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Productivity Growth and Capacity Utilization

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^{r}	revised
x	suppressed to meet the confidentiality requirements of the *Statistics Act*
^{E}	use with caution
F	too unreliable to be published
*	significantly different from reference category (p < 0.05)

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Table of Contents

Abstract

This paper presents a non-parametric approach for adjusting the multifactor productivity growth (MFPG) measure for variations in capacity utilization over time. In the framework developed here, a capital utilization measure is derived from the economic theory of production and is estimated by comparing the *ex-post* return with the *ex-ante* expected return on capital. The non-parametric approach is then compared with the parametric approach and the standard growth accounting framework. Both the non-parametric and parametric approaches correct for the cyclical bias in the standard MFPG measure, but the non-parametric approach offers more practical adjustment for capacity utilization, because it is easier to implement and more in line with the non-parametric approach long used by statistical agencies and researchers. The results suggest that variable capacity utilization is the main source of the pro-cyclicality in the standard measure of MFPG in Canada's goods-producing industries, and that the post-2000 decline in the standard MFPG measure in Canadian manufacturing is largely due to the decline in capacity utilization.

Executive Summary

Official productivity estimates provide summary statistics for multifactor productivity growth (MFPG) that can help track technical progress. They do so by comparing actual growth rates in output with the expected increase in output from an increase in inputs using pre-existing or current production techniques. At any point in time, existing techniques allow additional factor inputs (labour, capital) that are applied to the production process to produce additional output. Additional factors that are added to the production process multiplied by the existing marginal product of those factors provides an estimate of the expected amount of output in a given period that should have resulted from the use of these factor inputs. If actual output exceeds this, productivity is said to have increased and one of the sources is disembodied technical progress.

For such productivity residuals to provide an unbiased estimate of technical progress, all inputs should be measured at their levels in-use instead of in-place. But because inputs-in-use are typically unobservable, measures of factor inputs are generally derived from inputs in place. Some inputs, such as capital, are fixed in the short run, so their levels-in-use and their levels-in-place often differ as capacity utilization (the ratio of factors used to factors in place) changes. As a result, output growth can be generated by a short-run increase in capacity utilization rather than by an increase in capital stock and MFPG measures that are derived from factors in place will be biased. Without adjusting for the impact of cyclical variations in capacity utilization, reported MFPG can be pro-cyclical, that is, simply due to measurement error.

Since Solow (1957), numerous studies have tried to adjust the MFPG measure for capacity utilization, but the resulting MFPG measures generally remain pro-cyclical. Solow (1957) used unemployment rates to adjust for the changes in the utilization of both capital and labour. Jorgenson and Griliches (1967) used an index of electric motor utilization in U.S. manufacturing to adjust capital utilization in the U.S. private economy. Other measures that have been used to adjust capacity utilization include growth of materials (Basu 1996), hours worked per worker (Basu and Fernald 2001), and profit shares (Denison 1979). However, the use of such *ad hoc* proxies lacks the justification derived from a theoretical framework (Berndt and Fuss 1986).

This paper offers a non-parametric approach to the measurement of MFPG. It uses the growth accounting framework, which more appropriately allows for the effect of cyclical fluctuations in the rate of capacity utilization. A capital utilization measure is derived from the economic theory of production and is estimated by comparing the *ex-post* return with the *ex-ante* expected return on capital. This is intuitively appealing, because changes in *ex-post* return on capital are likely to mainly reflect the variation in capacity utilization. A higher level of unused capital is associated with a lower *ex-post* rate of return, which is calculated on the actual level of capital.

This paper differs from the literature on direct measurement of capital utilization. Rather than focusing on capital stock available for production (capital-in-place), it examines the demand for capital—the optimal amount of capital input associated with the observed (appropriately measured) amount of output and other inputs. As a result, the rate of capital utilization can be endogenously derived instead of being selected arbitrarily. The rate of capital utilization can be expressed as the ratio of the one-period *ex-post* user costs of capital to the one-period *ex-ante* or market user costs of capital. This ratio is a variant of Tobin's *q* that is used in Berndt and Fuss (1986).

Three procedures for measuring MFPG are examined here. The first is the standard growth-accounting procedure introduced by Solow (1957) and elaborated by Jorgenson and Griliches (1967), Christensen and Jorgenson (1969, 1970), and Diewert (1976). This widely used procedure has been adopted by many international statistical agencies. The second approach is the non-parametric procedure proposed in this paper. The third is the parametric approach developed in Berndt and Hesse (1986). All three procedures are implemented using data for Canadian manufacturing over the 1961-to-2007 period, and the two non-parametric approaches are implemented using data for Canadian industries at the two-digit level of the North American Industry Classification System over the same period.

The results show that the MFPG measured by the standard procedure is pro-cyclical, largely reflecting cyclical variations in capacity utilization in goods-producing, but not service-producing, industries. On the other hand, the non-parametric measure developed here removes most of the cyclicality. The results suggest that variable capacity utilization is important in explaining the pro-cyclicality in the standard non-parametric measure of MFPG in Canadian goods-producing industries, especially manufacturing.

In addition, this paper suggests that the post-2000 decline in MFPG in manufacturing was largely a result of the decline in capacity utilization. Once the decline in capacity utilization is taken into account in this period, MFPG did not decline as is suggested by the official MFPG measure but grew at the same long-term rate experienced in earlier decades.

1 Introduction

Official productivity estimates provide summary statistics for multifactor productivity growth (MFPG) that can help track technical progress. They do so by comparing actual growth rates in output with the expected increase in output from an increase in inputs using pre-existing or current production techniques. At any point in time, existing techniques allow additional factor inputs (labour, capital) that are applied to the production process to produce additional output. Additional factors that are added to the production process multiplied by the existing marginal product of those factors provides an estimate of the expected amount of output in a given period that should have resulted from the use of these factor inputs. If actual output exceeds this, productivity is said to have increased and one of the sources is disembodied technical progress.

MFPG is calculated as a residual of output growth net of the measured contribution of growth of all measured inputs, based either on a growth accounting procedure or on a regression using a production or a cost function. Both methods were introduced by Solow (1957), and consequently, measured MFPG is sometimes referred to as the Solow residual. If residually-measured MFPG is to provide unbiased estimates of technical progress, all inputs should be measured at their levels in-use instead of in-place. As well, growth of inputs should be weighted by their respective output elasticities evaluated at the use levels in order to accurately measure the output that would be expected in the period, given the resources actually being used in the production process.

Practically, not all inputs to production are measurable, and not all measurable inputs are measured correctly. One problem involves the determination of the level of input utilization. Because the level of inputs used in production (which is determined by the rate of capacity utilization) is often not observed, the level of inputs that is in-place is employed to estimate MFPG. However, the rate of capacity utilization tends to be pro-cyclical,[1] and employing a measure of inputs in place rather than inputs used introduces a cyclical bias in the MFPG estimate.

