

CONSTRUCTION OF WORKING PROBABILITIES AND
JOINT SELECTION PROBABILITIES FOR FELLEGI'S
PPS SAMPLING SCHEME

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A FORTRAN Subroutine to obtain the "working probabilities" for Fellegi's (1963) method of unequal probability sampling is given. The solution is obtained by an iterative procedure where the starting values for the $(k+1)$ th draw "working probabilities" are the solutions for the k th draw "working probabilities" and the iterative procedure is terminated when a prespecified accuracy is achieved. The limitation is that the Subroutine can only be used to obtain upto and including the 5th draw "working probabilities". It was observed that the convergence occurs very fast in double precision. Therefore all real variables have been declared as double precision. The joint selection probabilities Π_{ij} 's i.e. the probability that both the i th and j th units are in the sample are obtained by summing the probabilities of selecting those samples that contain both the i th and j th units. The joint selection probabilities are required for the variance estimation of the Horvitz-Thompson estimator of population total of the characteristic of interest.

1. DESCRIPTION

Fellegi (1963) has proposed a method for selecting a sample of n (≥ 2) units draw by draw and without replacement out of N units in such a way that the probability for the i -th unit to be selected is equal to p_i at each of the n successive draws ($\sum_{i=1}^N p_i = 1$). This is achieved by determining $(n-1)$ sets of selection probabilities referred to as "working probabilities". Let the $(n-1)$ sets of "working probabilities" be

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$$p_i(k) = 0, i = 1, 2, \dots, N; k=2, 3, \dots, n$$

$$\sum_{i=1}^N p_i(k) = 1, k = 2, 3, \dots, n.$$

The $p_i(k)$, $i = 2, 3, \dots, N$ are the "working probabilities" for selecting a unit at the k -th draw. The selection probabilities at the first draw $p_i(1)$ are given by

$$p_i(1) = p_i, \quad i = 1, 2, \dots, N.$$

Then the overall (unconditional) probability $\delta_i(k)$ of selecting i -th unit at the k -th draw is given by

$$\delta_i(k) = \sum_{(k-1; i)} [p_{i_1}(1) \times \frac{p_{i_2}(2)}{1-p_{i_1}(2)} \times \dots \times \frac{p_{i_{k-1}}(k-1)}{1-p_{i_1}(k-1)-p_{i_2}(k-1)-\dots-p_{i_{k-2}}(k-1)} \\ \times \frac{p_i(k)}{1-p_{i_1}(k)-p_{i_2}(k)-\dots-p_{i_{k-1}}(k)}]$$

$$i = 1, 2, \dots, N;$$

$$k = 1, 2, \dots, n$$

where $\sum_{(k-1; i)}$ denotes the summation over all possible ordered $(k-1)$ -tuples of $(i_1, i_2, \dots, i_{k-1})$ such that i_1, i_2, \dots, i_{k-1} are different integers between 1 and N , and none of them is equal to i . The condition that the i -th unit be selected with probability p_i at each of the n successive draws is satisfied by setting

$$\delta_i(k) = p_i, \quad i = 1, 2, \dots, N; \quad k = 1, 2, \dots, n.$$

We have $p_i(1) = p_i$, $i=1, 2, \dots, N$. Given that $p_i(2), \dots, p_i(k-1)$ have already been found, then approximate $p_i^{(0)}(k)$ by $p_i(k-1)$ and obtain $p_i^{(1)}(k)$ from the following formula

$$p_i^{(m)}(k) = p_i \times \left\{ \frac{\sum_{i=1}^{k-1} [p_i(1) \times \frac{p_i(2)}{1-p_i(2)} \times \dots \times \frac{p_i(k-1)}{1-p_i(1)-p_i(2)-\dots-p_i(k-1)}]}{1-p_i^{(m-1)}(k)-p_i^{(m-1)}(k)-\dots-p_i^{(m-1)}(k)} \right\}^{-1}$$

by setting $m = 1$ for $i = 1, 2, \dots, N$. Repeat for $m = 2, 3, \dots$, etc. until $p_i^{(m)}(k) = p_i^{(m-1)}(k)$ for all i up to the required number of decimal places. The procedure is carried out for $k = 2, 3, \dots, n$, thus obtaining the $(n-1)$ sets of "working probabilities" $p_i(2), p_i(3), \dots, p_i(n)$. Since i -th unit is selected with probability equal to p_i at each of the n successive draw, this property of the scheme makes it very attractive for rotating sample designs.

Bayless and Rao (1977) excluded Felleig's (1963) method from their study for $n=4$ due to convergence problems with the routine they used for obtaining the "working probabilities". They were not getting satisfactory answers even after a large number of iterations especially when $c.v.(x)^*$ was not small, where x -values are the sizes of the units in the population.

$$* c.v.(x) = \frac{\frac{N}{\sum_{i=1}^N x_i^2} - (\frac{\sum_{i=1}^N x_i}{N})^2 / (N-1)}{\frac{N}{\sum_{i=1}^N x_i} / N}$$

We have used Fellegi's (1963) example for which C.V. (x) is small and two populations [Cochran (1978) and Kish (1965)] with larger values for C.V.(x) to obtain the "working probabilities" for selecting upto 4 units. The iterative procedure was terminated when the change between two successive iterations was less than 10^{-6} for each element of the solution vector. The description of the populations and the number of iterations required to obtain the "working probabilities" at each of the draws is given below:

Pop. No.	Source	N	C.V. (x)	No. of iterations at draw		
				2	3	4
1	Fellegi [1963, p. 198]	6	0.25	5	7	12
2	Cochran [1978, p. 152]	20	1.03	4	5	7
3	Kish [1965, p. 42]	20	1.19	4	6	8

It is noticed that for the three populations we have used, the convergence at each of the draws is obtained in a very few number of iterations although the number of iterations required at each successive draw increases. It should be remarked that the values of "working probabilities" obtained for Fellegi's (1963) example agree with his values.

