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Current Issues on Seasonal Adjustment

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ABSTRACT

This paper discusses three problems that have been a major preoccupation among researchers and practitioners of seasonal adjustment in statistical bureaus for the last ten years. These problems are: (1) the use of concurrent seasonal factors versus seasonal factor forecasts for current seasonal adjustment; (2) finding an optimal pattern of revisions for series seasonally adjusted with concurrent factors; and (3) smoothing highly irregular seasonally adjusted data.

KEY WORDS: Concurrent vs forward seasonal factors; Revisions; Trend-cycle filters; Smoothing.

1. INTRODUCTION

During the last decade, within the domain of seasonal adjustment, statistical bureaus have focused their attention on three important issues: (1) the seasonal adjustment of a current value; (2) the revisions of concurrent seasonally adjusted data; and (3) the smoothing of highly irregular seasonally adjusted series.

The main purpose of this article is to discuss each of the above problems with respect to the X-11-ARIMA seasonal adjustment program developed by Dagum (1980) and which is applied by Statistics Canada and other statistical bureaus of the world.

The four modes in which the X-11-ARIMA computer package can be used to produce a current seasonally adjusted value are discussed in Section 2. In Section 3, the focus is on analysis of the revisions of concurrent seasonally adjusted data based on the linear filters of X-11-ARIMA. Section 4 deals with the nature and characteristics of the smoothing (trend-cycle) filters available in X-11-ARIMA.

2. SEASONAL ADJUSTMENT OF CURRENT VALUES

The seasonal adjustment of a current value can be done using either a "concurrent" seasonal estimate or a seasonal "forecast".

A "concurrent" seasonal estimate (factor or effect depending on whether a multiplicative or additive model is assumed) is obtained by seasonally adjusting, each time a new observation is available, all the data available up to and including that observation. On the other hand, a seasonal "forecast" is obtained from a series that ended in the previous year. A common practice is to generate these seasonal forecasts, say for year $t + 1$, from data that ended in December of the previous year t.

There are four modes in which the X-11-ARIMA computer program can be applied to produce a current (last observation) seasonally adjusted value. These four modes are: (i) using ARIMA extrapolations and concurrent seasonal factors; (ii) using ARIMA extrapolations and seasonal factor forecasts; (iii) using concurrent seasonal factors without the use of ARIMA extrapolations; and (iv) using seasonal factor forecasts without the use of ARIMA extrapolations.

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While statistical bureaus use the four modes to obtain current seasonally adjusted values, not all of them do so with the same frequency. Thus, for example, the dominant mode in Statistics Canada is (i) followed by mode (iii) whereas in the U.S. Bureau of Labor, the dominant mode is (ii) followed by mode (iv). The current seasonally adjusted value produced by each type of seasonal adjustment varies and is subject to different degrees of error.

Under the assumption of an additive decomposition model, the seasonal adjustment of a current value X_t can be obtained by

$$\hat{X}_t^{(\ell)} = X_t - \hat{S}_t^{(\ell)}, \tag{1}$$

where $\hat{S}_t^{(\ell)}$ denotes a forward seasonal estimate; or by

$$\hat{X}_t^{(0)} = X_t - \hat{S}_t^{(0)}, \tag{2}$$

where $\hat{S}_t^{(0)}$ denotes a concurrent seasonal estimate.

The current seasonally adjusted value will become "final" in the sense that it will no longer be revised after m more observations are added. Thus,

$$\hat{X}_t^{(m)} = X_t - \hat{S}_t^{(m)}, \tag{3}$$

where $\hat{S}_t^{(m)}$ denotes a final seasonal estimate.

Therefore, the total revision of a concurrent and of a forward seasonal estimate can be written as

$$r_t^{(0,m)} = \hat{S}_t^{(0)} - \hat{S}_t^{(m)}, \quad m > 0; \tag{4}$$

$$r_t^{(\ell,m)} = \hat{S}_t^{(\ell)} - \hat{S}_t^{(m)}, \quad m > 0 > \ell. \tag{5}$$

Under the assumption of an additive decomposition and no replacement of extreme values, $\hat{S}_t^{(m)}$, the final seasonal estimate from a series $X_{t-m}, \ldots, X_t, \ldots, X_{t+m}$ can be expressed by

$$\hat{S}_t^{(m)} = \sum_{j=-m}^{m} h_{m,j} X_{t-j} = h^{(m)}(B) X_t, \tag{6}$$

where $h_{m,j} = h_{m,-j}$ are the symmetric moving average weights to be applied to the series. $h^{(m)}(B)$ denotes the corresponding linear filter using the backshift operator B, such that $B^n = X_{t-n}$. Young (1968) showed that the length of this symmetric filter $h^{(m)}(B)$, for monthly series, is 145 but that it can be well approximated by 85 weights because the values of the weights attached to distant observations are very small and, thus, $m = 42$.

