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Some Aspects of Nonresponse Adjustments

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ABSTRACT

Unit and item nonresponse almost always occur in surveys and censuses. The larger its size the larger its potential effect will be on survey estimates. It is, therefore, important to cope with it at every stage where they can be affected. At varying degrees the size of nonresponse can be coped with at design, field and processing stages. The nonresponse problems have an impact on estimation formulas for various statistics as a result of imputations and weight adjustments along with survey weights in the estimates of means, totals, or other statistics. The formulas may be decomposed into components that include response errors, the effect of weight adjustment for unit nonresponse, and the effect of substitution for nonresponse. The impacts of the design, field, and processing stages on the components of the estimates are examined.

KEY WORDS: Nonresponse; Imputation; Estimation.

1. INTRODUCTION

As survey data are gathered from sampled unit, unit and item nonresponse will occur for at least some units despite all efforts to avoid it. The problem of dealing with nonresponse and the resultant missing data is two-fold. First, the effort through callbacks, repeated mailings etc. must be determined to the extent that it is cost-effective in reducing the mean square error of survey data and second, for the remaining nonresponse, the adjustments for the missing data must be obtained in order to reduce the nonresponse bias.

The field or survey centre effort to reduce or minimize unit nonresponse often means repeated attempts to contact selected units until a responsible person is available to reply to the survey questionnaire. The attempts pertain either to personal or telephone interview. In the case of mail surveys, repeated attempts mean successive mailings of a survey questionnaire to nonresponding units. In some cases, the repeated attempts may result in telephone or personal follow-ups. Some nonresponse is inevitable although every reasonable attempt should be made to minimize its levels. Thus, there will always remain some nonrespondents for whom all the efforts to convert them seem insufficient or inappropriate. The result is some imputation procedure to account for the missing data. This paper addresses the problems of controlling nonresponse at the design and field stage, followed by an examination of nonresponse adjustments at the processing stage. The examination will consider the feasibility and the practical as well as the methodological issues pertaining to the nonresponse adjustments.

Item nonresponse is often a more complex problem to deal with than unit nonresponse which is the type mostly referred to above. The most important factors which may reduce item nonresponse are good questionnaire design and a high quality of interviewers through proper hiring and training. A poorly designed questionnaire may also result in problems of following or completing the proper sequence of questions, whether by an interviewer or in a self-interview situation. Consequently, item nonresponse may occur in a questionnaire without the interviewer or respondent being aware of it. In addition, respondents may be willing to answer some but not all questions in a survey. Whatever the reason for missing items, the problems of substituting for them remains. Usually, a survey organization is unwilling to throw out whatever information

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has been obtained unless of course the responses to major items appear very faulty or illogical. Thus, other means of imputing for missing items while maintaining the partial information on the records are usually undertaken.

Various statistics are required from a survey or census to explain social phenomena, determine socio-economic policies, etc. These include means, totals, ratios, distributions, percentiles and graphs. The statistics are assumed to be based on a universe of N units that belong to the target population; where N may or may not be known.

It may be demonstrated that all of the statistics mentioned above may be expressed in terms of totals or counts. Consequently, the remainder of the article will deal with missing data as they affect estimates of totals and counts in surveys. Some references to censuses will also be made.

2. ESTIMATION FORMULA

In the presence of unit and item nonresponse, the estimate of the total of characteristic y may be given by the general expression as in (2.1) below.

$$\tilde{Y} = \sum_{i=1}^{N} t_i \pi_i^{-1} \left\{\delta_i[\delta_{iy} y_i + (1 - \delta_{iy}) z_{iy}] + (1 - \delta_i) z_i\right\}, \text{ where} \tag{2.1}$$

t_i = 1 or 0 according as unit i is selected or not,
π_i = probability that unit i is selected.
δ_i = 1 or 0 according as unit i responds or not,
δ_{iy} = 1 or 0 according as responding unit i responds to item or characteristic y or not,
y_i = observed response for characteristic y when $\delta_{iy} = \delta_i = 1$; y_i may or may not $= Y_i$, the true value,
z_{iy} = imputed value for item nonresponse, when $\delta_i = 1$, $\delta_{iy} = 0$.
z_i = imputed value for unit nonresponse when $\delta_i = 0$.

The above estimate may pertain to a class a of units, when one inserts the indicators variable β_{ia} equal to 1 or 0 after π_i^{-1} to indicate whether or not unit i belongs to class a (e.g., age-sex class a).