The joint selection probabilities are required for estimating the variance of the Horvitz-Thompson estimator

$$\hat{Y} = \frac{1}{n} \sum_{i \in s} \frac{y_i}{p_i}$$

of the total $Y = \sum_{i=1}^N y_i$ of y - variate of interest, where y_i is the value of y - variate pertaining to the i -th unit. Let π_{ij} denote the probability that both the i -th and j -th units are included in the sample, then π_{ij} , $i=1, 2, \dots, N-1; j = i+1, i+2, \dots, N$ can be obtained as follows:

Let $\delta_{ij}(k, \ell)$ denote the probability that the i -th unit was selected at the k -th draw and the j -th unit was selected at the ℓ -th draw ($\ell > k$). The probability $\delta_{ij}(k, \ell)$ is given by:

$$\delta_{ij}(k, \ell) = \sum_{(l-2; i, j)} [p_{i1}(1) \times \frac{p_{i2}(2)}{1-p_{i1}(2)} \times \dots \times \frac{p_{ik-1}(k-1)}{1-p_{i1}(k-1)-p_{i2}(k-1)-\dots-p_{ik-1}(k-1)} \times$$

$$\frac{p_i(k)}{1-p_{i1}(k)-p_{i2}(k)-\dots-p_{ik-1}(k)} \times \frac{p_{ik+1}(k+1)}{1-p_{i1}(k+1)-p_{i2}(k+1)-\dots-p_{ik-1}(k+1)-p_i(k+1)} \times$$

$$\dots \times \frac{p_{i\ell-1}(\ell-1)}{1-p_{i1}(\ell-1)-p_{i2}(\ell-1)-\dots-p_{ik-1}(\ell-1)-p_i(\ell-1)-p_{ik+1}(\ell-1)-\dots-p_{i\ell-2}(\ell-1)} \times$$

$$\frac{p_j(\ell)}{1-p_{i1}(\ell)-p_{i2}(\ell)-\dots-p_{ik-1}(\ell)-p_i(\ell)-p_{ik+1}(\ell)-\dots-p_{i\ell-1}(\ell)}],$$

$$i \neq j = 1, 2, \dots, N;$$

$$k = 1, 2, \dots, n-1;$$

$$\ell = k+1, k+2, \dots, n$$

where $\sum_{(\ell-2; i, j)}$ denotes the summation over all possible ordered $(\ell-2)$ -tuples of $(i_1, i_2, \dots, i_{k-1}, i_{k+1}, \dots, i_{\ell-1})$ such that $i_1, i_2, \dots, i_{k-1}, i_{k+1}, \dots, i_{\ell-1}$ are different integers between 1 and N, and none of them is equal to i or j. Then π_{ij} , the probability that both i-th and j-th units are included in the sample, is given by

$$\pi_{ij} = \sum_{k=1}^{n-1} \sum_{\ell=k+1}^n [\delta_{ij}(k, \ell) + \delta_{ji}(k, \ell)],$$

i = 1, 2, ..., N-1;

j = i+1, i+2, ..., N.

Structure

SUBROUTINE WKPROB (N, NS, MA, P, P1, P2, P3, P4, Q1, Q2, DEL,
MAX, ACC, PI, TOL, IFAULT)

Formal parameters - all real parameters in double precision.

N Integer Input: number of units in the population
NS Integer Input: sample size, $2 \leq NS \leq 5$
MA Integer Input: dimension of PI in the calling program
P Real Array(N) Input: contains the relative measure of sizes
of units in the sequence P_1, P_2, \dots, P_N ;

$$\sum_{i=1}^N p_i = 1$$

P1 Real Array (N) Output: working probabilities for selecting a unit
at the 2nd draw
P2 Real Array (N) Output: working probabilities for selecting a unit
at the 3rd draw
P3 Real Array (N) Output: working probabilities for selecting a unit
at the 4th draw

P4	Real Array(N)	Output: working probabilities for selecting a unit at the 5th draw.
Q1	Real Array(N)	Workspace
Q2	Real Array(N)	Workspace
DEL	Real Array (MA,NS)	Workspace
MAX	Integer	Input: maximum number of interations allowed for obtaining each set of working probabilities
ACS	Real	Input: desired accuracy of the working probabilities
P1	Real Array (MA,MA)	Output: matrix returning the joint selection probabilities π_{ij} , $i = 1, 2, \dots, N-1$ $j = i + 1, i + 2, \dots N$
TOL	Real	Input: maximum allowed value for the absolute difference between $\sum_{i=1}^N p_i$ and the number one
IFault	Integer	Output: failure indicator