Following equation (6) we can express a concurrent seasonal estimate $\hat{S}_t^{(0)}$ and a forward seasonal estimate $\hat{S}_t^{(\ell)}$ by:

$$\hat{S}_t^{(0)} = \sum_{j=-2m}^{0} h_{0,j} X_{t-j} = h^{(0)}(B) X_t, \quad m = 42, \tag{7}$$

where $h^{(0)}(B)$ denotes the asymmetric *concurrent* seasonal filter; and

$$\hat{S}_t^{(\ell)} = \sum_{j=-2m}^{\ell} h_{\ell,j} X_{t-j} = h^{(\ell)}(B) X_t, \ m = 42, \tag{8}$$

where $h^{(\ell)}(B)$ denotes the asymmetric *forecasting* seasonal filter and $\ell = 1,2, \ldots, 12$ for a monthly series.

The revision of a concurrent seasonal estimate depends on the distance between the concurrent and the final filter, that is, $d[h^{(0)}(B), h^{(m)}(B)]$, and on the innovations of the new observations $X_{t+1}, X_{t+2}, \ldots, X_{t+m}$.

Similarly, the revision of a forward seasonal estimate depends on $d[h^{(\ell)}(B), h^{(m)}(B)]$ and on the new innovations introduced by $X_{t-\ell}, \ldots, X_t, X_{t+1}, \ldots, X_{t+m}$.

Theoretical studies by Dagum (1982a and 1982b) have shown that

$$d[h^{(0)}(B), h^{(m)}(B)] < d[h^{(\ell)}(B), h^{(m)}(B)] \text{ for } \ell = 1, 2, \ldots 12. \tag{9}$$

The distance between the two filters is defined as the mean squared difference between the frequency response function of the filters over all the seasonal frequencies; a similar definition is given in the next section (equation (17)) using the root mean squared difference.

Relation (9) is true whether ARIMA extrapolations are used or not. Furthermore, the two studies also showed that

$$\begin{aligned} & d[h^{(0)}(B), h^{(m)}(B)] \quad \text{using ARIMA extrapolations} \\ < & d[h^{(0)}(B), h^{(m)}(B)] \quad \text{without ARIMA extrapolations,} \end{aligned} \tag{10}$$

and similarly

$$\begin{aligned} & d[h^{(\ell)}(B), h^{(m)}(B)] \quad \text{using ARIMA extrapolations} \\ < & d[h^{(\ell)}(B), h^{(m)}(B)] \quad \text{without ARIMA extrapolations,} \end{aligned} \tag{11}$$

$$\text{for } \ell = 1,2, \ldots, 12.$$

Studies by Dagum (1978), Bayer and Wilcox (1981), Kenney and Durbin (1982), McKenzie (1984), Dagum and Morry (1984), Pierce (1980) and Pierce and McKenzie (1985) have shown that

$$r^{(0,m)} < r^{(\ell,m)} \tag{12}$$

except in a few cases where

$$r^{(0,m)} > r^{(\ell,m)}. \tag{13}$$

The relationship (13) can be observed when the current observations of the latest year are strongly revised since X_t gets the largest weight in the estimations of $\hat{S}_t^{(0)}$.

From the viewpoint of the total revisions of the seasonal estimates, the results of the above empirical studies permit the ranking of the four modes as follows: mode (i) (ARIMA extrapolations with concurrent seasonal estimates) gives the smallest total revision; mode (iii) (no ARIMA extrapolations with concurrent seasonal estimates) ranks second; mode (ii) (ARIMA extrapolations with forward seasonal estimates) ranks third and mode (iv) (ARIMA extrapolations with forward seasonal estimates) ranks fourth.

3. REVISIONS OF CONCURRENT SEASONALLY ADJUSTED DATA

Statistics Canada's practice of using concurrent seasonal adjustment was first established in 1975 for the Labour Force Survey series. Gradually other foreign statistical agencies followed it. The use of concurrent seasonal factors for current seasonal adjustment poses the problem of how often should the series be revised. Kenny and Durbin (1982) recommended that revisions should be made after one month and thereafter each calendar year. Dagum (1982c) supported these conclusions and furthermore, recommended an additional revision at six months if the seasonal adjustment method is the X-11-ARIMA without the ARIMA extrapolation option.

