Telephone Sample Designs for the U.S. Black Household Population¹

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ABSTRACT

The two-stage rejection rule telephone sample design described by Waksberg (1978) is modified to improve the efficiency of telephone surveys of the U.S. Black population. Experimental tests of sample design alternatives demonstrate that: a) use of rough stratification based on telephone exchange names and states; b) use of large cluster definitions (200 and 400 consecutive numbers) at the first stage; and c) rejection rules based on racial status of the household combine to offer improvements in the relative precision of a sample, given fixed resources. Cost and error models are examined to simulate design alternatives.

KEY WORDS: RDD samples; Telephone surveys; Rare population samples.

1. INTRODUCTION

Surveys of rare populations lacking special frames often entail large per-unit costs relative to similar designs for the full population. When the rare population is a small subgroup of a readily identifiable population, the sample of that subgroup is often obtained by screening the larger population. Household surveys of demographic subgroups such as the U.S. Black population typically use such screening to locate eligible sample units; however, extensive screening to identify a rare population sample results in high costs per interview. In recent years telephone-sampling methods have been proposed as cost-efficient tools for sampling and interviewing rare populations. The cost of telephone interviewing is often less than face-to-face interviewing (Groves and Kahn 1979), and when screening is required to identify an eligible respondent, the cost-efficiency of telephone interviewing becomes even more marked. Still, the screening costs of telephone surveys of rare populations can be high in absolute terms.

This paper presents ways in which the screening method for telephone surveys can be refined to reduce costs while achieving desired levels of precision. In this paper we examine a variety of telephone sample designs for the U.S. Black household population. The telephone survey experiments described in this paper were conducted as part of a study of Black political attitudes and electoral behavior in the 1984 U.S. presidential election.

The use of telephone sampling and interviewing implies that Blacks living in households without telephones (about 15 percent of the U.S. Black household population) are not covered by the survey procedures. Such persons tend to be poorer and younger than those living in households with telephones (Thornberry and Massey 1983). To the extent that Blacks without telephones have attitudes and voting behaviors that are different from those with telephones, the survey estimates would differ from Black household population parameters. While not wanting to discount noncoverage error associated with telephone surveys of the Black population, this paper focuses on differential cost efficiencies and sampling error that might result from alternative approaches to telephone samples of Black households.

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The telephone sample designs presented here are extensions of a design described by Waksberg (1978). That random digit dialing (RDD) design (commonly referred to as the Waksberg-Mitofsky design) is a two-stage cluster sample of telephone numbers. U.S. telephone numbers contain 10 digits, a three-digit area code, a three-digit central office code or "prefix", and a four-digit suffix in the range 0000-9999 (e.g., 313-764-4424). At the primary stage, a stratified sample of 10-digit telephone numbers is randomly generated, and each such "'primary number" is linked to a block of 100 consecutive numbers (e.g., 313-764-4424 would be linked to the "100-series", 313-764-4400 to 313-764-4499). For household surveys, if the primary number is found to be a working household number, then its cluster of 100 consecutive telephone numbers is retained at the first stage for further sampling. If not, its "100-series" is discarded. Therefore, the probability of selection of a first stage 100-series is proportional to the number of working household numbers in that 100-series. In the second stage of sampling, equal numbers of working household numbers are selected from each of the 100-series retained at the primary stage. Therefore, the second stage sampling of households is performed with conditional probabilities of selection inversely proportional to the number of working household numbers in the 100-series. Thus, the design yields an equal probability (epsem) sample of household numbers, and clusters them so that the proportion of total numbers selected which reach households is higher than that obtained by a stratified random RDD sample. To clarify the discussion here, we refer to the 100-series banks of consecutive numbers as the primary stage unit (PSU) of the two-stage RDD design. The term "cluster" is reserved for the fixed set of working household numbers that is selected from the PSUs at the design's second stage.

In this research the sample design modifications aimed at reducing screening costs take three forms: a) stratification of telephone exchange units by proportion Black, and disproportionate allocation of the sample to high density Black strata; b) use of two-stage rejection rules based on both residential status and race of the household; and c) increase in PSU size (from 100 consecutive numbers to 200 and 400).