Failure Indications

IFault = 0 normal termination
= 1 one or more of $p_i > (1/NS)$
= 2 $DABS(\sum_{i=1}^N p_i - 1.0) > TOL$
= 3 both conditions 1 and 2 occur
= 4 sample size greater than 5
= 5 desired accuracy was not obtained in maximum allowed number of interations

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RESUME

L'auteur expose un sous-programme FORTRAN visant à obtenir les "probabilités de travail" à l'aide de la méthode d'échantillonnage à probabilités inégales de Fellegi (1963). On obtient la solution par une méthode itérative dans laquelle les valeurs de départ des "probabilités de travail" du $(k-1)$ -ième tirage sont la solution du k -ième tirage des "probabilités de travail"; ce calcul prend fin lorsque l'on atteint un niveau de précision déterminé à l'avance. Le sous-programme est limité car son utilisation ne peut dépasser le 5^e tirage des "probabilités de travail". On a observé que la convergence se produit très rapidement en double précision. Par conséquent, toutes les variables réelles ont été déclarées en double précision. Les probabilités conjointes de sélection, c.-à-d. la probabilité que les i -ième et j -ième unités fassent toutes deux partie de l'échantillon, s'obtiennent par sommation des probabilités de sélection des échantillons contenant les deux unités en cause. Les probabilités conjointes de sélection sont nécessaires à l'estimation de la variance de l'estimateur Horvitz-Thompson du total de la caractéristique à l'étude dans la population.

REFERENCES

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C.....SUBROUTINE TO OBTAIN WORKING PROBABILITIES AND 00001000
C.....JOINT SELECTION PROBABILITIES FOR FELLEGI'S PPS 00002000
C.....SAMPLING SCHEME. REF: 1963 JASA 58 , PP 183-201 . 00003000
C..... 00004000
C.....
1 SUBROUTINE WKPROB(N,NS,MA,P,P1,P2,P3,P4,Q1,Q2,DEL,MAX,ACC, 00005000
PI,TOL,IFault) 00006000
IMPLICIT REAL*8(A-H,O-Z) 00007000
DIMENSION P(N),P1(N),P2(N),P3(N),P4(N),Q1(N),Q2(N),DEL(MA,NS), 00008000
1 PI(MA,MA) 00009000
C.....N IS POPULATION SIZE. 00010000
C.....NS IS SAMPLE SIZE AND CAN HAVE VALUES 2 , 3 , 4 , 5 . 00011000
C.....MA IS MAXIMUM DIMENSION OF PI IN THE MAIN PROGRAM. 00012000
C.....P IS THE VECTOR OF GIVEN PROBABILITIES . SUM P = 1.0 00013000
C.....P , P1 , P2 , P3 , P4 AND P5 ARE SELECTION PROBABILITIES AT 00014000
C.....1ST , 2ND , 3RD , 4-TH AND 5-TH DRAWS RESPECTIVELY . 00015000
C.....Q1 , Q2 , DEL WORK SPACE . 00016000
CONTINUE 00017000
C.....MAX IS THE MAXIMUM NUMBER OF ITERATIONS ALLOWED TO OBTAIN 00018000
C.....THE WORKING PROBABILITIES. 00019000
C.....ACC IS THE DESIRED ACCURACY OF THE WORKING PROBABILITIES. 00020000
C.....PI IS THE OUTPUT RETURNING THE JOINT SELECTION PROBABILITIES . 00021000
C.....TOL IS THE PARAMETER SO THAT SUM P CANNOT DEVIATE FROM 1.0 BY 00022000
C.....MORE THAN THE VALUE OF TOL. 00023000
C.....IFault IS FAILURE INDICATOR TAKING THE FOLLOWING VALUES: 00024000
C..... 00025000
C..... 0 IF PI COMPUTED, NORMAL TERMINATION. 00026000
C..... 1 IF NS*P .GE.1.0 FOR ONE OR MORE P VALUES . 00027000
C..... 2 IF DABS(SUM P - 1.0) IS GREATER THAN TOL. 00028000
C..... 3 IF BOTH OF THE ABOVE TWO CONDITIONS. 00029000
C..... 4 IF NS , THE SAMPLE SIZE, IS GREATER THAN 5 . 00030000
C..... 5 IF DESIRED ACCURACY NOT OBTAINED IN MAXIMUM 00031000
C..... ALLOWED NUMBER OF ITERATIONS. 00032000
C..... 00033000
C.....
1 IFAULT=4 00034000
IF(NS.GT.5) RETURN 00035000
IFAUL1=0 00036000
IFAUL2=0 00037000
IDRAW=1 00038000
XNS=NS 00039000
SUMP=0.0 00040000
DO 1 I=1,N 00041000
SUMP=SUMP+P(I) 00042000
Q1(I)=P(I) 00043000
DEL(I,1)=P(I) 00044000
IF(XNS*P(I).GT.1.0) IFAUL1=1 00045000
1 CONTINUE 00046000
IF(DABS(SUMP-1.0).GT.TOL) IFAUL2=2 00047000
IFault=IFAUL1+IFAUL2 00048000
IF(IFault.NE.0) RETURN 00049000
C..... 00050000