Correction for the effect of variations in capacity utilization is important when rates of capacity utilization (the ratio of inputs used to inputs in place) change. Canada has recently experienced a resource boom and an upward appreciation of the Canada–United States exchange rate (US$/CAN$). Based on micro-data on plant adjustments to pressures arising from changes in export markets and resulting declines in capacity utilization, Baldwin, Gu and Yan (2011) show that that the decline in standard measures of MFPG during this period was primarily, if not completely, due to the decline in capacity utilization. Consequently, the standard non-parametric estimates of MFPG for this period require careful examination and evaluation.

Since Solow (1957), numerous studies have tried to adjust the MFPG measure for capacity utilization, but the adjusted measures generally remain pro-cyclical. Solow (1957) used unemployment rates to adjust for changes in the utilization of both capital and labour. Jorgenson and Griliches (1967) employed an index of electric motor utilization in U.S. manufacturing to adjust capital utilization in the U.S. private economy. Other measures that have been used to adjust capacity utilization include growth of materials (Basu 1996), hours worked per worker (Basu and Fernald 2001), and profit shares (Denison 1979). But as Berndt and Fuss (1986) noted, the use of *ad hoc* proxies is unsatisfactory, because it lacks a theoretical framework.

1. Basu (1996) observed that the pro-cyclicality of measured MFPG may be explained by three competing hypotheses. First, exogenous changes in production technology are the main driving force of business cycles, as is assumed in some real business cycle models (Cooley and Prescott 1995). Second, an economy will become more efficient endogenously by producing more output because of increasing returns to scale. Third, cyclical variations in the rate of capacity utilization, which is high in booms and low in downturns, are not appropriately accounted for when measuring MFPG. Basu (1996) concluded that the pro-cyclicality of MFPG most likely arises from cyclical variation in the rate of capacity utilization.

A non-parametric procedure is presented here that uses the growth accounting framework to measure MFPG. The procedure more appropriately accounts for the effect of cyclical fluctuations in the rate of utilization of capital input. A capital utilization measure is derived from the economic theory of production, and is estimated by comparing the *ex-post* return with the *ex-ante* expected return on capital. This is intuitively appealing, because changes in the *ex-post* return on capital should mainly reflect the variation in capacity utilization. A higher level of unused capital is associated with a lower *ex-post* rate of return, which is calculated on the actual level of capital.

This paper focuses on developing a measure of the utilization of capital input; the measurement of utilization of other inputs is less problematic. For materials, the amount purchased is likely to reflect the optimal level of materials needed for production, especially when annual data are used. For labour, hours worked is used to account for cyclicality in the utilization of labour.[2]

The approach presented here differs from the literature on direct measurement of capital utilization, which uses proxies such as unemployment rates and hours worked per worker.[3] Rather than focusing on capital stock available for production (capital-in-place) and adjusted for utilization, this analysis examines the demand for capital input—the optimal amount of capital input associated with the observed (appropriately measured) amount of output and other inputs. The rate of capital utilization can then be endogenously derived; it can be expressed as the ratio of the one-period *ex-post* user costs of capital to the one-period *ex-ante* or market user costs of capital. This ratio is a variant of Tobin's *q* found in Berndt and Fuss (1986),[4] who used it to adjust the rental price of capital input and to modify the weight of capital input growth in the calculation of MFPG so as to consider changes in capacity utilization.

This paper shows that, in order to take the rate of capital utilization into account when measuring MFPG, the ratio of *ex-post* to *ex-ante* return to capital should be used to adjust the quantity of capital input rather than the price of capital input. Berndt and Fuss (1986) used the actual level of capital-in-place in their measure of MFPG. They concluded that when capital-in-place is valued at its value of marginal product, the MFPG measure corrects for the variation in capacity utilization. But as observed by Basu and Fernald (2001) and Hulten (2010), the Berndt and Fuss procedure does not provide a solution for the issue of capacity utilization in the measurement of MFPG. The present analysis shows that in reaching their conclusion, Berndt and Fuss implicitly assume full utilization of capital.

Three approaches to measuring MFPG are examined in this paper. The first is the widely used growth-accounting procedure introduced by Solow (1957) and elaborated by Jorgenson and Griliches (1967), Christensen and Jorgenson (1969, 1970), and Diewert (1976). The second is the non-parametric procedure proposed in this study. The last is the parametric approach developed in Berndt and Hesse (1986). All three are implemented using data for the Canadian manufacturing industry for the 1961-to-2007 period; the two non-parametric approaches are

2. Employment may not be fully adjusted in the short run because of labour-hoarding behaviour and quasi-fixed adjustment costs of labour, such as hiring, training or redundancy. Nonetheless, in most cases, employers can reduce hours worked per worker during economic downturns.
3. It is important to distinguish the economic and engineering-type concepts of capacity utilization. The former is optimization-based. Specifically, capital-in-place is fully utilized economically when the optimization condition on capital is satisfied (the value of marginal products of capital is equal to its market user costs). Therefore, the amount of capital in use relies on relative user costs of inputs. By contrast, the engineering-type of capacity output is based on running machinery at normal or full schedule, and hence, is independent of relative user costs of inputs. If not otherwise noted, the economic concept is used in this paper.
4. Tobin's *q* is the ratio of the market value to the replacement cost of a firm. Given that a one-period rate of return on capital is calculated as the ratio of capital income to either the market value (*ex-ante*) or the replacement cost (*ex-post*) of capital, a variant of Tobin's *q* can be written as the ratio of the *ex-post* rate to *the ex-ante* rate. When the *ex-ante* rate is higher (lower) than *ex-post* rate, the replacement cost of a firm is higher (lower) than its market value, which implies that capital is under- (over-) utilized.

implemented using data for 18 Canadian industries at the two-digit level of the North American Industry Classification Stystem (NAICS) over the same period.

The remainder of this report is organized into five sections. The literature on the measurement of MFPG and capacity utilization is briefly reviewed in Section 2. A non-parametric procedure for measuring MFPG with correction for the effect of variations in capacity utilization is developed in Section 3. In Section 4, the procedure for estimating the one-period *ex-ante* (market) user cost of capital is described. In Section 5, the new procedure is implemented empirically using Canadian industry data, and the results are compared with those of the standard procedure and the parametric procedure of Berndt and Hesse (1986). Section 6 concludes.

2 A brief review of literature

Since Solow (1957), MFPG has commonly been calculated as the difference between actual output growth and output growth expected due to input growth. The resulting productivity "residual" reflects the effects of unmeasured inputs.

A feature of measured MFPG is its pro-cyclicality. Many economists believe that this is largely caused by mismeasurement related to unobserved cyclical variations in capacity utilization; some macroeconomists consider it as a stylized fact and an essential feature of business cycles.

Understanding the nature of technical progress requires appropriate correction for the effect of variations in capacity utilization in the measurement of MFPG. Many studies since Solow (1957) have tried to account for this effect by using *ad hoc* proxies, but the results have been less than satisfactory.