Stratification of the telephone population by race attempts to isolate exchange areas with high proportions of telephone subscribers who are Black. Higher sampling fractions are then applied to those strata, relative to strata with lower proportions Black. Under this disproportionate sample design, the total number of households that have to be contacted in order to obtain one interview with an eligible Black household is smaller than that for an epsem sample of the household population. Consequently, the screening costs for locating a sample of Black households are reduced. In telephone samples, the basic geographical unit for stratification is the wire center or telephone exchange, to which one or more three-digit prefixes (central office codes) may be assigned. In general, no counts of the subscriber population by racial characteristics are available for these sampling units. Thus, proxy indicators of high density Black exchanges must be used. The experiments described in this paper examined the value of such proxy indicators.

Blair and Czaja (1982) present an alteration of the Waksberg-Mitofsky RDD design which incorporates two-stage rejection rules based on both residential status and race eligibility of the household. For the Black population this method includes, at the first stage, only 100-series whose primary number was assigned to a Black household and then samples a fixed total of Black household numbers within those PSUs. In a U.S. national sample survey, Blair and Czaja found that using this design, the percentage of Black households among all household numbers chosen increased from 9 percent for the first stage to 25 percent for the second stage numbers. Given the compensating probabilities of selection in the two stages, this epsem design greatly reduces the level of screening required to obtain any given sample size of Black households. A similar alteration of the rejection rules for the two-stage Waksberg-Mitofsky design was employed in the experiments described in this paper.

In the Blair and Czaja design some of the primary stage 100-series contained too few Black household numbers to yield the number of elements per cluster required (10 in their case) for an epsem sample of Black households. In addition, relatively large screening costs are incurred at the first stage of selection for this design; over 44 primary numbers must be dialed to locate one Black household. The joint solution to these two problems is to both increase the size of the PSU and to select larger numbers of second stage elements per PSU. The analyses reported here examined the use of primary stage units of 100, 200, and 400 consecutive numbers each. The extension of the PSU definition beyond the standard 100 consecutive numbers was suggested by observations on the assignment of telephone numbers within prefixes. The following appears to be the most common pattern: 1) almost all household numbers within a prefix serve units located within the geographical boundaries of the exchange; 2) there is little geographical clustering of assignments within exchanges (i.e., neighbors do not tend to have consecutive telephone numbers, nor need they have numbers in the same prefix); and 3) there is more diversity in the percentage of household numbers among 1000-series than among 100-series within the same 1000-series of numbers. These impressions are the result of several years of household telephone sampling at the Survey Research Center. Observations 1) to 3) suggest that the expansion of the PSU definition from 100 consecutive numbers to a larger number might permit the use of larger clusters of secondary numbers with little reduction in the proportion of those numbers which are Black households.

2. THE PILOT STUDY

In two integrated experiments imbedded in a pilot survey, several design alternatives were tested. One purpose of the pilot study was to examine the ability of stratification based on civil government units, with only rough correspondence to telephone exchanges, to isolate sets of telephone numbers densely filled with black household numbers. For this, three strata of exchanges were defined:

- 1. "High density" Exchanges corresponding to the central cities of large Standard Metropolitan Statistical Areas (e.g., Chicago city, for the Chicago SMSA). This identification was based on the name of the telephone exchanges in these areas.
- 2. "Medium density" All other exchanges in selected southern states (Virginia, North Carolina, South Carolina, Florida, Georgia, Alabama, Mississippi, Louisiana). The vast majority of exchanges lie in only one state; those serving two states were associated with the state given in the exchange name.
- 3. "Low density"- The balance of exchanges in the coterminous United States.

An equal probability sample of 1400 six-digit area code/central office code prefix combinations was then systematically selected from the 34,389 such combinations listed as active on a frame which can be purchased from American Telephone & Telegraph (AT&T). Four-digit random numbers were appended to each selected six-digit stem to yield a sample of 1400 ten-digit primary numbers.

The results of the pilot study demonstrated that the three strata had vastly different proportions of Black telephone numbers. The low density stratum was found to require over six times as much screening to locate a black household as was required in the high density stratum. (This result was confirmed with more precision in the production study, discussed in the next section).