In the case of item nonresponse, z_{iy} is nearly always an explicit *imputed value* for the missing information. The imputed value may be obtained by (i) a hot deck procedure i.e., substitution of an available response of characteristic y from the survey questionnaire of another unit that responded with respect to the characteristic and that is as similar as possible to unit i according to a decision table, (ii) substitution from other sources of data from the same unit such as an earlier survey, census, or administratrive data if such data are available, (iii) by regression methods or (iv) by logical deduction and the list is by no means exhaustive. In some cases, systematic errors may occur from, for example, faulty coders or keypunchers. In such cases one attempts to change the codes to logical values relative to other information on the questionnaire in place of imputation. In any case, one hopes to achieve an imputed value or altered code as close to the true value Y_i as possible. In the case of continuous surveys, with characteristics that are stable over a long period of time (such as employment in some industries and occupations), the response or earlier survey data may be considered almost as good as that of current survey data for the same unit. This would be especially when the reference periods of the current and earlier survey data are not too far apart in time. This may be also true in the case of survey data one year apart in the case of seasonal characteristics such as, for example, those related to the fishing industry. Sometimes the imputation of earlier survey data may be used also for unit nonrespondents that were respondents previously and with stable characteristics.

Usually, in the case of unit nonresponse, the imputation is undertaken by weight adjustment by the inverse response rate in a cell or area. The estimate of total is then given by:

$$\tilde{Y} = \sum_{i=1}^{N} t_i \pi_i^{-1} (wa)_i \delta_i [\delta_{iy} y_i + (1 - \delta_{iy}) z_{iy}] \tag{2.2}$$

where $(wa)_i$ = weight adjustment for unit i to compensate for the deficient sample due to unit nonresponse. In the above expression, it is assumed that all item nonresponse has already been imputed for by z_{iy} in the case of responding unit i when $\delta_{iy} = 0$.

The estimates of the cumulative distribution function from the sample in the context of potential missing data may be obtained by replacing the observed value y_i by the indicator variable $c(y_i, Y) = 1$ or 0 according as $y_i \leq$ or $> Y$ and similarly for z_{iy} and z_i. The estimated c.d.f.'s corresponding to (2.1) and (2.2) are respectively given by (2.3) and (2.4) below.

$$\tilde{F}(Y) = \frac{1}{\hat{N}} \sum_{i=1}^{N} t_i \pi_i^{-1} \{\delta_i [\delta_{iy} c(y_i, Y) + (1 - \delta_{iy}) c(z_{iy}, Y)] + (1 - \delta_i) c(z_i, Y)\} \tag{2.3}$$

where $\hat{N} = \sum_{i=1}^{N} t_i \pi_i^{-1}$ denotes the estimated or the true count of units in the universe. Thus, depending upon the frame, sample design, and listings of units, $\hat{N}$ may or may not $= N$.

$$\tilde{F}(Y) = \frac{1}{\hat{N}} \sum_{i=1}^{N} t_i \pi_i^{-1} (wa)_i \delta_i [\delta_{iy} c(y_i, Y) + (1 - \delta_{iy}) c(z_{iy}, Y)] \tag{2.4}$$

While $\tilde{Y}$, as defined in (2.1) and (2.2), is identical according as to whether imputation for unit nonresponse is regarded as a substitution of mean values of respondents or as a weight adjustment, the c.d.f. estimates, $\tilde{F}(Y)$ as defined in (2.3) and (2.4), are not identical. When the mean of respondents, either overall or in adjustment cells defined for compensation of nonresponse, is substituted for each missing value as in (2.1) or (2.3), there results a spiking of such mean values in the estimated c.d.f., not reflecting the real shape of the c.d.f. in the population. The use of the weight adjustment $(wa)_i$, to inflate the sample weight π_i^{-1} in (2.4) avoids this spiking effect, yielding a different but more realistic estimate of the c.d.f.

Under full unit and item response, the estimates (2.1) and (2.2) simplify to the Horvitz-Thompson (1952) estimate of the total, which is unbiased apart from response errors. In the presence of missing data and imputation for them, the estimates (2.1) and (2.2) however are likely to be biased for reasons other than response errors unless z_{iy}'s and z_i's tend to equal y_i's when imputation for either item or unit nonresponse is required.

In the next section, the estimates (2.1) and (2.2) are decomposed into various components due to response error, imputation error due to item nonresponse, imputation error due to unit nonresponse and the effect of weight adjustments exceeding one.

3. Components of the Estimate

The estimate $\tilde{Y}$ given by (2.1) or (2.2) may be split up into 5 components, beginning with the Horvitz-Thompson estimate using the true values of the characteristic as in Table 1. The estimated c.d.f. $\tilde{F}(Y)$ as in (2.4) may be similarly split up but will be omitted in this paper.