Solow (1957) realized that both capital and labour may, at times, be under-utilized and incorporated what he defined as "capital- and labour-in-use" in his calculation of MFPG. He obtained his measure of capital- and labour-in-use by adjusting capital- and labour-in-place by the unemployment rate. Based on the cyclical-adjusted data, he concluded that MFPG accounted for more than 80% of labour productivity growth in the U.S. during the 1909-to-1949 period.

A decade later, Jorgenson and Griliches (1967) used an index of electric motor utilization in American manufacturing to adjust capital-in-place in the private economy. They concluded that MFPG accounted for less than 5% of output growth over the 1945-to-1965 period. Denison (1969) criticized this engineering-type approach, arguing that it is not appropriate to correct for capital inputs of various assets in the entire economy by using a single electric motor utilization index in manufacturing. The results in Christensen and Jorgenson (1970) support Denison's critique: with only non-residential structures and producers' durable equipment adjusted by the single electric motor utilization index, the reported contribution of MFPG to output growth became about 40%.

Other applications of *ad hoc* proxies to correct for the effect of cyclical capacity utilization include the ratio of energy use to capital stock (Burnside, Eichenbaum and Rebelo 1995), growth of materials (Basu 1996), hours worked per worker (Basu and Fernald 2001), and profit shares (Denison 1979). These measures are empirically driven rather than theoretically based.

A variety of economic approaches (non-parametric and parametric) to adjust data for cyclicality have been proposed since the late 1970s. Among them, Berndt and Fuss (1986) developed a non-parametric approach to adjust the productivity residual by using the weights based on *ex-post* residual capital income to evaluate the impact of capital and labour in the estimating formula for MFPG. Hulten (1986, p. 43) summarized their contribution as showing that:

> The bias in the estimate of multifactor productivity resulting from the erroneous assumption of long-run equilibrium is due to the mismeasurement of the weights in the calculation, and is not due to the mismeasurement of the flow of capital services.

Christensen and Jorgenson (1969) and Gollop and Jorgenson (1980) developed an *ex-post* procedure for estimating the shadow price of capital that corrects the weights applied to capital over the cycle without having to adjust the measured capital-in-place. This approach (referred to as the Jorgenson approach in this paper) has been widely adopted by international statistical agencies and in productivity studies. The Berndt-Fuss proposition can be regarded as providing a theoretical justification for the Jorgenson approach.

Despite the use of this correction for capacity utilization, the MFPG measure generated by the technique remains sensitive to business cycles. This suggests that it only imperfectly corrects for the cyclical effect—unless technological change itself ebbs and flows with the cycle. Most measures of MFPG based on the Berndt-Fuss approach or the Jorgenson approach still have cyclicality embedded in them.

Two types of parametric approaches developed to adjust for capacity utilization—one by Morrison in a series of studies,[5] and another by Berndt and Hesse (1986)—have several aspects in common. Capital input is assumed to be quasi-fixed, and production technology is specified as a translog short-run cost function. Long-run equilibrium conditions are imposed on the short-run cost function by applying Hotelling's lemma for deriving the demand functions (or share functions) for variable inputs; applying Shephard's lemma for deriving the price-setting equations for quasi-fixed inputs; and equating the marginal revenue and the marginal cost of output for deriving the output price-setting equation. These equations are then jointly estimated to obtain parameter values that are used to calculate MFPG.

The Morrison and Berndt-Hesse approaches differ in the number of issues addressed. Morrison tried to simultaneously capture the capacity utilization effect, non-constant returns to scale, and price mark-up behaviour in an integrated framework, since the normal MFPG estimate encompasses the impact of these factors, along with technical progress. She specified a form of inverse demand function for output where the price of output and the marginal revenue of output do not equate. Her estimated system of equations includes the demand functions for variable inputs, the price-setting equations for output and quasi-fixed inputs, but not the original cost function and the inverse demand function themselves. This strategy may not be efficient, because cross-equation restrictions on parameters do not necessarily hold when the cost function and the inverse demand function are excluded from the system of equations to be estimated. Also, because the derived functions are based on long-run equilibrium conditions, the estimated parameters may not fully reflect short-run optimization behaviour during economic downturns. Hence, it is inappropriate to use them to pin down the short-run elasticities during economic cycles. In addition, as noted by Fousekis (1999) and Hauver and Yee (1992), the prior one-to-one relationship between the capacity utilization effect and the elasticity of cost in Morrison's theoretical framework may not exist because of the mix of long-run and short-run conditions when deriving the relationship.

The Berndt-Hesse (1986) approach focuses only on the impact of capacity utilization on MFPG estimates. They imposed conditions for constant returns to scale in the short-run variable cost

5. See Morrison (1985, 1986, 1988, 1992a, 1992b).

function and eliminated price mark-up behaviour by equating marginal cost and output price. The modified cost function is jointly estimated with a set of "share" functions to obtain more efficient estimates. The parameter values derived from estimating the system of equations are used to solve for potential output, which is defined as the point at which minimal average total cost is reached. MFPG is then measured by potential output growth net of growth of total inputs. Imposing the long-run equilibrium conditions into the short-run variable cost function would be valid if the purpose is to calculate comparative statics between potential equilibriums.

Both of these parametric approaches involve complex multivariate analysis. An advantage of a non-parametric approach is that it can be implemented easily using observed quantities and prices of output and inputs. A disadvantage is that the nature of production technology is locally revealed using only two observations. As a result, measured MFPG generally remains volatile. By contrast, a parametric approach uses time-series data to reveal the information on production technologies globally, but suffers from the imprecisions associated with regression analysis—not the least of which come from errors-in-variables in the underlying data.[6]

The two parametric approaches have been found to produce similar estimates (Slade 1986; Morrison 1986), except in times of dramatic changes in production technology such as the period in the 1970s after the oil shock.

Official productivity statistics issued by statistical agencies that use constant factor shares often do not explicitly correct for cyclical variations.[7] Given the short-run cyclical variations in MFPG, analysis of productivity growth and technical progress often focuses on longer-term and peak-to-peak-comparisons. By comparing productivity growth from peak to peak, analysts can minimize the impact of changes in capacity utilization (Baldwin et al. 2011; Barnes 2011).

3 Non-parametric approach for measuring multifactor productivity growth with variable capital utilization

This section extends the traditional growth accounting framework to allow for short-run changes in capital utilization. It then compares this approach with the traditional growth accounting approach, which is appropriate for measuring MFPG in the long run.

In practice, many international statistical agencies make no attempt to adjust their MFPG measures for changes in the rate of capital utilization (OECD 2001).[8] The non-parametric approach described in this paper can be easily adopted by statistical agencies.