Another purpose of the pilot study was to test the use of rejection rules based on racial composition and working household status of sample numbers from PSUs of differing size. To provide increased precision in analyses related to this objective, an additional 500 primary

numbers were selected from the high- and medium- density strata. The 1900 primary numbers in the combined pilot study sample were then dialed and screened for their Black household status. If the sampled primary number reached a Black household, it simultaneously identified three different PSUs. As shown in Table 1, every individual number can be viewed as belonging to a single 100-series, a single 200-series, and a single 400-series. For example, the number 313-764-4424 is a member of the 4400-4499 100-series, the 4400-4599 200-series, and the 4400-4799 400-series. To test the feasibility of expanding the PSU size, the pilot study sampled secondary numbers from each of these three hundred series. The second stage cluster sizes of Black households were set at 3 for the 100-series of the primary number, 6 for the 200-series, and 9 for the 400-series clusters. In both the primary and secondary stages of selection, if the race of the household was not known, it was assumed to be a non-Black household.

Table 1 presents the disposition of the secondary numbers by PSU type and stratum. Of most interest is the proportion of secondary numbers assigned to Black households for the different PSU definitions. For the 100-series, .134 of all secondary numbers are Black household numbers. This implies that .223 of the households sampled were Black, compared to the .25 Black households found by Blair and Czaja. For the 200-series PSUs, .124 of all secondary numbers are Black household numbers. For the 400-series, .115 of all second stage sample telephone numbers are assigned to Black households. These proportions are all within sampling error of each other (the standard error of each estimate is at least .02). That is, no significant decrease in the proportion eligible was observed when the PSU definition was expanded from 100 to 400 consecutive numbers. These rates imply that while 100-series PSUs on the average can support second stage clusters of 13 or 14 sample Black households, the 400-series might on the average support cluster sizes of 46 sample Black households. The ability to increase the Black household cluster size at the second stage of sampling enables the researcher to greatly reduce sample screening costs.

Table 1 also compares the proportion of eligible secondary numbers for PSUs sampled from the three different strata used in the pilot study. For all the PSU definitions (100, 200, 400) the same result applies — the large SMSA telephone exchanges in the high Black density stratum offer close to a doubling of the eligibility rate when compared to the rate for the overall population (.21 versus .12 or .13). The medium density stratum, consisting of non-SMSA exchanges in selected Southern states, has eligibility rates below that of the nation as a whole (between .08 and .10). The low density stratum, the remainder of the country, also has lower than average eligibility rates (between .07 and .085). Since the high density stratum covers about 36 percent of the Black household population with telephones, the chosen stratification, in combination with disproportionate allocation of the primary stage samples, is an effective tool for reducing screening costs.

3. THE PRODUCTION STUDY

The production study used the stratification plan that was developed and tested in the pilot study. A disproportionately allocated sample of 11,223 primary numbers was selected from the three Black-density strata using sampling fractions in the ratio 3:2:1 (High:Medium: Low). Although the pilot study found no significant difference in the working household rate for PSUs of 200 and 400 consecutive numbers, a conservative decision was made to use the smaller 200-series PSUs in the production study. The expected second stage cluster size for each PSU was set at 5.5 Black households (not counting the primary number). Primary and secondary stage rejection rules for the modified two-stage Waksberg-Mitofsky design were identical to those used for the pilot study. Since much larger sample sizes were used in the production study, questions about precision and relative efficiencies of the design can be addressed with more confidence.

Table 1

Pilot Study

Disposition of Secondary Numbers Selected within 100-, 200- and 400-Series by Stratum

	Proportion of All Numbers Selected			
Stratum and Disposition	100- Series	200- Series	400- Series*	
High Density Black Stratum				
Black Households	.205	.201	.214	
Don't Know Race	.028	.029	.032	
Non-Black Households	.316	.279	.275	
Nonresidential/Nonworking	.451	.491	.479	
Number of Cases	(395)	(806)	(1163)	
Medium Density Black Stratum				
Black Households	.104	.080	.076	
Don't Know Race	.030	.018	.020	
Non-Black Households	.494	.443	.420	
Nonresidential/Nonworking	.372	.459	.484	
Number of Cases	(231)	(560)	(878)	
Low Density Black Stratum				
Black Households	.085	.084	.069	
Don't Know Race	.014	.028	.027	
Non-Black Households	.532	.577	.607	
Nonresidential/Nonworking	.369	.311	.297	
Number of Cases	(141)	(286)	(491)	
Total				
Black Households	.134	.124	.115	
Don't Know Race	.024	.025	.026	
Non-Black Households	.442	.431	.448	
Nonresidential/Nonworking	.400	.420	.411	
Number of Cases	(767)	(1652)	(2532)	

^{*} Weighted estimate to compensate for the disproportionate allocation of the cluster of 9 secondary numbers across the separate 100-number ranges of the 400-series.