C.....SELECTING UNIT 2 . 00051000
C..... 00052000
IDRAW=IDRAW+1 00053000
A=0.0 00054000
DO 20 J=1,N 00055000
20 A=A+F0(N,J,P,Q1) 00056000
ICOUNT=0 00057000
21 ICOUNT=ICOUNT+1 00058000
IF(ICOUNT.GT.MAX) GO TO 999 00059000
DMAX=0.0 00060000
DO 22 I=1,N 00061000
DEN=A-F0(N,I,P,Q1) 00062000
Q2(I)=P(I)/DEN 00063000
DIFF=DABS(Q2(I)-Q1(I)) 00064000
IF(DIFF.GT.DMAX) DMAX=DIFF 00065000
Q1(I)=Q2(I) 00066000
A=DEN+F0(N,I,P,Q1) 00067000
22 CONTINUE 00068000
IF(DMAX.GT.ACC) GO TO 21 00069000
WRITE(3,24) IDRAW,ICOUNT 00070000
24 FORMAT(1H1,///,20X,'WORKING PROBABILITIES AT DRAW : ',I5, 00071000
1 ///,20X,'NUMBER OF ITERATIONS FOR CONVERGENCE = ',I6, 00072000
2 ///) 00073000
DO 25 I=1,N 00074000
P1(I)=Q1(I) 00075000
DEL(I,2)=P1(I) 00076000
WRITE(3,26) I,P1(I) 00077000
25 CONTINUE 00078000
26 FORMAT(1H0,20X,' PROB (',I2,') = ',D14.6) 00079000
IF(IDRAW.EQ.NS) GO TO 550 00080000
C..... 00081000
C.....SELECTING UNIT 3. 00082000
C..... 00083000
IDRAW=IDRAW+1 00084000
A=0.0 00085000
DO 30 J=1,N 00086000
DO 30 K=1,N 00087000
30 A=A+F1(N,J,K,P,P1,Q1) 00088000
ICOUNT=0 00089000
37 ICOUNT=ICOUNT+1 00090000
IF(ICOUNT.GT.MAX) GO TO 999 00091000
DMAX=0.0 00092000
DO 31 I=1,N 00093000
S1=0.0 00094000
DO 32 J=1,N 00095000
S1=S1+F1(N,I,J,P,P1,Q1)+F1(N,J,J,P,P1,Q1)+F1(N,J,I,P,P1,Q1) 00096000
32 CONTINUE 00097000
DEN=A-S1+2.0*F1(N,I,I,P,P1,Q1) 00098000
Q2(I)=P(I)/DEN 00099000
DIFF=DABS(Q2(I)-Q1(I)) 00100000