Statistical agencies generally use the traditional growth accounting framework to measure MFPG. The approach, developed by Solow (1957), Jorgenson and Griliches (1967), and Diewert (1976), assumes the existence of an aggregate production function.[9] The framework decomposes output growth into two main components: the contribution of combined capital and labour inputs, and the contribution of MFPG. MFPG is the difference between output growth and the combined effect of input growth on expected economic growth. The framework provides a powerful analytical tool for analyzing factors contributing to long-run economic growth. A key

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6. See Macdonald (2007) for a discussion of these problems in standard multivariate analysis using the KLEMS database.
7. Canadian Productivity Accounts of Statistics Canada use changing factor shares in the measurement of MFPG.
8. This is especially the case for those that use capital stock in place and average income shares of capital or income shares derived from exogenous rates of return.
9. The debate on the existence of the aggregate production function can be traced back to the 1960s, largely between Robinson and Sraffa at the University of Cambridge in the United Kingdom and Samuelson and Solow at Massachusetts Institute of Technology in the United States, known as the Cambridge Controversy. Nonetheless, the theory of production functions is widely adopted in productivity analysis. Jorgenson (1966) developed a less restrictive approach using the production possibility frontier.

assumption is that producers are in long-run equilibrium: the optimal levels of labour and capital inputs utilized are assumed to be equal to the actual levels of capital and labour in place.

In the short run, however, capital is quasi-fixed and cannot be instantaneously adjusted to changes in demand; it can be adjusted only in the long run. Consequently, capital stock that is utilized (capital-in-use) differs from the actual capital stock (capital-in-place) that producers possess. For many reasons, a firm's capital utilization rate varies in the short run: a change in demand conditions, seasonal variations, interruptions in the supply of intermediate products, or a breakdown of machinery (OECD 2001).

Consider an economy with one output (Y), one variable input—labour (L), and one quasi-fixed capital (K). A Hicks' neutral production function with productivity parameter (A) can be written as

$$Y = Af(K, L). \tag{1}$$

For the production function to be well-defined, it is assumed that $\partial f / \partial L \geq 0$, $\partial f / \partial K \geq 0$, $\partial^2 f / \partial L^2 \leq 0$, $\partial^2 f / \partial K^2 \leq 0$, and $\partial^2 f / \partial L \partial K \geq 0$.

Capital is quasi-fixed in the short run, and hence, cannot be adjusted instantaneously. Therefore, capital-in-place and capital-in-use may differ in the short run. Capital-in-place reflects the supply of capital, while capital-in-use reflects the demand for capital. The two must be distinguished explicitly to avoid notational confusion. Let K^u be capital-in-use and K^s be capital-in-place. Then, $K^u = K^s$ in the long-run and $K^u \neq K^s$ in the short run. Logarithmically differentiating (1) yields:

$$\frac{d\ln(Y)}{dt} = \frac{\partial \ln(Y)}{\partial \ln(A)}\frac{d\ln(A)}{dt} + \frac{\partial \ln(Y)}{\partial \ln(L)}\frac{d\ln(L)}{dt} + \frac{\partial \ln(Y)}{\partial \ln(K^u)}\frac{d\ln(K^u)}{dt}. \tag{2}$$

Note that $\partial \ln(Y)/\partial \ln(A) = (A/Y)(\partial Y/\partial A) = (1/f)f = 1$. K is replaced in equation (1) by K^u to capture the possibility that capital-in-use may differ from capital-in-place in the short run. Denoting α_L and α_K the elasticities of output with respect to labour and capital, respectively, then (2) can be rewritten as:

$$\frac{\dot{A}}{A} = \frac{\dot{Y}}{Y} - \alpha_L \frac{\dot{L}}{L} - \alpha_K \frac{\dot{K}^u}{K^u}, \tag{3}$$

where $d\ln(X)/dt = (d\ln X / dt)/X = \dot{X}/X$.

Equation (3) shows that the short-run MFPG is the difference between output growth and the effects of combined input growth (labour, capital, etc.), where capital input is capital-in-use rather than capital-in-place, and the weight for capital input is the elasticity of output with respect to capital evaluated at capital-in-use. This equation provides a valid measure of MFPG, even when labour and capital inputs are not at cost-minimizing levels.

However, capital-in-use and the input elasticities in equation (3) are not observable. To measure MFPG using equation (3), additional assumptions must be made about the production function and producer behavior: namely, that perfect competition in output and input markets exists, and the production function is characterized by constant returns to scale.[10]

10. Whether these assumptions significantly distort the resulting estimates is a matter for empirical research, which has been investigated for Canada (Baldwin et al. 2001). The paper found that up to 20% of MFPG in Canada over the 1961-to-1995 period is due to the exploitation of economies of scale and fixed capital.

First, consider the firms' short-run optimization problem without assuming perfect competition in output markets and constant returns to scale in production. Let $P(Y)$ denote the inverse demand function for firms' output and assume $\partial P/\partial Y \leq 0$. Assume perfect competition in input markets such that the supply of inputs is unlimited at fixed market rental prices. Firms choose the optimal amount of labour and capital they need to rent from input markets to minimize the cost of producing a given amount of output (Y^0). The firms' cost-minimization problem can be written as

$$\begin{aligned} &\underset{K,L}{Min}\ P_L L + P_K K \\ &\quad s.t.\quad Af(L,K) = Y^0, \end{aligned} \tag{4}$$

where P_L and P_K are the one-period market user costs of labour and capital, respectively. The associated Lagrange function is

$$\Lambda(L,K,\lambda) = P_L L + P_K K + \lambda(Y^0 - Af(L,K)). \tag{5}$$

The Lagrange multiplier (λ) refers to the shadow cost of output. The first-order conditions for cost minimization are

$$\begin{cases} \dfrac{\partial\Lambda}{\partial L} = P_L - \lambda\dfrac{A\partial f}{\partial L} = 0 \\ \dfrac{\partial\Lambda}{\partial K} = P_K - \lambda\dfrac{A\partial f}{\partial K} = 0 \\ \dfrac{\partial\Lambda}{\partial\lambda} = Y^0 - Af(L,K) = 0 \end{cases} \quad\Rightarrow\quad \begin{cases} \dfrac{A\partial f(L,K)}{\partial L} = \dfrac{P_L}{\lambda} \\ \dfrac{A\partial f(L,K)}{\partial K} = \dfrac{P_K}{\lambda} \\ Y^0 = Af(L,K) \end{cases} \tag{6}$$

The optimal demand for labour and capital can be derived using equation (6). When labour can be adjusted instantaneously, the observed labour input (L^0) can be regarded as reflecting the optimal demand for labour. The optimal amount of capital that firms need to rent is then K^u. Assume the production function is homogenous of degree γ. Euler's theorem implies that

$$\gamma Y^0 = \frac{A\partial f}{\partial L}L^0 + \frac{A\partial f}{\partial K}K^u, \tag{7}$$

imposing the first-order conditions in (7) gives

$$\gamma\lambda Y^0 = P_L L^0 + P_K K^u = C(Y^0) \quad\Rightarrow\quad AC_{Y^0} \equiv C/Y^0 = \gamma\lambda. \tag{8}$$