Table 2 presents the results from both the primary and secondary number screening for the production study. The unbiased weighted estimate for an "epsem" two-stage RDD design suggests that 13 percent of all secondary numbers were Black households (the standard error about this estimate is .6 percent). This is in close agreement with the 12 percent secondary number eligibility rate observed in the pilot study. A comparison of the results for the primary stage of selection with those of the secondary stage illustrates the large gains possible by using a two-stage design for telephone sampling of Black households. The gains under the two-stage design are most dramatic in the low density Black stratum where there is nearly a nine-fold increase in the proportion of Black household numbers from the primary to secondary stage (.011 to .090). In the high density stratum the increase is closer to a twofold one (.072 to .190). For the disproportionate allocation design, the unweighted proportions of Black households at the two stages are 3 percent (primary stage) and 15 percent

(secondary stage). Comparison of these figures with the estimates for the epsem design (i.e., 2 percent and 13 percent) indicates the reduction in screening achieved by disproportionate allocation.

As in the pilot study, the percentage of Black households varies over the three strata, although the advantage to distinguishing the medium and low density strata is more evident. Across the three strata, the Black household eligibility rate for secondary numbers varies in an approximate 2:1.5:1 ratio. The three strata also differ in the total proportion of secondary numbers that are assigned to residences. The high density Black stratum has larger proportions of secondary numbers assigned to nonresidential units, probably reflecting the urbanization levels of the exchanges in that stratum.

Table 2
Production Study
Disposition of Numbers Selected by Stratum

Stratum and Disposition	Primaries	Secondaries
High Density Stratum		
Black Households	.072	.190
Don't Know Race	.035	.027
Non-Black Households	.219	.352
Nonresidential/Nonworking	.674	.431
Number of Cases	(3,128)	(6,671)
Medium Density Stratum		
Black Households	.032	.141
Don't Know Race	.020	.018
Non-Black Households	.188	.469
Nonresidential/Nonworking	.760	.372
Number of Cases	(1,879)	(2,375)
Low Density Stratum		
Black Households	.011	.090
Don't Know Race	.019	.023
Non-Black Households	.199	.505
Nonresidential/Nonworking	.771	.382
Number of Cases	(6,116)	(3,987)
Estimate for "Epsem Design"*		
Black Households	.021	.129
Don't Know Race	.021	.023
Non-Black Households	.200	.454
Nonresidential/Nonworking	.758	.394
Proportion Black Households		
for Disproportionate Design	.031	.150
Number of Cases	(11,123)	(13,033)

^{*} Weighted estimates of "epsem design" rates. Weights compensate for disproportionate sampling rates used to select the Production Study sample from the three density strata.

Each PSU of 200 consecutive numbers can be viewed as two half-PSUs of 100 numbers each. Table 3 demonstrates that proportions of nonresidential numbers (.378) found in the half-PSU (100-series) in which the sample primary number fell are lower than in the other half-PSU (.409), but this difference is not statistically significant at the .05 level (standard error about .02). Similarly, the proportion of Black households is somewhat larger in the 100-series of the primary number (.133) than in the adjacent 100-series (.125). Again, this difference is not likely to be found in most replications of the experiment. Table 3 provides another perspective on the results in Table 2, showing only a negligible reduction in the proportion eligible in 100-series adjacent to that of the primary numbers.