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IF(DIFF.GT.DMAX) DMAX=DIFF          00101000
Q1(I)=Q2(I)                         00102000
S1=0.0                               00103000
DO 33 J=1,N                          00104000
S1=S1+F1(N,I,J,P,P1,Q1)+F1(N,J,J,P,P1,Q1)+F1(N,J,I,P,P1,Q1) 00105000
33 CONTINUE                           00106000
A=DEN+S1-2.0*F1(N,I,I,P,P1,Q1)     00107000
31 CONTINUE                           00108000
IF(DMAX.GT.ACC) GO TO 37            00109000
WRITE(3,24) IDRAW,ICOUNT             00110000
DO 36 I=1,N                          00111000
P2(I)=Q1(I)                         00112000
DEL(I,3)=P2(I)                      00113000
WRITE(3,26) I,P2(I)                 00114000
36 CONTINUE                           00115000
IF(IDRAW.EQ.NS) GO TO 550          00116000
C.....                                00117000
C.....SELECTING UNIT 4.              00118000
C.....                                00119000
IDRAW=IDRAW+1                        00120000
A=0.0                                 00121000
DO 40 J=1,N                          00122000
DO 40 K=1,N                          00123000
DO 40 L=1,N                          00124000
40 A=A+F2(N,J,K,L,P,P1,P2,Q1)      00125000
ICOUNT=0                             00126000
49 ICOUNT=ICOUNT+1                   00127000
IF(ICOUNT.GT.MAX) GO TO 999         00128000
DMAX=0.0                             00129000
DO 41 I=1,N                          00130000
S1=0.0                               00131000
S2=0.0                               00132000
DO 42 J=1,N                          00133000
DO 43 K=1,N                          00134000
S1=S1+F2(N,I,J,K,P,P1,P2,Q1)+F2(N,J,J,K,P,P1,P2,Q1)        00135000
1      +F2(N,J,I,K,P,P1,P2,Q1)+F2(N,J,K,I,P,P1,P2,Q1)        00136000
2      +F2(N,J,K,J,P,P1,P2,Q1)+F2(N,J,K,K,P,P1,P2,Q1)        00137000
43 CONTINUE                           00138000
S2=S2+2.0*F2(N,I,I,J,P,P1,P2,Q1)+F2(N,J,J,I,P,P1,P2,Q1)    00139000
1      +2.0*F2(N,I,J,I,P,P1,P2,Q1)+F2(N,J,I,J,P,P1,P2,Q1)    00140000
2      +2.0*F2(N,J,I,I,P,P1,P2,Q1)+F2(N,I,J,J,P,P1,P2,Q1)    00141000
3      +2.0*F2(N,J,J,J,P,P1,P2,Q1)                                00142000
42 CONTINUE                           00143000
DEN=A-S1+S2-6.0*F2(N,I,I,I,P,P1,P2,Q1) 00144000
Q2(I)=P(I)/DEN                      00145000
DIFF=DABS(Q2(I)-Q1(I))               00146000
IF(DIFF.GT.DMAX) DMAX=DIFF          00147000
Q1(I)=Q2(I)                         00148000
S1=0.0                               00149000
S2=0.0                               00150000
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DO 44 J=1,N                               00151000
DO 45 K=1,N                               00152000
S1=S1+F2(N,I,J,K,P,P1,P2,Q1)+F2(N,J,J,K,P,P1,P2,Q1) 00153000
1      +F2(N,J,I,K,P,P1,P2,Q1)+F2(N,J,K,I,P,P1,P2,Q1) 00154000
2      +F2(N,J,K,J,P,P1,P2,Q1)+F2(N,J,K,K,P,P1,P2,Q1) 00155000
45 CONTINUE                                00156000
S2=S2+2.0*F2(N,I,I,J,P,P1,P2,Q1)+F2(N,J,J,I,P,P1,P2,Q1) 00157000
1      +2.0*F2(N,I,J,I,P,P1,P2,Q1)+F2(N,J,I,J,P,P1,P2,Q1) 00158000
2      +2.0*F2(N,J,I,I,P,P1,P2,Q1)+F2(N,I,J,J,P,P1,P2,Q1) 00159000
3      +2.0*F2(N,J,J,J,P,P1,P2,Q1)                00160000
44 CONTINUE                                00161000
A=DEN+S1-S2+6.0*F2(N,I,I,I,P,P1,P2,Q1)    00162000
41 CONTINUE                                00163000
IF(DMAX.GT.ACC) GO TO 49                  00164000
WRITE(3,24) IDRAW,ICOUNT                 00165000
DO 47 I=1,N                               00166000
P3(I)=Q1(I)                             00167000
DEL(I,4)=P3(I)                           00168000
WRITE(3,26) I,P3(I)                      00169000
47 CONTINUE                                00170000
IF(IDRAW.EQ.NS) GO TO 550               00171000
C.....                                     00172000
C.....SELECTING UNIT 5.                  00173000
C.....                                     00174000
IDRAW=IDRAW+1                            00175000
A=0.0                                     00176000
DO 50 J=1,N                               00177000
DO 50 K=1,N                               00178000
DO 50 L=1,N                               00179000
DO 50 M=1,N                               00180000
50 A=A+F3(N,J,K,L,M,P,P1,P2,P3,Q1)    00181000
ICOUNT=0                                  00182000
59 ICOUNT=ICOUNT+1                        00183000
IF(ICOUNT.GT.MAX) GO TO 999             00184000
DMAX=0.0                                 00185000
DO 51 I=1,N                               00186000
S1=0.0                                     00187000
S2=0.0                                     00188000
S3=0.0                                     00189000
DO 52 J=1,N                               00190000
DO 53 K=1,N                               00191000
DO 54 L=1,N                               00192000
S1=S1+F3(N,J,K,I,L,P,P1,P2,P3,Q1)+F3(N,J,K,J,L,P,P1,P2,P3,Q1) 00193000
1      +F3(N,J,K,K,L,P,P1,P2,P3,Q1)+F3(N,I,J,K,L,P,P1,P2,P3,Q1) 00194000
2      +F3(N,J,J,K,L,P,P1,P2,P3,Q1)+F3(N,J,I,K,L,P,P1,P2,P3,Q1) 00195000
3      +F3(N,J,K,L,I,P,P1,P2,P3,Q1)+F3(N,J,K,L,J,P,P1,P2,P3,Q1) 00196000
4      +F3(N,J,K,L,K,P,P1,P2,P3,Q1)+F3(N,J,K,L,L,P,P1,P2,P3,Q1) 00197000
54 CONTINUE                                00198000
S2=S2+2.0*F3(N,I,J,I,K,P,P1,P2,P3,Q1)+F3(N,J,J,I,K,P,P1,P2,P3,Q1) 00199000
1      +2.0*F3(N,J,I,I,K,P,P1,P2,P3,Q1)+F3(N,J,I,J,K,P,P1,P2,P3,Q1) 00200000
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2   +2.0*F3(N,J,J,J,K,P,P1,P2,P3,Q1)+F3(N,I,J,J,K,P,P1,P2,P3,Q1) 00201000
3   +2.0*F3(N,I,I,J,K,P,P1,P2,P3,Q1)+F3(N,J,K,J,I,P,P1,P2,P3,Q1) 00202000
4   +2.0*F3(N,J,K,I,I,P,P1,P2,P3,Q1)+F3(N,J,K,K,I,P,P1,P2,P3,Q1) 00203000
5   +2.0*F3(N,I,J,K,I,P,P1,P2,P3,Q1)+F3(N,J,J,K,I,P,P1,P2,P3,Q1) 00204000
6   +2.0*F3(N,J,I,K,I,P,P1,P2,P3,Q1)+F3(N,J,K,I,J,P,P1,P2,P3,Q1) 00205000
7   +2.0*F3(N,J,K,J,J,P,P1,P2,P3,Q1)+F3(N,J,K,K,J,P,P1,P2,P3,Q1) 00206000
8   +2.0*F3(N,J,J,K,J,P,P1,P2,P3,Q1)+F3(N,J,I,K,J,P,P1,P2,P3,Q1) 00207000
9   +   F3(N,J,K,I,K,P,P1,P2,P3,Q1)+F3(N,J,K,J,K,P,P1,P2,P3,Q1) 00208000
A   +2.0*F3(N,J,K,K,K,P,P1,P2,P3,Q1)+F3(N,I,J,K,J,P,P1,P2,P3,Q1) 00209000
B   +   F3(N,I,J,K,K,P,P1,P2,P3,Q1)+F3(N,J,J,K,K,P,P1,P2,P3,Q1) 00210000
C   +   F3(N,J,I,K,K,P,P1,P2,P3,Q1)                                00211000
53 CONTINUE
      S3=S3+6.0*F3(N,I,I,I,J,P,P1,P2,P3,Q1)                      00212000
1   +6.0*F3(N,I,J,I,I,P,P1,P2,P3,Q1)                            00213000
2   +2.0*F3(N,J,J,I,I,P,P1,P2,P3,Q1)                            00214000
3   +6.0*F3(N,J,I,I,I,P,P1,P2,P3,Q1)                            00215000
4   +2.0*F3(N,J,J,J,I,P,P1,P2,P3,Q1)                            00216000
5   +2.0*F3(N,J,I,J,I,P,P1,P2,P3,Q1)                            00217000
6   +2.0*F3(N,I,J,J,I,P,P1,P2,P3,Q1)                            00218000
7   +6.0*F3(N,I,I,J,I,P,P1,P2,P3,Q1)                            00219000
8   +2.0*F3(N,J,J,I,J,P,P1,P2,P3,Q1)                            00220000
9   +2.0*F3(N,J,I,I,J,P,P1,P2,P3,Q1)                            00221000
A   +6.0*F3(N,J,J,J,J,P,P1,P2,P3,Q1)                            00222000
B   +2.0*F3(N,J,I,J,J,P,P1,P2,P3,Q1)                            00223000
C   +2.0*F3(N,I,J,I,J,P,P1,P2,P3,Q1)                            00224000
D   +2.0*F3(N,I,J,J,J,P,P1,P2,P3,Q1)                            00225000
E   +2.0*F3(N,I,I,J,J,P,P1,P2,P3,Q1)                            00226000
      00227000
52 CONTINUE
      DEN=A-S1+S2-S3+24.0*F3(N,I,I,I,I,P,P1,P2,P3,Q1)          00228000
      Q2(I)=P(I)/DEN                                         00229000
      DIFF=DABS(Q2(I)-Q1(I))                                00230000
      IF(DIFF.GT.DMAX) DMAX=DIFF                           00231000
      Q1(I)=Q2(I)                                         00232000
      S1=0.0                                           00233000
      S2=0.0                                           00234000
      S3=0.0                                           00235000
      DO 55 J=1,N                                         00236000
      DO 56 K=1,N                                         00237000
      DO 57 L=1,N                                         00238000
      S1=S1+F3(N,J,K,I,L,P,P1,P2,P3,Q1)+F3(N,J,K,J,L,P,P1,P2,P3,Q1) 00239000
1   +F3(N,J,K,K,L,P,P1,P2,P3,Q1)+F3(N,I,J,K,L,P,P1,P2,P3,Q1) 00240000
2   +F3(N,J,J,K,L,P,P1,P2,P3,Q1)+F3(N,J,I,K,L,P,P1,P2,P3,Q1) 00241000
3   +F3(N,J,K,L,I,P,P1,P2,P3,Q1)+F3(N,J,K,L,J,P,P1,P2,P3,Q1) 00242000
4   +F3(N,J,K,L,K,P,P1,P2,P3,Q1)+F3(N,J,K,L,P,P1,P2,P3,Q1) 00243000
      00244000
57 CONTINUE
      S2=S2+2.0*F3(N,I,J,I,K,P,P1,P2,P3,Q1)+F3(N,J,I,K,P,P1,P2,P3,Q1) 00245000
1   +2.0*F3(N,J,I,I,K,P,P1,P2,P3,Q1)+F3(N,J,I,J,K,P,P1,P2,P3,Q1) 00246000
2   +2.0*F3(N,J,J,J,K,P,P1,P2,P3,Q1)+F3(N,I,J,J,K,P,P1,P2,P3,Q1) 00247000
3   +2.0*F3(N,I,I,J,K,P,P1,P2,P3,Q1)+F3(N,J,K,J,I,P,P1,P2,P3,Q1) 00248000
4   +2.0*F3(N,J,K,I,I,P,P1,P2,P3,Q1)+F3(N,J,K,K,I,P,P1,P2,P3,Q1) 00249000
      00250000