When the weight adjustment $(wa)_i = 1$, the last line cancels out and the first 4 lines (3.1) to (3.4) total the estimate as given by (2.1). When the unit nonresponse is compensated for by a weight adjustment $(wa)_i > 1$, there is no direct substitution z_i for the missing value

Table 1:
Components of the Estimate $\tilde{Y}$

$\tilde{Y} = \sum_{i=1}^{N} t_i \pi_i^{-1} Y_i$	..	unbiased estimate based on full response, with true values	(3.1)
$+ \sum_{i=1}^{N} t_i \pi_i^{-1} (y_i - Y_i)$	..	effect of response error	(3.2)
$+ \sum_{i=1}^{N} t_i \pi_i^{-1} \delta_i (1 - \delta_{iy})(z_{iy} - y_i)$	..	effect of item nonresponse	(3.3)
$+ \sum_{i=1}^{N} t_i \pi_i^{-1} (1 - \delta_i)(z_i - y_i)$	..	effect of unit nonresponse	(3.4)
$+ \sum_{i=1}^{N} t_i \pi_i^{-1} [(wa)_i - 1] \delta_i [\delta_{iy} y_i + (1 - \delta_{iy}) z_{iy}]$	..	effect of weight adjustment for unit nonresponse	(3.5)

and z_i is taken to be 0 in (3.4). In that case, the 5 lines total the estimate as given by (2.2) and the negative effect of unit nonresponse in (3.4) is compensated for by the positive effect of weight adjustment in (3.5).

(a) Response error

The sum of the 1st and 2nd lines of the estimate $\tilde{Y}$ (See 3.1 and 3.2) equal the desired Horvitz-Thompson estimate of total under full response. The observed response y_i for unit i may not equal the true value Y_i so that a response error at unit i level may result. The response error, which is not the real subject of this paper, can only be reduced, though not likely eliminated, by proper interviewer training, good questionnaire design with unambiguous definitions of characteristics and questions and without cluster that would confuse the interviewer and/or respondent.

When the sampled weighted response errors of (3.2) do not cancel out, the estimate of the total $\hat{Y}$ under full response, contains response error and upon taking expected value over all possible samples and response E_1 and E_3 (See Platek and Gray 1983), it may be found to be subject to response bias B_r and response variance in addition to sampling variance (SV). The response variance may be decomposed into simple (SRV) and correlated response variance (CRV) components.

The response bias, and all of the variance components (SV), (SRV) and (CRV) for the above estimate are derived in Platek and Gray (1983), subsection 2.2, pp. 257-8.

Response errors are usually studied by means of a reconciled reinterview program, whereby a subsample of responding units are reinterviewed and any observed differences between the original and reinterview data pertaining to the sample reference period are reconciled to determine which of the original or reinterview is the correct response. Reconciled reinterview surveys are undertaken in both the Canadian Labour Force Survey and the U.S. Current Population Surveys (CPS), two similar monthly surveys to measure unemployment,employment, etc.

For example, Poterba and Summers (1984), present in Table 2 some CPS results for a reconciled Reinterview Survey of May, 1976, based on a subsample of 3,329 men and 3,750 women. By means of reconciliation of a reinterviewed subsample, the *true* status of an individual is obtained so that it can be determined whether or not that individual responded correctly or not in the original survey, which in this case is CPS. Thus,the number of individuals with the true characteristics *Employed* in the reconciled interview sample who were actually reported as Employed, Unemployed, or Not in the LF in the original survey may be determined. From the three numbers, the proportion (or the probability) of correct and incorrect responses by true LF status may be estimated as in the table below.

Thus, for all of the men who were actually unemployed, 0.8720 is the estimated proportion of such men according to the reconciled reinterview study, who were accurately reported as unemployed while (0.0474 + 0.0806) or 0.1280 of the unemployed men were incorrectly reported as either *Employed* or *not in the Labour Force*. Thus, if y denotes characteristic *unemployed* i.e. $Y_i = 1$ when individual no. i is actually unemployed and a male then $y_i = 1$ correctly with probability 0.8720 while $y_i = 0$, incorrectly with probability 0.1280.

In the Canadian Labour Force Survey, the reconciled reinterview study sample during Jan.-Nov., 1984 covered 7,148 individuals and the corresponding probabilities of reporting labour force status as employed, unemployed or NILF in the regular LFS by *true* status as determined by the reinterview during 1984 are given in Table 3 below.

Thus the probability of correctly labelling an individual as unemployed, given that he/she actually unemployed is estimated to be .8691 in LFS compared with .8602 in CPS, almost

Table 2

Probabilities of Reporting Labour Force Status as Employed, Unemployed, or NILF in the Regular CPS, by *True* Status as Determined by the Reinterview Survey, May 1976.

	Status as Reported in the Regular CPS		
True Status	Employed	Unemployed	NILF
Total[1]			
Employed	0.9905	0.0016	0.0079
Unemployed	0.0356	0.8602	0.1041
NILF	0.0053	0.0025	0.9923
Men[2]			
Employed	0.9922	0.0013	0.0065
Unemployed	0.0474	0.8720	0.0806
NILF	0.0062	0.0048	0.9890
Women[3]			
Employed	0.9892	0.0019	0.0089
Unemployed	0.0194	0.8442	0.1363
NILF	0.0049	0.0015	0.9936

[1] Sampling size = 7,079
[2] Sampling size = 3,329
[3] Sampling size = 3,750

Source: Tables were computed from "General Labour Force Status in the CPS Reinterview by Labour Force Status in the Original interview.
Both Sexes. Total. After Reconciliation.
May 1976, Bureau of the Census (unpublished)