The average eligibility rate - proportion of Black households - across PSUs should not be the only criterion for evaluating the sample design. In order to implement an epsem design within strata, each PSU in the design must have a sufficient number of Black households to support the designated number of second stage sample Black households. Thus, the distribution over PSUs of the proportion eligible is also of interest. Figures 1, 2 and 3 contain histograms describing the distribution over all the PSUs of the proportion of Black households by stratum. The stability of the three distributions varies because the number of sample PSUs is about four times greater in the high density stratum than the other two (224 PSUs in the high density stratum to about 60 in the medium and low density strata). The shapes of the distributions, however, appear to be very different for the three strata. The distributions for the low and medium density strata are highly skewed, with 60 percent of PSUs in the medium density stratum and 65 percent of PSUs in the low density stratum having 5 to 20 percent Black households. These eligibility rates correspond to a maximum of 10 to 40 sample Black households for the 200-series PSUs from the low and medium density stratum. In the production study the low density stratum contained several PSUs that would not permit those cluster sizes (6 of the 63 PSUs in that stratum are estimated to have fewer than 10 Black households). The distribution in the high density stratum is much more uniform (4 of the 224 PSUs estimated to have fewer than 10 Black households).

These distributions of percentage Black households by PSU deserve more discussion. Given our current understanding of the assignment of residential numbers to available banks of numbers, there is no reason to believe that within an exchange (or a prefix) there are general tendencies to assign different residential areas to different 100-series. That is, within an exchange serving both Black and non-Black households the hypothesis of assignment of numbers without regard to the race of the subscriber is a strong one. Stated alternatively,

Table 3

Production Study
Disposition of Secondary Numbers by Whether in Same 100-Series as Primary Numbers

	Disposition	n	
Status	Same 100-Series as Primary Number	Adjacent 100-Series	
Black Households	.133	.125	
Don't Know Race	.024	.022	
Non-Black Households	.465	.444	
Nonresidential/Nonworking	.378	.409	
Number of Cases	(6,522)	(6,511)	

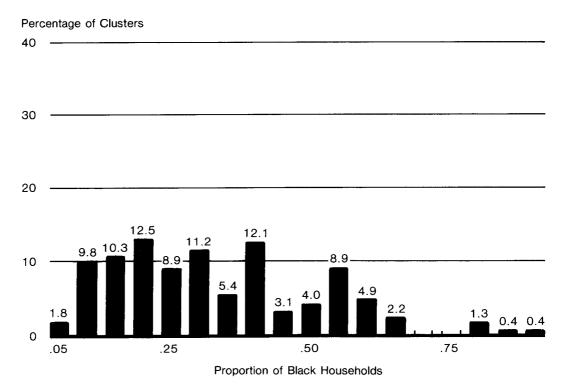


Figure 1. Percentage of High Density Clusters By Proportion of Black Households

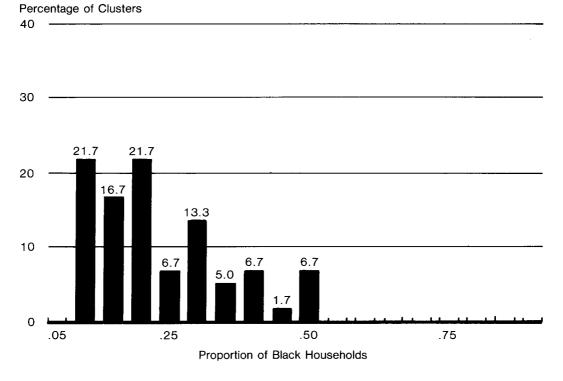


Figure 2. Percentage of Medium Density Clusters By Proportion of Black Households

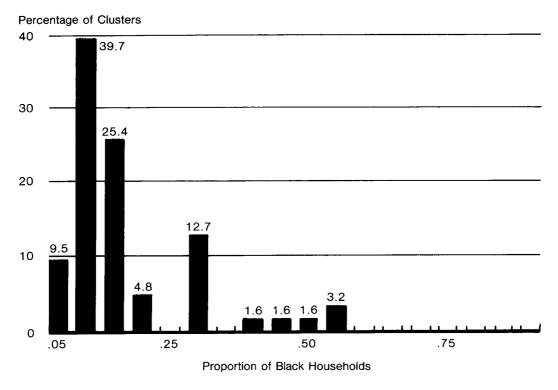


Figure 3. Percentage of Low Density Clusters By Proportion of Black Households

unless the exchanges are subdivided into wire centers that correspond to the residential locations of Black households, there is no *a priori* reason for large amounts of clustering of Black households within 200-series. Following this logic, the more uniform distribution in the high density stratum reflects, we believe, the variability in proportions of Blacks among the telephone populations in the different exchanges in the stratum.