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5      +2.0*F3(N,I,J,K,I,P,P1,P2,P3,Q1)+F3(N,J,J,K,I,P,P1,P2,P3,Q1) 00251000
6      +2.0*F3(N,J,I,K,I,P,P1,P2,P3,Q1)+F3(N,J,K,I,J,P,P1,P2,P3,Q1) 00252000
7      +2.0*F3(N,J,K,J,J,P,P1,P2,P3,Q1)+F3(N,J,K,K,J,P,P1,P2,P3,Q1) 00253000
8      +2.0*F3(N,J,J,K,J,P,P1,P2,P3,Q1)+F3(N,J,I,K,J,P,P1,P2,P3,Q1) 00254000
9      +      F3(N,J,K,I,K,P,P1,P2,P3,Q1)+F3(N,J,K,J,K,P,P1,P2,P3,Q1) 00255000
A      +2.0*F3(N,J,K,K,K,P,P1,P2,P3,Q1)+F3(N,I,J,K,J,P,P1,P2,P3,Q1) 00256000
B      +      F3(N,I,J,K,K,P,P1,P2,P3,Q1)+F3(N,J,J,K,K,P,P1,P2,P3,Q1) 00257000
C      +      F3(N,J,I,K,K,P,P1,P2,P3,Q1) 00258000
56 CONTINUE 00259000
S3=S3+6.0*F3(N,I,I,I,J,P,P1,P2,P3,Q1) 00260000
1      +6.0*F3(N,I,J,I,I,P,P1,P2,P3,Q1) 00261000
2      +2.0*F3(N,J,J,I,I,P,P1,P2,P3,Q1) 00262000
3      +6.0*F3(N,J,I,I,I,P,P1,P2,P3,Q1) 00263000
4      +2.0*F3(N,J,J,J,I,P,P1,P2,P3,Q1) 00264000
5      +2.0*F3(N,J,I,J,I,P,P1,P2,P3,Q1) 00265000
6      +2.0*F3(N,I,J,J,I,P,P1,P2,P3,Q1) 00266000
7      +6.0*F3(N,I,I,J,I,P,P1,P2,P3,Q1) 00267000
8      +2.0*F3(N,J,J,I,J,P,P1,P2,P3,Q1) 00268000
9      +2.0*F3(N,J,I,I,J,P,P1,P2,P3,Q1) 00269000
A      +6.0*F3(N,J,J,J,J,P,P1,P2,P3,Q1) 00270000
B      +2.0*F3(N,J,I,J,J,P,P1,P2,P3,Q1) 00271000
C      +2.0*F3(N,I,J,I,J,P,P1,P2,P3,Q1) 00272000
D      +2.0*F3(N,I,J,J,J,P,P1,P2,P3,Q1) 00273000
E      +2.0*F3(N,I,I,J,J,P,P1,P2,P3,Q1) 00274000
55 CONTINUE 00275000
A=DEN+S1-S2+S3-24.0*F3(N,I,I,I,I,P,P1,P2,P3,Q1) 00276000
51 CONTINUE 00277000
IF(DMAX.GT.ACC) GO TO 59 00278000
WRITE(3,24) IDRAW,ICOUNT 00279000
DO 60 I=1,N 00280000
P4(I)=Q1(I) 00281000
DEL(I,5)=P4(I) 00282000
WRITE(3,26) I,P4(I) 00283000
60 CONTINUE 00284000
550 CONTINUE 00285000
C.....CALCULATE THE JOINT SELECTION PROBABILITIES . 00286000
DO 551 I=1,N 00287000
DO 552 J=1,N 00288000
IF(J.EQ.I) GO TO 552 00289000
S1=0.0 00290000
S2=0.0 00291000
S3=0.0 00292000
T1=DEL(I,1)*DEL(J,2)/(1.0-DEL(I,2)) 00293000
IF(NS.EQ.2) GO TO 590 00294000
DO 553 K=1,N 00295000
IF(K.EQ.I.OR.K.EQ.J) GO TO 553 00296000
SN=DEL(I,1)*DEL(K,2)*DEL(J,3) 00297000
SD=(1.0-DEL(I,2))*(1.0-DEL(I,3)-DEL(K,3)) 00298000
T2=SN/SD 00299000
SN=DEL(K,1)*DEL(I,2)*DEL(J,3) 00300000