For any two points in time $t + k$, $t + \ell$ $(k < \ell)$, the revisions of the seasonal estimates and consequently of the seasonally adjusted value is given by

$$r_t^{(\ell,k)} = \hat{X}_t^{(\ell)} - \hat{X}_t^{(k)}, \; k < \ell. \tag{14}$$

This revision reflects: (1) the innovations introduced by the new observations X_{t+k+1}, $X_{t+k+2}, \ldots, X_{t+k+\ell}$; and (2) the differences between the two asymmetric seasonal adjustment filters $Y^{(\ell)}(B)$ and $Y^{(k)}(B)$. If one fixes $k = 0$ and lets ℓ vary from 1 to m, then relation (14) gives a sequence of revisions of the concurrent seasonally adjusted values for different time spans or lags. The *total revision* of the concurrent estimate is given for $\ell = m$. If one fixes $\ell = k + 1$ and lets k take values from 0 to $m - 1$, then relation (14) gives the sequence of *single period revisions* of each estimated seasonally adjusted value and in particular, if one starts at $k = 0$ one obtains the $m - 1$ successive single period revisions of each estimated seasonally adjusted value before it becomes final. If one fixes $\ell = k + 12$ and lets k take values from 0 to $m - 12$, then equation (14) gives the sequence of annual revisions.

The revisions in which we are interested here are those introduced by filter discrepancies, and these can be studied by looking at the frequency response functions of the corresponding filters. Similarly to equation (6), we can approximate the seasonally adjusted value for recent years from the X-11-ARIMA program (with or without ARIMA extrapolations) by

$$\hat{X}_t^{(n)} = \sum_{j=n}^{m} Y_{n,j} X_{t-j} = Y^{(n)}(B)\, X_t. \tag{15}$$

Equation (15) represents a linear system where $\hat{X}_t^{(n)}(n)$ is the convolution of the input X_t and a sequence of weights $Y_{n,j}$ called the *impulse response function* of the filter. The properties of this function can be studied using its Fourier transform which is called the *frequency response function*, defined by

$$\Gamma^{(n)}(\omega) = \sum_{j=-n}^{m} Y_{n,j} e^{-2\pi\omega j}, \; -\tfrac{1}{2} \leq \omega \leq \tfrac{1}{2}, \tag{16}$$

where ω is the frequency in cycles per unit time. $\Gamma^{(n)}(\omega)$ fully describes the effects of the linear filter on the given input. Monthly and annual revisions of the concurrent filter of X-11-ARIMA with and without the ARIMA extrapolations have been calculated by Dagum (1987) based on the mathematical distance between the various frequency response functions of the filters. The pattern is characterized by a rapid decrease in the size of the monthly revisions of the concurrent filter for $\ell = 1, 2$, and 3; and a slow decrease thereafter until $\ell = 11$; then a large increase occurs at $\ell = 12$ followed by a decrease at $\ell = 13$ and then another large increase at $\ell = 24$ followed by a decrease at $\ell = 25$. Dagum (1987) showed that this pattern of monthly revisions is the same whether ARIMA extrapolations are used or not.

The significant decreases for the first three consecutive revisions are due to the improvement of the Henderson (trend- cycle) filter weights. The reversal of direction in the size of the filter revisions at $\ell = 12$ and $\ell = 13$, is due to the improvements of the seasonal filter that becomes less asymmetrical from year to year until three full years are added to the series. The two largest revisions occur at $\ell = 1$ and $\ell = 12$. *Given the non-monotonicity of single monthly revisions, it is not advisable to revise the concurrent estimate any time a new observation is added to the series.*

A revision scheme often used by statistical bureaus for their concurrent seasonally adjusted series consists of keeping constant the concurrent estimate from the time it appears until the end of the year and then revising annually the current and earliest years. Therefore, first year revisions due to filter discrepancies are given by $R^{(0,0)}$, $R^{(1,0)}$, ..., $R^{(11,0)}$; second year revisions by $R^{(12,0)}$, $R^{(13,1)}$, ..., $R^{(23,11)}$ third-year revisions by $R^{(24,12)}$ $R^{(25,13)}$ and so on where $R^{(\ell,k)}$ is defined by

$$R^{(\ell,k)} = [2\int_0^{1/2} \| \Gamma^{(\ell)}(\omega) - \Gamma^{(k)}(\omega) \|^2 \, d\omega]^{1/2}, \tag{17}$$

$$\ell = 1, 2, \ldots, n, k = 0, 1, 2, \ldots, n - 12,$$

and $n = 42$ for the X-11-ARIMA seasonal adjustment filters.

Table 1 shows the first-, second- and third-year revisions of the concurrent seasonal adjustment filter for X-11-ARIMA without extrapolation and with extrapolations from one ARIMA model and two sets of parameter values (other cases are shown in Dagum 1987). The ARIMA model chosen is the classical (0,1,1) $(0,1,1)_{12}$ model that is $(1 - B)(1 - B^{12}) X_t = (1 - \theta B)(1 - \Theta B^{12}) a_t$ where X_t denotes the original series, B is the backshift operator such that $B^n X_t = X_{t-n}$, a_t is a purely random process that represents the innovations and θ and Θ are the non-seasonal and seasonal parameters, respectively.