4. SAMPLING VARIANCE PROPERTIES

To achieve greater cost-efficiency in the RDD sampling of Black households it is advantageous to use both large clusters of sample households per PSU (i.e., for a fixed sample size, a smaller number of PSUs) and disproportionate allocation of PSUs to strata of exchanges which vary in their proportion of Black telephone households. While both greater clustering and disproportionate allocation of the sample improve cost-efficiency, the overall precision of the sample is affected by the increased clustering effects and added design effects due to the non-optimal weighting that is required to compensate for the unequal selection probabilities for households from the three density strata. Increased design effects of sample estimates due to non-optimal weighting are described in Kish (1976). The clustering influence on the design effect for the modified RDD procedures is developed in the following paragraphs.

Ceteris paribus, the larger the number of sample elements chosen per PSU the higher the design effect (the ratio of the sampling variance of the given design to that of a simple random sample with the same number of elements). The model often used is $Deff = 1 + \rho(b - 1)$, where Deff is the design effect, ρ is the intracluster correlation for

the statistic, and b is the number of sample elements per PSU. Others have shown for many variables on the total U.S. household population that the intracluster correlations for the 100-series tend to be smaller than those generally found in area probability sample clusters (see Groves, 1978). This may not be the case for the Black population for 100-series, and there are no empirical estimates available concerning intracluster correlations for 200-series clusters. The expectation prior to estimating sampling errors was that there would be no change in the intracluster correlations between the 100- and 200-series. This hypothesis reflects the understanding of the assignment of telephone numbers within exchanges that was described above.

Based on sampling errors estimated from the production study data set, the average design effect for a selected set of seven survey statistics is 1.28 for the 100-series and 1.30 for the 200-series. The 100-series average design effect was estimated from those cases which fell into the 100-series of the primary number, while the cases from the entire 200-series were used in computing the average 200-series design effect. Thus, the average cluster size of completed interviews is 2.0 for the 100-series (coefficient of variation, .043) and 3.4 for the 200-series (coefficient of variation, .029). These design effects reflect all the stratification, clustering and weighting in the design and also the fact that the variability in the cluster sizes in the 100-series is greater. (The rejection rule forced an equal number of sample Black households at the 200-series but not necessarily at the 100-series level.) Given that the average design effects for the 100-series and the 200-series are close to one another (1.28 to 1.30), the dominant influence on the sampling variance appears to be non-optimal weighting required by the disproportionately allocated sample design, with little loss in precision due to PSU size alone (moving from the 100- to the 200-series clusters).

Table 4 (page 11) presents the synthetic intracluster correlations by stratum for the seven survey statistics used to compute the estimate of average design effect. The estimates of synthetic intracluster correlations were obtained from the design effect, following Kish's model of Rho = (Deff - 1)/(b - 1), and are unweighted so as to remove the confounding effect of weighting on the synthetic estimates. The estimates in the table tend to be unstable due to the small number of clusters in each stratum, the small average cluster size of completed interviews, and its associated coefficient of variation. These sample design features complicate our inference about clustering effects in the 100- versus the 200-series. Overall, the 100-series estimates of intracluster correlation are somewhat higher than those in the 200-series. We believe that this reflects more an instability in the estimated synthetic correlation than a real difference in clustering effects. We believe that these estimates provide little evidence that there is a change in the intracluster correlation between the 100- and 200-series.

5. OPTIMAL DESIGN FEATURES

The previous sections of the paper address the effect of alternative sample features on cost-efficiency and sampling variance. Survey costs and errors are often combined at the design step to address whether "optimal" features of the survey can be identified. This approach attempts to identify the design which offers minimum variance for a fixed set of resources allocated to the survey. Given the data in this research we can estimate the optimal choices of two design attributes: a) number of sample elements per PSU, and b) allocation of the sample across the three "Black-density" strata.