Table 3

Number of Individual and Probabilities of Reporting LF Status (in brackets) by *True* Characteristic. Jan.-Nov. 1984

True LF Characteristic (Reconciled reinterview)	Regular LFS			
	Employed	Unemployed	NILF	Total
Employed	4,082 (0.9831)	19 (0.0046)	51 (0.0123)	4,152
Unemployed	8 (0.0122)	571 (0.8691)	78 (0.1187)	657
NILF	28 (0.0120)	30 (0.0128)	2,281 (0.9752)	2,339
Total	4,118	620	2,410	7,148

the same. The corresponding probabilities for *Employed* and *Not in the Labour Force* in LFS are estimated during 1984 to be .9831 and .9752 compared with .9905 and .9923 for CPS, both somewhat lower in LFS. The reason for the difference cannot be determined at this stage. In any case, the response errors are likely more serious at national than at small area levels. For example, at national levels the response biases may be larger in magnitude relative to their sampling errors while a small area level estimate may be subject to response biases of about the same percent as at national level, but which may be much smaller than the sampling errors.

(b) Item Nonresponse and Imputation Error

The third line (3.3) of the estimate $\tilde{Y}$ in Table 1 showed the deviation from the desired estimate $\hat{Y}$ as a result of imputation for item nonresponse when the imputed value $z_{iy} \neq y_i$ and when the sampled weighted differences $(z_{iy} - y_i)$ over the sampled units with imputations for item nonresponse do not cancel out. Item nonresponse results from a respondent refusing to answer certain questions on the questionnaire may have been inadvertently left incompleted by either the respondent (in the case of self-enumeration) or by the interviewer. The second of the two causes of item nonresponse may result from similar causes as for response errors; i.e. complex questions with ambiguous definitions and/or an involved or cluttered questionnaire with a tendency for potential errors in following the proper path, depending upon replies to filter questions.

When item nonresponse does occur, an imputation strategy as described earlier may be undertaken, which almost always results in an explicit substitution. Crucial to data analysis at micro-levels is the need to obtain a value z_{iy} as close to the true value Y_i or at least as close to what would be the observed y_i , if the unit had responded to the question(s) that determine(s) characteristic y. There is unfortunately no way of knowing how close z_{iy} agrees with y_i except through re-enumeration of the unit, or a review and study of external sources or earlier survey data (which may not be available). The further danger of item nonresponse and the imputation for it may be the false sense of security to the data user who may not be aware or who may not be informed of the substituted value z_{iy} in place of a bonafide response at the micro-data level. The imputed value z_{iy} will tend to deviate in either direction from the true value Y_i to a greater extent than the potential response error y_i if that unit responds to the characteristic. This may not always be the case. Unfortunately, it usually cannot be determined at the micro-level whether or not z_{iy} is less accurate than y_i would be. Even if the imputation error may sometimes be lower than the potential response error, it may further deteriorate the quality of the published statistics because of the presence of additional variance components.

Item nonresponse and response errors are often detected in the LFS by a monthly project Field Edit Module which analyzes questionnaires that failed edit for one or more questions. The distinction between response errors and item nonresponse however is often quite blurred in the analysis without probing into the individual questionnaires in detail. The common type of discrepancy is a miscoding of a question rather than item nonresponse per se. Many questions are split up into 5 or 6 different sub-categories and a miscoding may be interpreted as an item nonresponse for one sub-category and a response error for another sub-category pertaining to the same question. The analysis of the Field Edit Module deals with items (questions) but not sub-categories of the questions. The item discrepancy rate is thus difficult to define unambiguously. It pertains to a subset of questionnaires for which a specific question, say, No. q is relevant according to filter questions and decision tables. Let us suppose that out of a responding sample size of m questionnaires, question No. q is relevant for $m_q \leq m$ questionnaires. Then the discrepancy rate is the proportion of m_q questionnaires that failed edit, whether by item nonresponse or faulty coding. The ambiguity in the definition lies in whether the subset m_q should include those questionnaires with the question completed in error, those with the question left blank in error or merely those questionnaires with the question coded correctly or incorrectly. Notwithstanding the possible ambiguity in the definition, the item discrepancy rates for about 50 items as analysed for calendar year 1984 should indicate an upper bound to the fractional error in the estimates of statistics based on the items. A sample of item (defined in Table 4a) discrepancy rates for 1984 is given in Table 4 below.

Thus, for a straightforward item like (10) "Did the respondent do any work last week? Yes or No," the discrepancy rate is only 0.2%, much lower than even the national standard error. For more complex items likes Nos. 12, 36, 41, 54 and 77 the discrepancy rate averages more than 10% with ranges 2 to 6% in either direction from the mean over the year. The discrepancies are corrected for, by hot deck procedures, use of last survey's responses (if available) or by logical deduction from other questionnaire data. Thus, in many instances an item discrepancy may be altered to a response subject to response rather than imputation error so that the discrepancy rates should be construed as an upper bound to the overall imputation error rates for the items.