Since the largest single period revisions occur at $\ell = 1$ and $\ell = 12$ as mentioned above, a better revision scheme would be to incorporate monthly and annual revisions. It is expected that (1) adjusting concurrently each month, say from January to November and revising only once when the next month is available, and (2) adjusting concurrently December when it first appears and then revising the first year and earlier years when January is added, should improve the reliability of the filter applied during the current year while maintaining simultaneously the filter's homogeneity for month-to-month comparisons.

The first-year revisions of the first-month revised filter would then be $R^{(1,1)}$, $R^{(2,1)}$, ..., $R^{(11,1)}$. Table 2 shows these revisions and although the pattern is very similar to that of the concurrent filter, *the size of the revisions are much smaller if no extrapolations are used.* On the other hand, *the improvement is less important if ARIMA extrapolations are used.* Similarly, no major differences were observed for the second- and third-year revisions.

3.1 Estimation of Trading Day Variations and ARIMA Models with Concurrent Seasonal Adjustment

Besides the type of revisions scheme to be applied, there are two other problems posed by concurrent seasonal adjustment associated with trading day variations and ARIMA modelling.

Table 1
First-, Second- and Third-Year Revisions of the Concurrent Seasonal Adjustment Filter of X-11-ARIMA

Revisions $R^{(\ell,k)}$	Without ARIMA Extrapolations	With ARIMA Extrapolations from a (0,1,1) $(0,1,1)_{12}$ Model: $\theta = .40$ $\Theta = .80$	With ARIMA Extrapolations from a (0,1,1) $(0,1,1)_{12}$ Model: $\theta = .80$ $\Theta = .80$
$R^{(1,0)}$	.12	.12	.06
$R^{(2,0)}$	.13	.13	.08
$R^{(3,0)}$	.13	.13	.08
$R^{(4,0)}$	.13	.13	.09
$R^{(5,0)}$	.15	.13	.09
$R^{(6,0)}$	.17	.13	.09
$R^{(7,0)}$	.16	.13	.09
$R^{(8,0)}$	.16	.13	.09
$R^{(9,0)}$	.16	.13	.09
$R^{(10,0)}$	.16	.14	.09
$R^{(11,0)}$	.16	.14	.09
$R^{(12,0)}$	.29	.28	.26
$R^{(13,1)}$	.27	.27	.26
$R^{(14,2)}$	.27	.27	.26
⋮	⋮	⋮	⋮
$R^{(23,11)}$	.27	.26	.26
$R^{(24,12)}$	.20	.16	.16
$R^{(24,13)}$	.18	.17	.16
$R^{(36,24)}$	.16	.17	.16
⋮	⋮	⋮	⋮

Table 2
First-Year Revisions of the First-Month Revised Seasonal Adjustment Filter

Revisions $R^{(\ell,1)}$	Without ARIMA Extrapolations	With ARIMA Extrapolations from a (0,1,1) $(0,1,1)_{12}$ Model: $\theta = .40$ $\Theta = .80$	With ARIMA Extrapolations from a (0,1,1) $(0,1,1)_{12}$ Model: $\theta = .80$ $\Theta = .80$
$R^{(2,1)}$	.07	.10	.06
$R^{(3,1)}$	.07	.10	.06
$R^{(4,1)}$	.07	.10	.07
$R^{(5,1)}$	.08	.10	.08
$R^{(6,1)}$	.10	.11	.08
$R^{(7,1)}$	.11	.11	.08
$R^{(8,1)}$	.11	.11	.08
$R^{(9,1)}$	.11	.11	.08
$R^{(10,1)}$	.12	.11	.08
$R^{(11,1)}$	.12	.12	.08

For series which are flows in the sense that they result from the accumulation of daily values over the calendar months, there is a systematic effect caused by trading day variations. Trading day variations arise mainly because the activity varies with the days of the week. Other sources are associated with accounting and reporting practices. For example, stores that do their bookkeeping activities on Friday tend to report higher sales in months with five Fridays than in months with four Fridays. The trading day effects are estimated in the X-11-ARIMA program using ordinary least squares on a simple deterministic regression model. Consequently, the weights estimated for each day change any time a new observation is added to the series. Since regression techniques are very sensitive to outliers, these changes can be sometimes unnecessarily large.

When the series are seasonally adjusted concurrently, the trading day estimates change all the time. In order to avoid unnecessary revisions, Statistics Canada's practice is to use the weights calculated by the program at the end of the previous calendar year or the weights provided by the users, as priors for the current year. The weights are then revised on an annual basis.

The effect of trading day variations must be removed from the series before ARIMA modelling, for these type of models cannot adequately handle trading day variations. In other words, if the X-11-ARIMA program is used with ARIMA extrapolations on series with trading day variations, these variations should be estimated *a priori* and if significant, they should be removed from the original series before the ARIMA modelling.