To determine the optimal cluster size we use a total cost model, $C = C_o + C_a a + C_b ab$, where C_o represents fixed costs, C_a is the sampling and screening cost for each sample cluster, of which a are selected, and C_b is the sampling, screening and interviewing cost

Table 4

Production Study
Synthetic Intracluster Correlations
for 100- and 200-Series Clusters for Seven Statistics by Stratum

	Synthetic Intracluster Correlation*					
Statistic	High Density Black Stratum		Medium Density Black Stratum		Low Density Black Stratum	
	100- Series	200- Series	100- Series	200- Series	100- Series	200- Series
Proportion Very Satisfied						
with Life as a Whole	.021	002	172	042	238	116
Proportion Who Think They						
Are Better Off Financially						
Than One Year Ago	.113	.075	.094	.069	.206	.049
Proportion Who Will Vote						
for Mondale	.189	.021	.086	087	436	046
Proportion Who Attend Church	.013	.017	009	078	.035	110
Proportion in Same City						
or Town All of Life	078	.001	.058	.114	.221	.248
Proportion Voted in 1980						
Presidential Election	045	035	101	013	.364	.356
Proportion Who Think Reagan						
Will Be Elected President	045	045	545	078	.124	105
Average	.024	.005	084	016	.039	.039

^{*} These estimates are unweighted.

associated with each interview obtained, of which there are b in each cluster. Because the proportions of Black households vary across the three strata in the design, the C_a and C_b parameters vary across strata (see Table 5). The optimal cluster size is computed as $\sqrt{C_a(1-\rho)/(C_b\rho)}$ (Kish, 1965). Using cost data from the production survey, Table 5 presents estimated optimal cluster sizes for overall means and proportions with three alternative levels of intracluster correlation; .005, .01, and .02. (These values are similar to those obtained for attitudinal and behavioral variables in the actual surveys.) The C_a and C_b cost estimates for each stratum also appear. The Table shows that the optimal cluster sizes are largest in the low density stratum, reflecting the high screening costs in that group. Note also that these optimal cluster sizes tend to be larger than those actually used in the survey, $\bar{b} = 6.5$.

Note further that the optimal cluster sizes are similar for 100- and 200-series PSUs and the loss of cost-efficiency of the 200-series relative to that of the 100-series is minor and similar optimal cluster sizes result. (The sampling variance estimates also imply that intracluster correlations in the 100- and 200-series clusters are similar.)

The optimal cluster sizes in Table 5 generally exceed the levels that could be supported with a 100-series PSU definition. That is, a large proportion of 100-series PSUs would not have a sufficient number of Black household numbers to fulfill the designated second stage cluster size. For that reason alone, the 200-series is favored. Even with 200-series, the specified second stage cluster sizes could not be obtained for some PSUs in the low density stratum. (This suggests the true optimal cluster size solution should be constrained to reflect the capacities of the PSUs and the approach used here is useful to guide practical decisions on cost-efficiency, but does not reflect some extreme conditions.)

Table 5

Cost Parameters and Optimal Number of Sample Elements Per Cluster, by Stratum for 100- and 200-Series Clusters and Different ρ Values

Stratum and	Optimal Cluster Size			Cost Parameters	
Cluster Definition	$\rho = .005$	$\rho = .01$	$\rho = .02$	C_{ha}	C_{hb}
	I	ligh Density S	tratum		
100	15.9	11.2	7.9	\$50.81	\$40.11
200	15.9	11.2	7.9		\$39.78
	Me	edium Density	Stratum		
100	22.4	15.8	11.1	\$114.09	\$45.18
200	21.3	15.0	10.6		\$50.00
	I	Low Density S	tratum		
100	29.8	21.0	14.8	\$309.98	\$69.52
200	29.9	21.1	14.8		\$69.18

The second design decision evaluated is the choice of sample allocation to strata. The survey used sampling fractions in the ratio of 3:2:1 from the high density to the low density stratum. We explored the optimal allocation across strata, assuming that the optimal cluster sizes were chosen in each stratum (as shown in Table 5). Given a fixed cluster size in each stratum, b_h , we set the sampling fraction in the h-th stratum, f_h , proportional to $\sqrt{(Deff_hS^2_h)/(C_{ha}/b_h)}$, where $Deff_h$ is the design effect for the statistic in the h-th stratum, S_h^2 is the element variance in the h-th stratum, C_{ha} is the sampling and screening costs for PSUs in the h-th stratum, and b_h is the number of sample elements per cluster in the h-th stratum.