A firm chooses the amount of output to be produced so as to equate marginal revenue and marginal cost, that is,

$$\begin{aligned} &MR_{Y^0} = P + Y^0\frac{\partial P}{\partial Y} = P\left(1 + \frac{Y^0}{P}\frac{\partial P}{\partial Y}\right) \equiv P(1+\varepsilon_{PY}) = MC_{Y^0} = \lambda, \\ &\quad\Rightarrow\quad \lambda = P(1+\varepsilon_{PY}) \end{aligned} \tag{9}$$

where P is the output price. Substituting λ into equation (8) gives

$$\gamma(1+\varepsilon_{PY})PY^0 = P_L L^0 + P_K K^u = C(Y^0). \tag{10}$$

The output elasticities with respect to labour and capital can then be approximated as

$$\begin{aligned}\alpha_L &\equiv \frac{\partial \ln(Y)}{\partial \ln(L)} = \frac{L^0}{Y^0}\frac{\partial Y}{\partial L} = \frac{L^0}{Y^0}\frac{P_L}{\lambda} = \frac{P_L L^0}{PY^0}\frac{1}{1+\varepsilon_{PY}} = \frac{s_L^0}{1+\varepsilon_{PY}} \\ \alpha_K &\equiv \frac{\partial \ln(Y)}{\partial \ln(K)} = \frac{K^u}{Y^0}\frac{\partial Y}{\partial K} = \frac{K^u}{Y^0}\frac{P_K}{\lambda} = \frac{P_K K^u}{PY^0}\frac{1}{1+\varepsilon_{PY}} = \frac{s_K^0}{1+\varepsilon_{PY}}.\end{aligned} \tag{11}$$

Also

$$\alpha_L + \alpha_K = \gamma \quad \Rightarrow \quad s_L^0 + s_K^0 = \gamma(1+\varepsilon_{PY}). \tag{12}$$

The optimal amount of capital that firms need to rent from the capital market can be calculated as

$$K^u = \frac{\gamma(1+\varepsilon_{PY})PY^0 - P_L L^0}{P_K}. \tag{13}$$

The superscript 0 is dropped hereafter to simplify the notation. Substituting (11) and (12) into (3) gives the formula for measuring short-run MFPG as

$$\frac{\dot{A}^{short}}{A^{short}} = \frac{\dot{Y}}{Y} - \alpha_L \frac{\dot{L}}{L} - \alpha_K \frac{\dot{K}^u}{K^u} = \frac{\dot{Y}}{Y} - \frac{s_L}{1+\varepsilon_{PY}}\frac{\dot{L}}{L} - (\gamma - \frac{s_L}{1+\varepsilon_{PY}})\frac{\dot{K}^u}{K^u}. \tag{14}$$

Equation (14) is still not measurable, because the price elasticity of output and the returns to scale parameter are not known. In practice, perfect competition in output markets and constant returns to scale in the production function are often assumed, that is,

$$\varepsilon_{PY} = 0 \quad \text{and} \quad \gamma = 1. \tag{15}$$

Under these conditions, output elasticities with respect to labour and capital can be approximated using their respective shares in nominal output. Equation (14) then becomes

$$\frac{\dot{A}^{short}}{A^{short}} = \frac{\dot{Y}}{Y} - s_L \frac{\dot{L}}{L} - s_K \frac{\dot{K}^u}{K^u} \ . \tag{16}$$

MFPG measured by (16) may include the effects of non-constant returns to scale and price mark-ups when equation (15) does not hold.

Equation (16) has been used in previous empirical studies to measure MFPG in the short run (Hulten 1986), except that capital-in-use is arbitrarily determined. In the literature, capital-in-use is typically approximated by multiplying capital stock with proxies for the rate of capital utilization. As discussed earlier, those proxies include the ratio of energy use to capital stock (Burnside, Eichenbaum and Rebelo 1995), hours worked per worker (Basu and Fernald 2001), profit shares (Denison 1979), unemployment rates (Solow 1957), and manufacturing electric motor utilization indices (Jorgenson and Griliches 1967).

This paper departs from previous studies that use proxies for capacity utilization to measure MFPG in the short run. The level of capital stock used in the short run is estimated based on optimization conditions that hold in the short run when capital is quasi-fixed. Under the assumptions of perfect competition in output markets and constant returns to scale, the actual level of capital stock used satisfies the following equation:

$$PY = P_L L + P_K K^u, \text{ or } K^u = \frac{PY - P_L L}{P_K}. \tag{17}$$

To impute the level of capital stock used in the short run, estimates of the *ex-ante* one-period user cost of capital P_K are required. The next section presents a practical approach for estimating the *ex-ante* user cost of capital that producers face.

Equations (16) and (17) are used to calculate a measure of short-run MFPG when capital-in-use differs from capital-in-place. The capital utilization ratio can be defined as $CU_K \equiv K^u / K^s$. Equation (16) can be rewritten in a format used in previous empirical studies of MFPG measurement with variable capital utilization[11]

$$\frac{\dot{A}^{short}}{A^{short}} = \frac{\dot{Y}}{Y} - s_L \frac{\dot{L}}{L} - s_K \left(\frac{\dot{K}^s}{K^s} + \frac{\dot{CU}_K}{CU_K} \right). \tag{18}$$

The measure of capital utilization employed here to adjust the actual level of capital stock in the measurement of short-run MFPG is related to the profit rates used by Denison (1979) to adjust the quantity of capital input for the rate of variation in its utilization in MFPG measurement. It is also related to a variant of Tobin's *q* used to adjust the income share of capital inputs for the rate of variation in the utilization of capital input by Berndt and Fuss (1986).

The one-period ex-post user cost of capital $\bar{Z}_K$ can be defined as:

$$\bar{Z}_K = \frac{PY - P_L L}{K^s}. \tag{19}$$

$\bar{Z}_K$ represents an average value and is residually calculated. Using equation (17), the following can be derived:

$$\begin{gathered} PY = P_L L + P_K K^u = P_L L + P_K CU_K K^s = P_L L + Z_K K^s \\ \Rightarrow \quad CU_K = \frac{Z_K}{P_K}. \end{gathered} \tag{20}$$

Equation (20) indicates that the utilization rate of capital is equal to the ratio of the *ex-post* returns on capital to the *ex-ante* (market) user cost of capital, or a variant of Tobin's *q*.[12] Because the *ex-post* return on capital is closely related to profit rates, the approach described here can also be viewed as a justification for Denison's procedure, which uses profit rates to adjust capital utilization.[13] This result also supports the approach in Baldwin et al. (2011) who use the share of capital income to adjust for the variation in the utilization of capital, since the *ex-post* return on the actual level of capital is positively related to the capital share of income.