Another problem associated with concurrent seasonal adjustment refers to how often the ARIMA models should be identified. The current practice at Statistics Canada is to use the automatic ARIMA model selection option once a year and if the model is accepted, then it is kept constant for a whole year, letting only the parameters change when more observations are added. In order to keep the model constant, the user's supplied model option should be applied. Maintaining the ARIMA model constant avoids unnecessary revisions that may result from changing of models back and forth simply because of the presence of outliers.

4. SMOOTHING OF VOLATILE SEASONALLY ADJUSTED SERIES

One of the main purposes of the seasonal adjustment of economic time series is to provide information on current economic conditions, particularly to determine the stage of the cycle at which the economy stands. Since seasonal adjustment means removing only seasonal variations, thus leaving trend-cycle variations together with irregular fluctuations, it is often difficult to detect the short-term trend or cyclical turning points for series strongly affected with irregulars. In such cases, it may be preferable to smooth the seasonally adjusted series using trend-cycle estimators which suppress as much as possible the irregulars without affecting the cyclical component.

The use of trend-cycle values has been discussed by several writers and recently by Moore *et al* (1981), Kenny and Durbin (1982), Maravall (1986) and Dagum and Laniel (1987). Although not yet practised widely, some statistical agencies such as Statistics Canada and the Australian Bureau of Statistics smooth some of their seasonally adjusted series, particularly those series that are strongly affected by irregulars.

The combined linear filters applied to the original series to generate a central (symmetric) estimate of the trend-cycle component have been calculated by Young (1968) for Census Method II-X-11 variant. This filter is similar to that of X-11-ARIMA with and without ARIMA extrapolations. Dagum and Laniel (1987) extended Young's (1968) results to include the estimation of the asymmetric trend-cycle filters of X-11-ARIMA with and without the ARIMA extrapolations.

Figure 1 shows the gain functions of the central (symmetric) seasonal adjustment filters and smoothed seasonally adjusted data (trend-cycle) filters. It is apparent that the trend-cycle filters suppress all the noise present in the series, where the noise is defined as the power present in all frequencies $\omega \leq .166$. This frequency corresponds to the first harmonic of the fundamental seasonal frequency of a monthly series. This pattern results from the convolution of the seasonal adjustment filters with the 13-term Henderson trend-cycle filter.

Figure 2a shows the gain functions of the concurrent and first-month revised trend-cycle filters of X-11-ARIMA *without* ARIMA extrapolations. Figure 2b shows their corresponding phase-shift functions expressed in months instead of radians. We can observe that the gain for all $\omega \leq .166$ is much larger for these two asymmetric filters as compared with the central filter. Furthermore, there are large amplifications for frequencies near the fundamental seasonal. All this means that the concurrent and first revised smoothed seasonally adjusted values will have more noise than the final estimates. On the other hand, it is apparent that the phase shifts are very small, less than one month for the most important cyclical frequencies $0 < \omega < .055$ (i.e., cycles of periodicities equal to and longer than 18 months).