Table 6 presents optimal ratios of sampling fractions for various combinations of element variances in the three strata and the various ρ values. The Table shows that the optimal allocations across strata are relatively insensitive to changes in ρ values (for the range of ρ values that are likely given this design). If the strata with higher densities of Black households have element variances at least equal to that of the low density stratum, an oversampling of those strata is desirable. (This reflects the much lower costs in those strata.) The 3:2:1 ratio of sampling fractions is best when the ratio of strata standard deviations is about 1.7:1.5:1. An examination of the data obtained from the survey suggests that many variables have ratios of standard deviations across the three strata close to 1:1:1. For such variables the optimal ratio of sampling fractions is 1.7:1.4:1, given the optimal cluster sizes shown in Table 5. (With the cluster size of 6.5 actually used in each stratum, the optimal fractions have the ratio 2.5:1.6:1.) Both these ratios of sampling fractions suggest that the oversampling actually used in the production study created a loss of precision per unit cost, relative to that corresponding to the optimal sampling fractions.

Table 6

Optimal Allocation of the Sample Across Strata for Overall Means, Given Optimal Cluster Sizes in Each Stratum, for Various Relative Standard Deviations Across Strata and Values of Intracluster Correlations

Ratios of Within Stratum Standard Deviations (High:Med:Low)	Ratios of Optimal Sampling Fractions (High:Med:Low)		
ρ =	.005		
3: 2:1 1.7:1.5:1 1: 1:1 .33: .5:1	5.2 : 2.7 : 1 3 : 2 : 1 1.7 : 1.4 : 1 .6 : .9 : 1		
ρ =	01		
3: 2:1 1.7:1.5:1 1: 1:1 .33: .5:1	5.2 : 2.7 : 1 3 : 2 : 1 1.7 : 1.4 : 1 .6 : .9 : 1		
ρ =	= .02		
3: 2:1 1.8:1.5:1 1: 1:1 .33: .5:1	5.1 : 2.7 : 1 3 : 2 : 1 1.7 : 1.3 : 1 .6 : .9 : 1		

6. SUMMARY

Rare population sampling forces the survey statistician to consider combinations of PSU and cluster definitions, stratification, and alterations of measures of size which are not typically found in cross-section samples. This research found that these traditional sample design techniques can be adapted to increase the efficiency of two-stage telephone samples for the Black household population with telephones.

First, this research found that even the rough correspondence between telephone exchanges and large cities and states permitted stratification that successfully discriminated exchange groups with vastly different eligibility rates. The high density stratum had over twice the proportion of Black households as did the low density stratum. This permits control over screening costs in sample implementation. With other rare populations which are residentially segregated, similar results are expected.

Second, the use of rejection rules based on subpopulation eligibility effectively reduced screening costs within PSUs. This increases the eligible proportion of secondary numbers from twofold to ninefold, depending on which density stratum was considered.

Third, use of a larger PSU (200-versus 100-series of consecutive numbers) produced no serious loss of eligibility. Hundred series densely filled with eligible numbers tend to be adjacent to others densely filled. This is a discovery concerning the practice of assigning numbers by telephone companies. This fact permits larger numbers of sample numbers per PSU, another key feature in reducing the costs of the Black population sample.

Despite great pressures for cost reduction in rare population samples, it is important to balance errors and costs explicitly in choosing the final design. In this research such cost

and sampling error modeling suggested that disproportionate allocation of the sample to Black-density strata is desirable. In addition, it is most efficient to select a relatively large set of secondary numbers per PSU. This set is sufficiently large that the 200- or 400-series PSU definition must be used.

Although we have applied this design only to the Black population, its performance should be similar for other residentially segregated populations. This includes income groups, certain occupational groups, and ethnic groups.

In addition, the discoveries of this research may also have implications for cross-section samples. Increasing the PSU size from 100 to 200 consecutive numbers may be advantageous in a two-stage RDD design for sampling the general telephone household population. The larger 200-series would provide twice as many numbers to select from and, as with the rare population, the proportion of eligible numbers would tend to be similar to that found in the 100-series. Therefore, given low intracluster correlation values, the cluster size of eligible numbers for a design could be set much closer to the optimal size. Because all PSUs selected would be able to support the chosen number of sample numbers, the achieved cluster size of eligible numbers should also be less variable over PSUs and therefore the impact of compensating weighting on the variance of estimates should not be great.

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