Capacity utilization measured as the ratio of the *ex-post* to *ex-ante* use cost of capital is positively related to the *ex-post* user cost, and is negatively related to the *ex-ante* user cost. The

11. Hulten (1986) derived the same equation (Equations (13) to (15) of this paper), but did not go further.
12. A firm's market value reflects the market value of tangible and intangible capital. When intangible capital is not included in the capital stock, the *ex-ante* user cost of capital will be inflated, and therefore, will bias the capacity utilization estimate downward. The *ex-ante* user cost is calculated as the nine-year moving average of the *ex-post* user cost (see next section), which allows us to ignore the issue of the omission of intangible capital if both types of capital have the same capacity utilization level.
13. See Denison (1979).

ex-post user cost is estimated as the ratio of the *ex-post* capital income to the actual level of capital stock. This estimated *ex-post* user cost of capital should be negatively related to the level of unutilized capital or positively related to the level of capital utilization. On the other hand, the *ex-ante* user cost of capital is employed to choose the optimal level of capital utilization that minimizes total input costs. The higher the *ex-ante* user cost of capital, the lower will be the level of capital utilization that is chosen to minimize cost.

This measure of capital utilization can be greater than or less than one: a value less than one indicates that capital is not fully utilized; a value exceeding one, indicates that the actual level of capital stock is used more intensively than normal, for example, by adding more shifts and working on weekends.

The non-parametric framework for the adjustment of capacity utilization is developed under assumptions of perfect competition and constant returns to scale. If imperfect competition and increasing returns to scale exist, changes in capacity utilization based on the *ex-post* user cost may reflect changes in price markup. However, the correlation between the measure of capacity utilization presented here and price mark-up is small and statistically non-significant for most Canadian industries, as shown in Appendix, Section 7.3.

In the empirical section, a discrete approximation of (18) is used to measure short-run MFPG:

$$\ln\left(\frac{MFP_t}{MFP_{t-1}}\right)=\ln\left(\frac{Y_t}{Y_{t-1}}\right)-\overline{s}_L\ln\left(\frac{L_t}{L_{t-1}}\right)-(1-\overline{s}_L)\left[\ln\left(\frac{K_t^s}{K_{t-1}^s}\right)+\ln\left(\frac{CU_{K,t}}{CU_{K,t-1}}\right)\right]$$
$$\text{with}\quad \overline{s}_L=\frac{1}{2}\left(\frac{P_{L,t}L_t}{P_tY_t}+\frac{P_{L,t-1}L_{t-1}}{P_{t-1}Y_{t-1}}\right)=\frac{1}{2}\left(s_{L,t}+s_{L,t-1}\right) \tag{21}$$
$$\text{and}\quad CU_{K,t}=\frac{P_tY_t-P_{L,t}L_t}{P_{K,t}K_t^s},\qquad CU_{K,t-1}=\frac{P_{t-1}Y_{t-1}-P_{L,t-1}L_{t-1}}{P_{K,t-1}K_{t-1}^s}.$$

The short-run MFPG measure can be compared with the traditional measure of MFPG that applies to the economy, which is characterized by the competitive equilibrium that is assumed to prevail in the long run. In the long run, inputs are fully adjusted to maximize profits. Capital-in-use is equal to capital-place, that is, $K^u=K^s=K$. Long-run MFPG can then be written as

$$\frac{\dot{A}}{A}=\frac{\dot{Y}}{Y}-s_L\frac{\dot{L}}{L}-s_K\frac{\dot{K}}{K}, \tag{22}$$

where $s_L+s_K=1$ under the assumption of constant returns to scale.

The main difference between the long- and short-run MFPG measures is that the profit-maximizing level of capital stock (capital-in-place) differs from the actual level of capital stock used (capital-in-use) in the short run. But in the long run, they are equal, because capital can be adjusted to the profit-maximizing level. It is noteworthy that, with constant returns to scale technology, the income shares of capital and labour are unchanged whether an economy is in a long-run equilibrium or at a short-run optimization point.

If the traditional growth accounting framework (22) is used to measure short-run MFPG with variable capital utilization, it will yield a biased estimate of true MFPG. The bias can be derived by subtracting (22) from (18):

$$\frac{\dot{A}^{short}}{A^{short}}=\frac{\dot{Y}}{Y}-s_L\frac{\dot{L}}{L}-s_K\left(\frac{\dot{K}^s}{K^s}+\frac{\dot{CU}_K}{CU_K}\right)=\frac{\dot{A}}{A}-s_K\frac{\dot{CU}_K}{CU_K}. \qquad (23)$$

The MFPG applicable to short-run circumstances that lead to excess (or inadequate) capital is equal to the MFPG that applies in the long run minus the rate of change in capital utilization, multiplied by the share of capital income.[14]

Statistical agencies and most previous researchers used Equation (22) to measure short-run MFPG with variable capital utilization. The analysis presented here shows that these measures leads to a biased estimate of true MFPG that is adjusted for capital utilization. When capital utilization declines, the true MFPG measure is higher than the biased MFPG measure. On the other hand, when capital utilization increases, the true MFPG measure is lower than the biased MFPG measure.

The approach for measuring MFPG with variable capital utilization proposed in this study applies when there is only one type of capital input. The approach can be extended to the economy with multiple capital inputs if the rate of capital utilization is assumed to be the same across different assets.

Comparison with Berndt-Fuss approach

Berndt and Fuss (1986) developed a non-parametric procedure for adjusting capacity utilization in MFPG measurement. They contend that the value of capital input should be adjusted, not the quantity of capital services. In their framework, the *ex-post* return to capital should be used in the MFPG measure to take changes in capacity utilization into account. By contrast, this paper argues that that the ratio of *ex-post* to *ex-ante* return to capital should be used to adjust the quantity of capital input rather the price of capital.

Berndt and Fuss (1986) define capacity output as the output at which capital and labour inputs used in production minimize costs and maximize profits. They define capacity utilization as the ratio of actual output to capacity output. In the short-run, capital stock is quasi-fixed and cannot be changed instantaneously. Capital input is said to be used more intensively when the fixed level of capital stock is combined with more labour inputs than are optimal in the long run. In this case, actual output exceeds capacity output, and capacity utilization is greater than one. On the other hand, if capital input is combined with fewer labour inputs than are optimal in the long run, actual output is less than capacity output and measured capacity utilization is less than one.

Berndt and Fuss (1986) argue that when capital is fixed in the short run, MFPG can be calculated as output growth minus the weighted sum of capital and labour input growth, where the weight for labour input is the share of labour costs in nominal output, and the weight for capital input is the share of the shadow value of capital in nominal output. The difference between Berndt and Fuss (1986) and the traditional growth accounting framework is in the weights used to calculate combined input growth. For the traditional growth accounting framework, the weights for both capital and labour are based on their market prices (labour compensation for labour input; market user cost of capital for capital input). For the Berndt and

14. Statistics Canada publishes the survey-based measure of capacity utilization. It is an engineering-type of measure of capacity utilization. Capacity is defined as the maximum production attainable under firms' normal operating practice with respect to the use of production facilities, overtime, work shifts, holidays, etc. We used this official measure to correct for variations in capacity utilization when measuring the MFPG for the Canadian manufacturing industry and found that the measured MFPG and output growth are still highly correlated (results available on request). By contrast, when the economic-type of capacity utilization measure developed in this paper is used, the two variables are no longer correlated. These results favour the measure presented in this analysis for the purposes examined here.