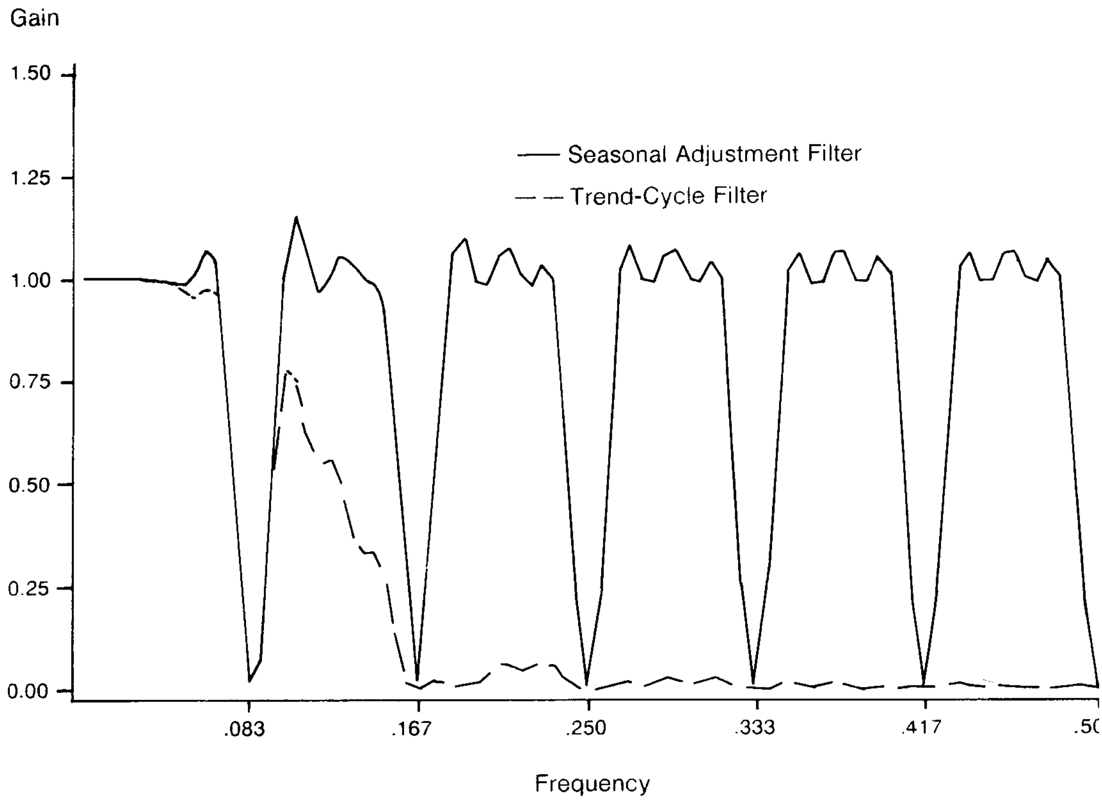

Figure 1. Gain Functions of the Central (Symmetric) Trend-Cycle and Seasonal Adjustment Filters of X-11-ARIMA.

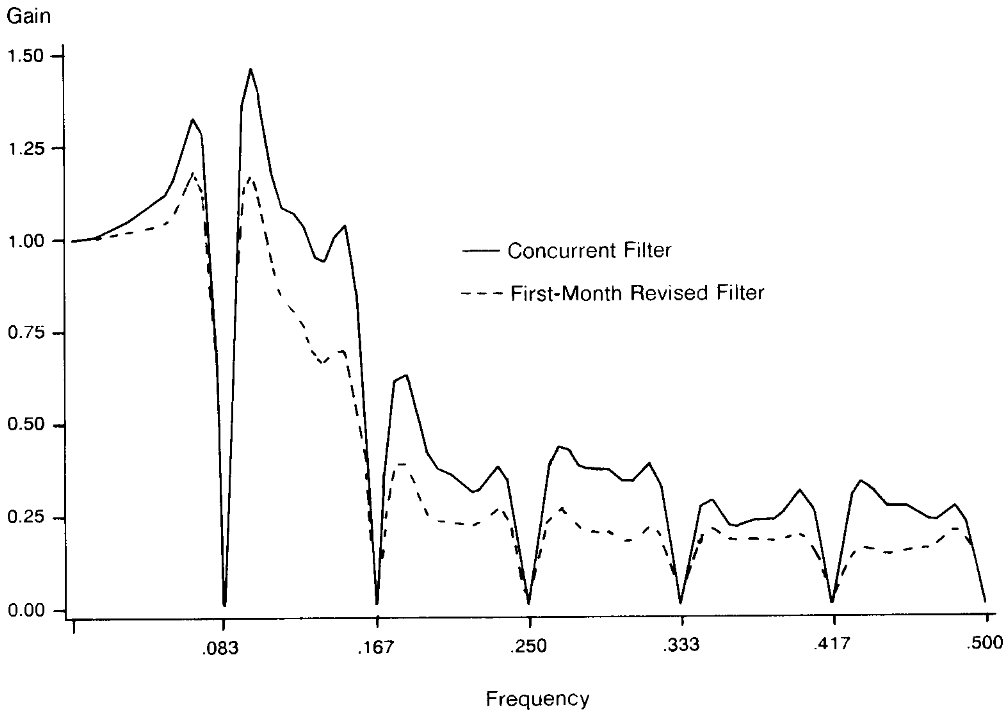

Figure 2a. Gain Functions of the Concurrent and First-Month Revised Filters of X-11-ARIMA without ARIMA Extrapolations.