(c) Unit Nonresponse and Weight Adjustment

In the case of unit nonresponse the two components of $\tilde{Y}$ given by (3.4) and (3.5) must be studied together since unit nonresponse is generally compensated for by a weight adjustment $(wa)_i$ rather than direct substitution z_i for a missing unit value. Weight adjustments are usually calculated by inverse rates in adjustment cells of which there are two basic types, balancing areas and weighting classes. Balancing areas are frequently design-dependent geographic areas such as a stratum, primary sampling unit, cluster, or a groups of strata or even the entire sample. Weighting classes are defined by post-strata (strata defined after sampling) formed on the basis of information available to both respondents and nonrespondents in the sample. The nonrespondent's information may be obtained from partial nonrespondents with some known characteristics even though the particular characteristic being estimated is not known for the partial nonrespondents. Alternatively, the information may be derived from external sources pertaining to the nonrespondents. Inverse response rates may be calculated for either balancing areas or weighting classes and used as weight adjustments to compensate for missing data in the cells.

Table 4
Average Discrepancy Rate by Item (defined in Table 4a)

Item	Average Discrepancy Rate	Range of Rates in 1984 (Min. to Max.)
10	0.2%	0.2% Every month
12	12.3%	10.4% to 14.3%
14	6.7%	5.7% to 8.4%
16	0.4%	0.3% to 0.5%
17	6.6%	2.0% to 9.9%
30	0.4%	0.3% to 0.5%
32	7.0%	3.0% to 11.6%
33	4.3%	1.8% to 6.0%
36	10.6%	8.1% to 12.7%
40	4.1%	1.5% to 6.8%
41	12.1%	6.2% to 19.7%
54	10.1%	7.9% to 12.1%
76	<0.1%	0.0% to 0.1%
77	15.0%	11.8% to 17.3%

Source: Internal report by Karen Switzer to P.D. Ghangurde March 4, 1985 "Some Findings on the Field Edit Module (FEM) Reports from 1984".

Table 4a
Definition of Items

(10)	Last week did (respondent) do any work at a job or business? Yes or No.
(12)	If yes to 11, "Did... have more than one job last week, was this a result of changing employers?" Yes or No.
(14)	What is the reason... usually works less than 30 hours per week, if actual response to (13) no. of hrs. worked 30.
(16)	Last week, how many hours was ... away from work for any reason whatsoever (holidays, vacations, illness, labour dispute, etc.) "00" should be filled in
(17)	What was the main reason for being away from work? (10 possible codes)
(30)	Last week did ... have a job or business at which he/she did not work? Yes or No.
(32)	Counting from the end of last week, in how many weeks will ... start to work at his/her new job? (Reply to Yes in (31), "Last week did ... have a job to start at a definite date in the future?")
(33)	Why was ... absent from work last week? (8 possible codes)
(36)	Identical to (14) but pertaining to *Unemployed* instead of *Employed* individuals.
(40)	Inthe past 4 weeks has ... looked for another job? Yes or No.
(41)	What has ... done in the past 4 weeks to find another job? (8 possible codes, 1 to 3 different codes in 1, 2, or 3 spaces).
(54)	What was the main reason why ... left that job? (9 possible codes) in response to yes to (50) has ... ever worked at a job or business (pert. to individuals permanently unable to work) and questions (51) to (53) dealing with date of last job and part/full time status. (54) is slipped if date of last job not too recent according to a pre-printed date in (52).
(76/77)	Class of worker and whether or not same as previous month, with respect to main job (76) and other job (77)

There are several types of weight adjustments available for inflation of the sample to compensate for unit nonresponse, the most common being the inverse response rate defined by the ratio of the sample size to the responding sample size in an adjustment cell. Thus, if the cell contains N_b units in its population and is represented by n_b selected units, where:

$n_b = \sum_{i \epsilon b} t_i$ the sample size in cell b which may or may not be a constant; depending on the definition of the cell,

$\hat{N}_b = \sum_{i \epsilon b} \pi_i t_i$, an estimate of the size of cell b in the population, usually N_b would not be known except in a census.

$m_b = \sum_{i \epsilon b} t_i \delta_i$ = no. of responding units in cell b, i.e., the responding sample size,

then, $(wa)_i = n_b / m_b$ when i lies in adjustment cell b. (3.6)