Fuss procedure, the weight for capital input is based on its shadow value rather than the market user cost of capital.

Unlike most empirical studies that adjust the quantity of capital inputs for variations in the rate of capital utilization, Berndt and Fuss (1986) argue that it is not the quantity of capital input that should be adjusted, but the cost share of capital input.

When capacity utilization (the ratio of actual output to capacity output) is greater than one, the shadow value of capital exceeds the market user cost of capital, and MFPG derived from the traditional growth accounting framework overstates true MFPG as measured by the Berndt and Fuss procedure. If capacity utilization is less than one, the shadow value of capital is below the market user cost of capital, and MFPG derived from the traditional growth accounting framework understates true MFPG. Therefore, the MFPG measure associated with the traditional growth accounting framework tends to show more cyclical fluctuations than the MFPG measure from the Berndt and Fuss procedure. In that sense, Berndt and Fuss correct for the cyclical fluctuation in the MFPG measure that is due to changes in capacity utilization.

More specifically, Berndt and Fuss (1986) indicate that short-run MFPG should be calculated as:

$$\frac{\dot{A}^{BF}}{A^{BF}} = \frac{\dot{Y}}{Y} - s_L \frac{\dot{L}}{L} - s_K^{BF} \frac{\dot{K}^s}{K^s}, \text{ with } s_K^{BF} = \frac{Z_K K^s}{P\,Y}, \tag{24}$$

where $Z_K \equiv P \;\partial Y / \partial K^s$ is the shadow value of capital input. The weight for capital input in equation (24) is based on the shadow value of capital.

Basu and Fernald (2001) note that the Berndt and Fuss procedure does not offer a solution for the issue of capacity utilization in MFPG measurement since they always use the actual level of capital-in-place in their measure of MFPG. To emphasize that distinction, capital-in-place K^s is used in their MFPG measure in equation (24). The contribution of Berndt and Fuss is to show that the elasticity of capital input that should be used for measuring MFPG is not necessarily equal to the cost share of capital input valued at the market price when capital input cannot be adjusted instantaneously.

Basu and Fernald (2001) use an economy with a Cobb-Douglas production to illustrate the difference between the contribution of Berndt and Fuss and other capacity utilization studies. Suppose that output is produced via a Cobb-Douglas production function. The output elasticities of labour and capital inputs are constant. Then the output elasticity used to weight capital input growth and to calculate MFPG in the short run, when capital is fixed, is the same as the output elasticity in the long run when capital input can be adjusted to its optimal level. MFPG derived from the Berndt and Fuss procedure is the same as the MFPG measure from traditional growth accounting. But a capacity utilization problem persists—for example, capital can be underutilized during a recession.

To implement the Berndt and Fuss procedure, it is necessary to measure the elasticity of capital input and weight for capital-input growth. This can be done by assuming that the production function is characterized by constant returns to scale, and there is just one capital input. The estimate of Z_K can be derived using

$$Z_K = \frac{PY - P_L L}{K^s} \quad \Rightarrow \quad s_K^{BF} = s_K = 1 - s_L. \tag{25}$$

In this case, the Berndt and Fuss measure of MFPG is identical to the one derived from the traditional growth accounting framework. Hulten (1986) claimed that Berndt and Fuss provide a theoretical justification for the Jorgenson approach. As an alternative, Berndt and Fuss use Tobin's *q* to estimate the elasticity of capital input, which is equal to the market price of capital input times Tobin's *q*: $Z_K^* = \hat{Z}_K \equiv qP_K$, where *q* is Tobin's *q*. In this case, $s_K^{BF} + s_L \neq 1$. While the Berndt and Fuss approach is valid when capital is fully utilized ($K^u = K^s$), it is not appropriate when capital is underutilized or $K^u < K^s$. In that situation, output is produced using labour input and capital-in-use. Output should be compared with the level of capital used K^u to estimate MFPG, as is done in most empirical studies on capacity utilization. More output would be produced if the actual level of capital stock K^s were used. In addition, the shadow value of capital input is used to weight the growth rates of capital stock in the Berndt and Fuss procedure. But when capital used is less than the actual level of capital stock, the shadow value of capital at the actual level of capital stock is zero, because the additional unit of capital stock adds nothing to output (Appendix, Section 7.1 contains a formal proof).

The Berndt and Fuss procedure does not provide a true measure of MFPG when capital utilization changes, but the approach derived here does. To see the difference, consider movements along a short-run unit cost curve or production function with capital fixed. Because there is no shift in the production function, MFPG that represents the shift in production function is zero, and capital-in-place remains unchanged:

$$\frac{\dot{A}}{A} = 0 \quad \text{and} \quad \frac{\dot{K}^s}{K^s} = 0. \tag{26}$$

The MFPG measure from Berndt and Fuss and the approach presented here can be written as:

$$MFPG = \begin{cases} \dfrac{\dot{Y}}{Y} - s_L \dfrac{\dot{L}}{L} & \text{Berndt-Fuss approach} \\ \dfrac{\dot{Y}}{Y} - \left\{ s_L \dfrac{\dot{L}}{L} + (1 - s_L)\left(\dfrac{\dot{Z}_K}{Z_K} - \dfrac{\dot{P}_K}{P_K} \right) \right\} & \text{Approach in this paper.} \end{cases} \tag{27}$$

Berndt and Fuss (1986) claimed that their measure of MFPG captures the true MFPG, which is zero because there is no shift in production function.[15] The Berndt and Fuss measure of MFPG differs from zero except when labour input growth is greater than output growth, $\left|\dot{L}/L\right| \gg \left|\dot{Y}/Y\right|$. This condition incorrectly implies that an increase (decrease) in capacity utilization is associated with a decline (rise) in labour productivity. This is not the case when MFPG is measured by the procedure developed in this paper.

4 Estimating the *ex-ante* user cost of capital

The *ex-ante* user cost of capital is a key parameter that is requied for implementing the parametric and non-parametric approaches for measuring MFPG.

The user cost of capital can be written as (Jorgenson and Griliches 1967):

$$P_{Kt} = P_{It} T_t (r_t + \delta_t - \frac{\dot{P}_{It}}{P_{It}}) + \varphi_t, \tag{28}$$

15. See footnote 14 in Berndt and Fuss (1986).

where P_I is the implicit price index of capital stock, T is the income tax factor, r is the internal rate of return on capital, δ is the implicit depreciation rate of capital, and φ is the indirect tax factor including royalty payments.[16] The term $\dot{P}_{It}/P_{It}$ represents gains or losses from holding capital assets. The user cost of capital includes the sum of the opportunity cost of capital and the depreciation of the asset, and a term that subtracts capital gains or losses from holding the assets.

