates in comparison with WLS estimates. However, given the instability problem common with cross-classified categorical survey data, the min $Q^{(T)}$ estimates are expected to provide a robust alternative to WLS estimates.

ACKNOWLEDGEMENT

The second author's research was supported by Statistics Canada and the Natural Sciences and Engineering Research Council of Canada.

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