Before defining other possible weight adjustments, we will concentrate on the frequently applied inverse unweighted response rate in a cell as in (3.6). The estimate of the total defined by (2.2) with $(wa)_i = n_b / m_b$ may be rewritten as a special case of (2.1), with z_i given by:

$$z_i = \hat{T}_b / \pi_i^{-1} m_b, \tag{3.7}$$

where $\hat{T}_b = \sum_{i \epsilon b} \pi_i^{-1} t_i \delta_i [\delta_{iy} y_i + (1 - \delta_{iy}) z_{iy}]$, sample weighted total of responding units in cell b. In the case of equal sample weights in a cell, the imputed value z_i simplifies to the mean value of m_b respondents in the cell. By substituting z_i given by (3.7), into (2.1), it may be shown that the estimate is identical to (2.2) with $(wa)_i = (n_b / m_b)$. Thus, one may regard imputation for unit nonresponse as a substitution of $z_i = \hat{T}_b / (\pi_i^{-1} m_b)$ in (2.1) or as a weight adjustment to the sample weights by $(wa)_i = n_b / m_b$ in (2.2). In the case of the weight adjustment, one would set $z_i = 0$ in (3.4) in $\tilde{Y}$ as split up into 5 components. Alternatively, one may employ the imputed value z_i as defined in (2.1) and in that case, one would set $(wa)_i = 1$ in (3.5) resulting in that component of $\tilde{Y} = 0$. Thus in order to consider the effect of weight adjustment $(wa)_i > 1$, both the negative component (3.4) and positive component (3.5) must be studied together; but to consider the effect of the implicit imputed value z_i, given by (3.7), one needs only to consider (3.4).

The weight adjustment (n_b / m_b) is used in LFS, where the adjustment cells are design-dependent psu's in non-self representing areas (NSR) and strata (subunits) of contiguous city blocks in self-representing areas (SR). In Table 5, the number of cells, the unweighted average of the weight adjustments and the frequency distribution of the weight adjustment in intervals 1-1.01, 1.01-1.02. ..., 1.10 and over are given by region/type of area for the survey, Jan. 1983.

The average weight adjustment of 1.0348 at Canada level is less than what one would expect with a nonresponse rate of about 5%. The reason for the apparent low average weight adjustment is that, for purposes of calculations of the inverse response rate, some unit nonrespondents with available responding data of the previous month for imputation purposes are treated like respondents. This applies to about 20 to 30% of the nonrespondents every month.

Table 5
Number of Adjustment Cells, Average and Frequency Distribution of the Weight Adjustments by Region/Type of Area. January, 1983

Region	Type of Area	No. Cells	Aver. $(wa)_i$	No. of cells in intervals of $(wa)_i$										
				1-1.01	1.01-1.02	1.02-1.03	1.03-1.04	1.04-1.05	1.05-1.06	1.06-1.07	1.07-1.08	1.08-1.09	1.09-1.10	1.10+
Atl.	NSR	254	1.0250	143	6	22	21	13	13	9	7	8	2	10
Atl.	SR	123	1.0246	58	5	11	15	14	4	3	6	4	1	2
Que.	NSR	126	1.0550	72	2	8	10	10	6	8	6	0	1	3
Que.	SR	185	1.0265	106	0	7	8	23	11	4	5	7	3	11
Ont.	NSR	120	1.0333	58	1	10	11	11	8	4	2	2	2	11
Ont.	SR	252	1.0416	116	1	13	24	21	16	9	9	8	10	25
Pr.	NSR	328	1.0348	167	5	17	22	23	24	15	12	10	8	25
Pr.	SR	149	1.0306	40	23	23	20	13	8	7	3	5	4	3
BC	NSR	85	1.0468	38	3	7	8	8	2	5	1	1	1	11
BC.	SR	119	1.0412	46	4	7	15	10	7	7	7	3	3	10
Can.	NSR	913	1.0358	478	17	64	72	65	53	41	28	21	14	60
Can.	SR	828	1.0337	366	33	61	82	81	46	30	30	27	21	51
Canada		1,741	1.0348	844	50	125	154	146	99	71	58	48	35	111

Without a knowledge of the nonrespondents' characteristics, it cannot be determined precisely the threshold level beyond which the weight adjustment would become critical to result in an unacceptable bias along with an increase in the variance due to a smaller effective sample size. If the threshold is arbitrarily set for LFS at 1.05 (a level sometimes assumed by survey practitioners) then about 1/4 of the balancing units (441 out of 1,741) across Canada had critical weight adjustments of 1.05 or more in Jan. 1983. In many other surveys such as those dealing with income and expenditure, the nonresponse rate is higher overall and would likely be critical in nearly all cells if the same threshold of 1.05 is assumed.

There are other types of weight adjustments in cells. For example, one could exclude from cell b as defined above, those units that contain item nonresponse for at least one question. Let us suppose there are m_{bQ} units in cell b free of item nonresponse for the whole set of questions on the questionnaire. For $(m_b - m_{bQ})$ responding units in the cell with some item nonresponse the weight $(wa)_i = 1$, and for the remaining m_{bQ} responding units, free of item nonresponse, the weight adjustment is given by:

$$(wa)_i = [n_b - (m_b - m_{bQ})]/m_{bQ}, \text{ which exceeds } n_b/m_b. \qquad (3.7a)$$

The following is the justification for applying no weight adjustment i.e., $(wa)_i = 1$, for those units in the cell with some item nonresponse but a larger weight adjustment (3.7a) than (n_b/m_b), for those units free of item nonresponse Records with item nonresponse likely contain response and imputation errors while records free of item nonresponse contain only response errors and with the large weight applied to records free of item nonresponse, it may be possible to obtain estimates with lower mean square error than by using the same weight adjustment for all m_b responding units in the cell. To our knowledge, weight adjustments such as described above have not been applied but they may be worthy of study if the decrease in the bias offsets the increase in the variance that would occur with the different weights.