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SD=(1.0-DEL(K,2))*(1.0-DEL(K,3)-DEL(I,3))	00301000
T3=SN/SD	00302000
S1=S1+T2+T3	00303000
IF(NS.EQ.3) GO TO 553	00304000
DO 554 L=1,N	00305000
IF(L.EQ.I.OR.L.EQ.J.OR.L.EQ.K) GO TO 554	00306000
SN=DEL(I,1)*DEL(K,2)*DEL(L,3)*DEL(J,4)	00307000
SD=(1.0-DEL(I,2))*(1.0-DEL(I,3)-DEL(K,3))	00308000
1 *(1.0-DEL(I,4)-DEL(K,4)-DEL(L,4))	00309000
T4=SN/SD	00310000
SN=DEL(K,1)*DEL(I,2)*DEL(L,3)*DEL(J,4)	00311000
SD=(1.0-DEL(K,2))*(1.0-DEL(K,3)-DEL(I,3))	00312000
1 *(1.0-DEL(K,4)-DEL(I,4)-DEL(L,4))	00313000
T5=SN/SD	00314000
SN=DEL(K,1)*DEL(L,2)*DEL(I,3)*DEL(J,4)	00315000
SD=(1.0-DEL(K,2))*(1.0-DEL(K,3)-DEL(L,3))	00316000
1 *(1.0-DEL(K,4)-DEL(L,4)-DEL(I,4))	00317000
T6=SN/SD	00318000
S2=S2+T4+T5+T6	00319000
IF(NS.EQ.4) GO TO 554	00320000
DO 555 M=1,N	00321000
IF(M.EQ.I.OR.M.EQ.J.OR.M.EQ.K.OR.M.EQ.L) GO TO 555	00322000
SN=DEL(I,1)*DEL(K,2)*DEL(L,3)*DEL(M,4)*DEL(J,5)	00323000
SD=(1.0-DEL(I,2))*(1.0-DEL(I,3)-DEL(K,3))	00324000
1 *(1.0-DEL(I,4)-DEL(K,4)-DEL(L,4))	00325000
2 *(1.0-DEL(I,5)-DEL(K,5)-DEL(L,5)-DEL(M,5))	00326000
T7=SN/SD	00327000
SN=DEL(K,1)*DEL(I,2)*DEL(L,3)*DEL(M,4)*DEL(J,5)	00328000
SD=(1.0-DEL(K,2))*(1.0-DEL(K,3)-DEL(I,3))	00329000
1 *(1.0-DEL(K,4)-DEL(I,4)-DEL(L,4))	00330000
2 *(1.0-DEL(K,5)-DEL(I,5)-DEL(L,5)-DEL(M,5))	00331000
T8=SN/SD	00332000
SN=DEL(K,1)*DEL(L,2)*DEL(I,3)*DEL(M,4)*DEL(J,5)	00333000
SD=(1.0-DEL(K,2))*(1.0-DEL(K,3)-DEL(L,3))	00334000
1 *(1.0-DEL(K,4)-DEL(L,4)-DEL(I,4))	00335000
2 *(1.0-DEL(K,5)-DEL(L,5)-DEL(I,5)-DEL(M,5))	00336000
T9=SN/SD	00337000
SN=DEL(K,1)*DEL(L,2)*DEL(M,3)*DEL(I,4)*DEL(J,5)	00338000
SD=(1.0-DEL(K,2))*(1.0-DEL(K,3)-DEL(L,3))	00339000
1 *(1.0-DEL(K,4)-DEL(L,4)-DEL(M,4))	00340000
2 *(1.0-DEL(K,5)-DEL(L,5)-DEL(M,5)-DEL(I,5))	00341000
TA=SN/SD	00342000
S3=S3+T7+T8+T9+TA	00343000
555 CONTINUE	00344000
554 CONTINUE	00345000
553 CONTINUE	00346000
590 PI(I,J)=T1+S1+S2+S3	00347000
552 CONTINUE	00348000
551 CONTINUE	00349000
N1=N-1	00350000