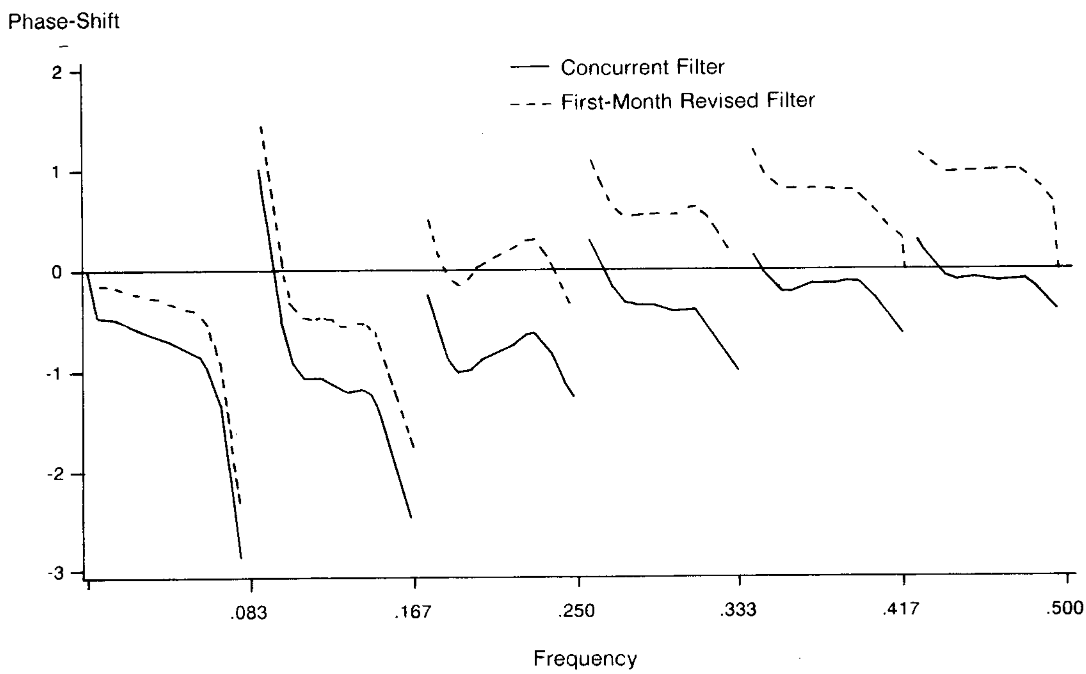

Figure 2b. Phase-Shift Functions of the Concurrent and First-Month Revised Filters of X-11-ARIMA without ARIMA Extrapolations.

Figures 3a and 3b show the gain and phase-shift functions of the concurrent and first-month revised trend-cycle filters of X-11-ARIMA with ARIMA extrapolations. The extrapolations are obtained from an IMA model $(0,1,1)(0,1,1)_{12}$ with $\theta = .40$ and $\Theta = .60$. The gain functions are closer to the symmetric (central) filter than those of X-11-ARIMA without the ARIMA extrapolations. There are no amplifications around the fundamental seasonal frequency and a similar attenuation of power at higher frequencies. On the other hand, there is more phase-shift (being near to one month) for low frequencies and less phase-shift for all high frequencies.

Dagum and Laniel (1987) studied the time path of the revisions of the trend-cycle filters and compared them with those of the seasonal adjustment filters. Their results, as summarized in Table 3, show that the total revisions of the trend-cycle asymmetric filters converge to zero much faster than those of the corresponding seasonal adjustment filters. In fact, the total revision of the trend-cycle filter three months after the concurrent filter is only .1, whereas a close value is achieved for the seasonal adjustment filter only after 24 months have been added to the series. Except for the total revisions of the concurrent filter which is larger for the trend-cycle filters compared with the corresponding seasonal adjustment filter, in all the other cases the total revisions are smaller for the trend-cycle filters. Furthermore, the trend-cycle filter revisions converge much faster to zero as compared with those of the seasonal adjustment filters.

Table 3

Time Path of the Total Revisions of the Trend-Cycle and the Seasonal Adjustment Asymmetric Filters of X-11-ARIMA

Revisions $R^{(\ell,k)}$*	Without Extrapolations		With Extrapolations from a $(0,1,1)\ (0,1,1)_{12}$ Model $\theta = .40$ $\Theta = .60$	
	Trend-Cycle Filter	Seasonal Adjustment Filter	Trend-Cycle Filter	Seasonal Adjustment Filter
$R^{(48,0)}$	.45	.36	.41	.32
$R^{(48,1)}$	.27	.33	.26	.32
$R^{(48,2)}$	.15	.32	.15	.32
$R^{(48,3)}$	.11	.32	.11	.31
$R(48,4)$	.12	.32	.11	.31
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$R^{(48,12)}$	.10	.23	.09	.20
$R^{(48,24)}$	.07	.13	.05	.10
$R^{(48,36)}$	.03	.05	.02	.04
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$R^{(48,47)}$	.01	.01	.01	.01

* $\ell = 48$ for the "final" trend-cycle filter and $\ell = 42$ for the final seasonal adjustment filter. However, the values shown for the revision of the seasonal adjustment filters are also calculated for $\ell = 48$ since after $\ell = 42$ the values are final and, thus, do not change.