In the case of units with unequal probability sampling, there exists a weight adjustment based on the weighted sample and responding units in a cell instead of the unweighted ones. In such as case,

$$(wa)_i = \hat{N}_b / \hat{M}_b, \tag{3.8}$$

where $\hat{M}_b = \sum_{i \epsilon b} \pi_i^{-1} t_i \delta_i$ is the sample weighted count of responding units in cell b. For the analogous case to the weight adjustment $(wa)_i$ in (3.7a) applied only to responding units free of item nonresponse,

$$(wa)_i = [\hat{N}_b - (\hat{M}_b - \hat{M}_{bQ})] / \hat{M}_{bQ} \tag{3.9}$$

where $\hat{M}_{bQ} = \sum_{i \epsilon b} \pi_i^{-1} t_i \delta_i \Pi_{q=1}^{Q} \delta_{iq}$, the weighted count of responding units in cell b, free of item nonresponse.

$\delta_{iq} = 1$ or 0 according as unit i responded or did not respond to question no. q of the survey questionnaire containing Q questions; thus, $\Pi_{q=1}^{Q} \delta_{iq} = 1$ only if responding unit i is free of item nonresponse.

The justification for using (3.9) in lieu of (3.8) may be similar to that for using (3.7a) instead of (3.6). The justification for using weighted in place of unweighted response rates needs explanation and is provided after Table (6).

One could derive separate $(wa)_i$ expressions as of (3.7a) or (3.9) for each question q or for each characteristic y, defined by a set of one or more questions. Unfortunately, one would be faced with different weight adjustments in an adjustment cell for different questions or characteristics resulting in inconsistencies among different characteristics in published tables. In order to ensure uniform survey weights and weight adjustments, $(wa)_i$ should depend only on the unit and not on the question or characteristic though one may permit imputations for some items while excluding them for other items such as major ones in the weight adjustments (3.7a) or (3.9) as long as the inclusions and exclusions are consistent in the adjustment cell. For example, one may consider an imputation for missing item by logical deduction rather than by hot decking as pertaining to a record free of item nonresponse for weight adjustment purposes.

For each of the above weight adjustments as in (3.6) to (3.9), it can be shown that (2.2) is a particular case of (2.1) with z_i given by a weighted or unweighted mean of respondents. Thus, the implicit imputed value z_i for nonresponding unit i for each of the four cases of weight adjustments cited above is given by the expressions in Table (6). Additional notation is required for the expressions as given below:

$$\hat{T}_b = \sum_{i=1}^{N_b} t_i \pi_i^{-1} \delta_i [\delta_{iy} y_i + (1 - \delta_{iy}) z_{iy}] \tag{3.10}$$

= sample weighted total of unit respondents including imputations for item nonresponse but excluding weight adjustments by inverse unit response rate.

$$\hat{T}_{by} = \sum_{i=1}^{N_b} t_i \pi_i^{-1} \delta_i \delta_{iy} y_i$$ = sample weighted total of unit and item respondents with respect to characteristic y, (3.11)

$$\hat{T}_{bQy} = \sum_{i=1}^{N_b} t_i \pi_i^{-1} \delta_i \prod_{q=1}^{Q} \delta_{iq} y_i$$ = sample weighted total of unit and item respondents with respect to characteristic y, but excluding those records in the cell with imputation for any item nonresponse (3.12)

Thus, $\hat{T}_{bQy} \leq \hat{T}_{by} \leq \hat{T}_b$.

The weight adjustment $(n_b - m_b + m_{bQ})/m_{bQ} = 1 + (n_b - m_{bQ})/m_{bQ}$ of (c) $\geq$ the weight adjustment of (n_b / m_b) of (a) since $m_{bQ} \leq m_b$(see Table 6). Hence, for a given response rate m_b/n_b in a cell, one may anticipate a larger variance of an estimate using (c) than one using (a). The larger variance may or may not counteract a potentially smaller imputation bias in the overall mean square error. The same holds true in the case of applying weighted response rates $(\hat{N}_b - \hat{M}_b + \hat{M}_{bQ})/\hat{M}_{bQ}$ in (d) as opposed to $\hat{N}_b/\hat{M}_b$ in (b) since $\hat{M}_{bQ} \leq \hat{M}_b$. When pps sampling is applied, the use of weighted vs. unweighted response rates leads to another interesting result. It is shown in Platek and Gray (1983), p. 264-265 that, when the response and selection probabilities, i.e., α_i and π_i, are positvely correlated, the weight adjustments with weighted response rates will tend to be higher than those with unweighted rates. Thus under the condition of positive c orrelation between α_i and π_i, $E(\hat{N}_b / \hat{M}_b) > E(n_b / m_b)$ and similarly, $E[(\hat{N}_b - \hat{M}_b + \hat{M}_{bQ})/\hat{M}_{bQ}] > E[(n_b - m_b + m_{bQ}) / m_{bQ}]$, where $E = E_1 E_2$, the expected value overall possible samples of units and subsamples of responding units as described by Platek and Gray (1983), p. 251.