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DO 556 I=1,N1          00351000
J1=I+1                  00352000
DO 557 J=J1,N          00353000
PI(I,J)=PI(I,J)+PI(J,I) 00354000
557 CONTINUE            00355000
556 CONTINUE            00356000
RETURN                  00357000
999 IFAULT=5            00358000
WRITE(3,1000) IDRAW,MAX 00359000
1000 FORMAT(1H1,///,20X,'DRAW ',I2,' DID NOT CONVERGE IN ',    00360000
1        I4,' ITERATIONS .')
      RETURN              00361000
      END                 00362000
      DOUBLE PRECISION FUNCTION FO(N,J,P,Q1) 00363000
      IMPLICIT REAL*8(A-H,O-Z) 00364000
      DIMENSION P(N),Q1(N) 00365000
      FO=P(J)/(1.0-Q1(J)) 00366000
      RETURN              00367000
      END                 00368000
      DOUBLE PRECISION FUNCTION F1(N,J,K,P,P1,Q1) 00369000
      IMPLICIT REAL*8(A-H,O-Z) 00370000
      DIMENSION P(N),P1(N),Q1(N) 00371000
      F1=P(J)*P1(K)/((1.0-P1(J))*(1.0-Q1(J)-Q1(K))) 00372000
      RETURN              00373000
      END                 00374000
      DOUBLE PRECISION FUNCTION F2(N,J,K,L,P,P1,P2,Q1) 00375000
      IMPLICIT REAL*8(A-H,O-Z) 00376000
      DIMENSION P(N),P1(N),P2(N),Q1(N) 00377000
      F2=P(J)*P1(K)*P2(L)/((1.0-P1(J))*(1.0-P2(J)-P2(K)) 00378000
1          *(1.0-Q1(J)-Q1(K)-Q1(L))) 00379000
      RETURN              00380000
      END                 00381000
      DOUBLE PRECISION FUNCTION F3(N,J,K,L,M,P,P1,P2,P3,Q1) 00382000
      IMPLICIT REAL*8(A-H,O-Z) 00383000
      DIMENSION P(N),P1(N),P2(N),P3(N),Q1(N) 00384000
      F3=P(J)*P1(K)*P2(L)*P3(M)/((1.0-P1(J))*(1.0-P2(J)-P2(K)) 00385000
1          *(1.0-P3(J)-P3(K)-P3(L)) 00386000
2          *(1.0-Q1(J)-Q1(K)-Q1(L)-Q1(M))) 00387000
      RETURN              00388000
      END                 00389000
                                         00390000
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