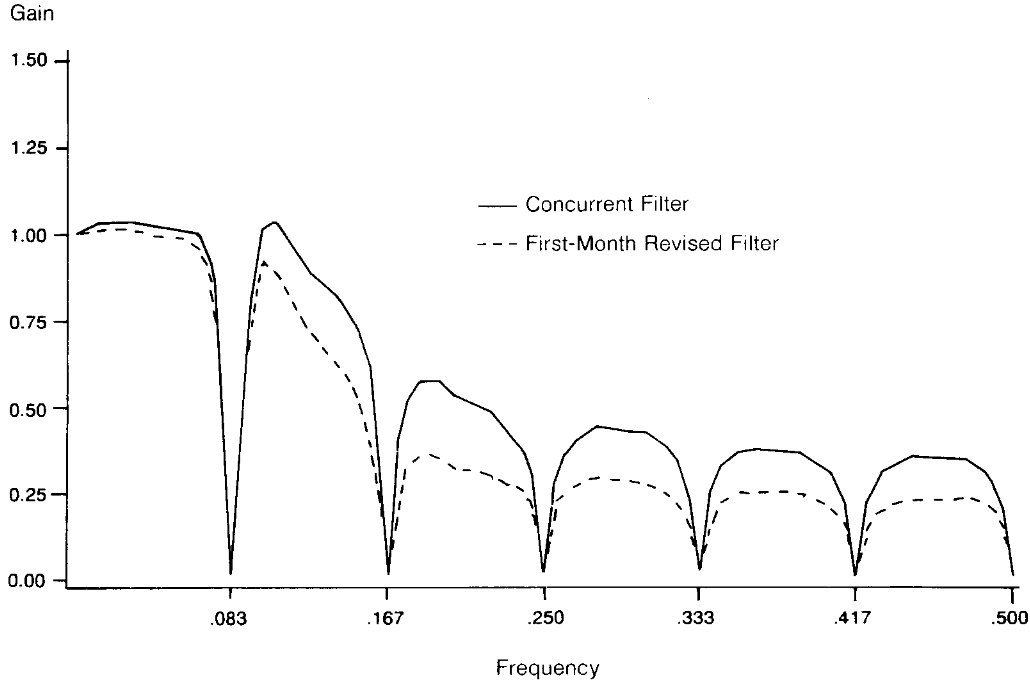

Figure 3a. Gain Functions of the Concurrent and First-Month Revised Filters of X-11-ARIMA with ARIMA Extrapolations (θ = .40, Θ = .60).

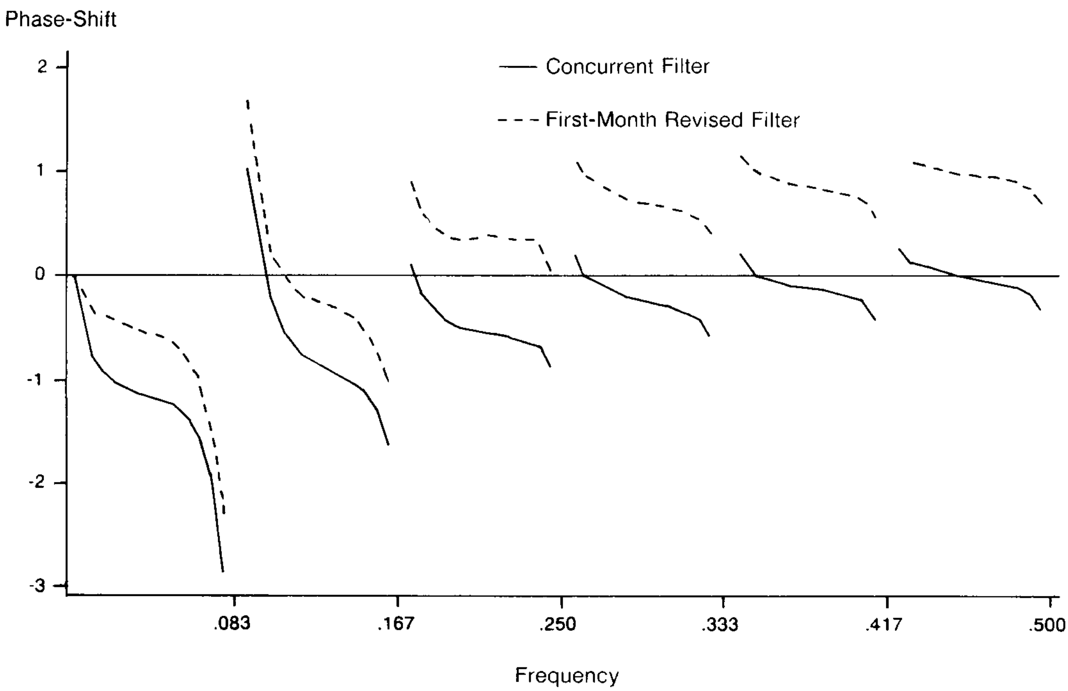

Figure 3b. Phase-Shift Functions of the Concurrent and First-Month Revised Filters of X-11-ARIMA with ARIMA Extrapolations (θ = .40, Θ = .60).

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