Table 6
Implicit Imputed Value for Unit Nonrespondent by Weight Adjustment (Cell Level)

	Weight Adjustment	Reference in text	Implicit Imputed value when $i=0$	Description
(a)	n_b/m_b	(3.6)	$\hat{T}_b/(\pi_i^{-1} m_b)$	Unweighted unit response rate
(b)	$\hat{N}_b/\hat{M}_b$	(3.8)	$\hat{T}_b/\hat{M}_b$	Weighted unit response rates
(c)	$\frac{n_b - m_b + m_{bQ}}{m_{bQ}}$	(3.7a)	$\hat{T}_{bQy} / \pi_i^{-1} m_{bQ}$	Unweighted unit response rates among units free of item nonresponse
(d)	$\frac{\hat{N}_b - \hat{M}_b + \hat{M}_{bQ}}{\hat{M}_{bQ}}$	(3.9)	$\hat{T}_{bQy} / \hat{M}_{bQ}$	Weighted unit response rates among units free of items nonresponse

Note: In the case of self-weighting sample (srswor as a particular case), the implicit imputed value z_i becomes the simple mean of respondents for both cases (a) and (b), and the simple mean of respondents (excluding those with some item nonresponse) in the cases of (c) and (d).

* See appendix I for derivation.

Whatever the weight adjustment used to compensate for unit nonresponse, it is doubtful that the individual values z_i *implicit imputed* would be close to the individual true values Y_i or even to the potential observed responses y_i. The best that can be achieved with the weight adjustment is to hope that adjustment cells formed to compensate for missing data due to unit nonresponse will ensure minimum differences between the characteristics of respondents and nonrespondents in the cells. Thus, the formation and delineation of adjustment cell is most crucial for compensation regardless of the type of weight adjustment that is applied.

7. FINAL REMARKS

As seen in the sections above, there is no ready-made solution to the missing data, whatever the types that occur. The initial strategy is to minimize the occurance of missing data to the extent possible, without incurring great cost or sacrificing the timeliness of the survey data. Every attempt should be made at the onset to prepare for some nonresponse and set up imputation strategies. If missing data occur in about the manner anticipated, then the survey data processing ought to proceed on schedule, with the appropriate substitutions or weight adjustments. Clearly, the scheduling of survey data collection, publishing, etc. can proceed in a more orderly fashion in continuous or repeated surveys than in ad hoc one-time surveys for which the survey designer may not realize, until after the fact, all the things that can go wrong such as unexpected refusals or lack of interest on the part of both interviewers and respondents.

In order to deal with the nonresponse problems it is essential to maintain a continuous study of nonresponse rates by the survey characteristic (in the case of item nonresponse), reason for nonresponse, and if possible, to extend the study to an analysis of item and unit response probabilities so that imputation biases may be estimated from the survey itself. Alternatively, model-based estimates may continue to be explored to examine the imputation bias and, furthermore, to strengthen the estimates by employing additional information.

APPENDIX

Derivation of Implicit Value z_i for Unit Nonresponse imputation

In the case of (c) and (d) of Table 6, the estimate of cell b level is given by:

$$\tilde{Y}_b = \hat{T}_{bQy}\,(wa)_i + (\hat{T}_b - \hat{T}_{bQy}) \tag{A.1}$$

$$= \hat{T}_b + [(wa)_i - 1]\,\hat{T}_{bQy}$$

In case (c), $(wa)_i - 1 = (n_b - m_b)/m_{bQ}$

$$= \sum_i t_i(1 - \delta_i)/m_{bQ}$$

or $\tilde{Y}_b = \hat{T}_b + \sum_i t_i\pi_i^{-1}(1 - \delta_i)\hat{T}_{bQy}/\pi_i^{-1}m_{bQ}$ (A.2) or by equating (A.2) to (A.1), noting the definitions of $\hat{T}_b$ in (3.10) and $\tilde{Y}$ in (2.1), one may see that the imputed value z_i is given by $\hat{T}_{bQy} / \pi_i^{-1} m_{bQ}$ as stated in (c) of Table (6).

Similarly, when weighted response rates are employed, the implicit imputed value z_i may be found to be $\hat{T}_{bQy} / \hat{M}_{bQ}$ as in (d) of Table (6). The results for (a) and (b) of Table (6) follow by setting $m_{bQ} = m_b$ and $\hat{M}_{bQ} = \hat{M}_b$.

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