The user cost of capital depends on the estimate of the rate of return or the opportunity cost of capital *r*. The empirical literature uses one of two alternatives to estimate the rate of return and the user cost of capital: 1) the rate calculated endogenously from the System of National Accounts and 2) a rate chosen exogenously from observed market rates (Schreyer, Diewert and Harrison 2005). Rates calculated endogenously use data from the national accounts on estimates of capital stock and capital income that consist of gross operating surplus and the portion of mixed income attributable to capital. Using the formula for the cost of capital and recognizing that capital income just pays for capital services make it possible to solve for the rate of return on capital.

Alternatively, the rate of return can be taken from other sources—a rate of return observed in financial markets, for example. Here, there are several choices: a risk-free rate of return such as a government bond rate; a corporate debt rate that takes the risk of the business sector into account; or a weighted average of corporate debt and equity rates that recognizes that the corporate sector is financed by a mixture of debt and equity.

For this paper, the endogenous rate is used to estimate MFPG with variable capital utilization for the manufacturing sector. The rate of return observed in the financial markets, such as the government bond rate, reflects the average return. Those rates should be adjusted for inherent risks for individual industries before they can be used to calculate the user cost of capital for those industries. Although the methods for this adjustment are well established, data for the adjustment are not readily available.

The endogenous rate of return is calculated using data on capital income and the actual level of capital stock. It can be highly volatile, especially at the industry level. The volatility partly reflects changes in capacity utilization. To implement the approach proposed in this study, the *ex-ante* user cost of capital that excludes the effect of capacity utilization is needed. It is assumed that the *ex-ante* user cost of capital is an average of the *ex-post* user cost over a period of time. It is calculated as the sum of smoothed real rate of return and depreciation minus smoothed capital gains:

$$P_{Kt} = P_{It} T_t (\overline{r}_t^* + \delta_t - \overline{\pi}_t^*) + \varphi_t, \tag{29}$$

where $\overline{r}_t^*$ is the nine-year moving average of the real endogenous rate of return and $\overline{\pi}_t^*$ is the nine-year moving average of real capital gains/losses on fixed capital assets. The real endogenous rate of return and the real capital gains/losses are calculated by deflating the nominal rate of return and the nominal capital gains/losses by the inflation rate based on the Consumer Price Index:

$$r_t^* (\text{or } \pi_t^*) = \frac{1 + r_t (\text{or } \pi_t)}{1 + f_t} - 1 \quad \text{and} \quad \overline{r}_t^* (\text{or } \overline{\pi}_t^*) = \frac{1}{9} \sum_{s=-4}^{4} r_{t+s}^* (\text{or } \pi_{t+s}^*). \tag{30}$$

16. The indirect tax factor is calculated as the ratio of the sum of net indirect taxes on production and royalty payments to real net capital stock. Data on indirect taxes on production and royalties come from the National Accounts.

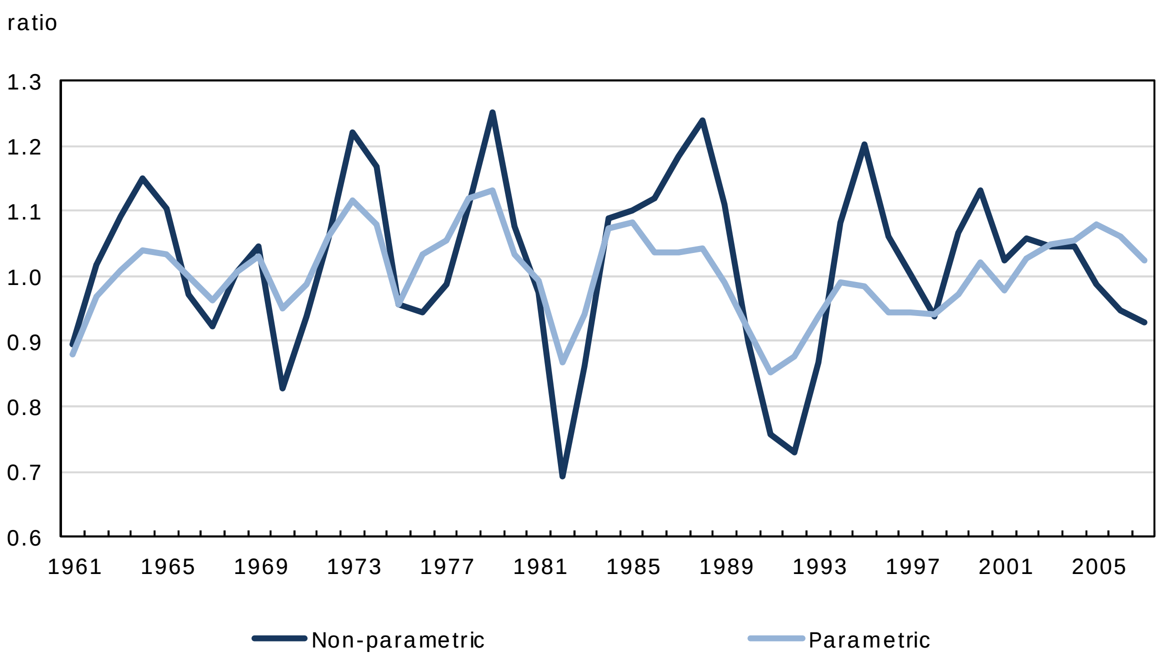
5 Empirical results for Canadian business sector industries

In this section, non-parametric and parametric approaches for measuring MFPG with variable capacity utilization are used to examine whether the estimate of MFPG that is derived from the traditional growth accounting framework is affected by the variation in capacity utilization over time. For the non-parametric approach, the one developed in this paper will be used. For the parametric approach, the one developed by Berndt and Hesse (1986) will be used (Appendix, Section 7.2). First, the results for the manufacturing sector will be presented, and then, the results for other industries. The data are from the Statistics Canada KLEMS database (CANSIM table 383-0022).

Before the effect of capacity utilization on MFPG is examined, alternate estimates of capacity utilization are presented. The non-parametric approach produces an estimate of capacity utilization as the ratio of capital-in-use to capital-in-place ($CU_K \equiv K^u / K^s$). The parametric approach produces an estimate of the ratio of actual output to capacity output ($CU_Y \equiv Y / Y^*$).

Chart 1

Alternative measures of capacity utilization, manufacturing



Source: Statistics Canada, authors' calculations based on KLEMS database.

Chart 1 displays the two measures of capacity utilization for the Canadian manufacturing industry from 1961 to 2007. They depict similar trends of and variations in capacity before 2000. Both declined significantly during the two economic downturns in this period. Capacity utilization increased after the early 1990s as a result of the high-tech boom and growing demand for Canadian manufacturing exports in the U.S. market with a depreciation of the Canadian dollar vis-à-vis the U.S. dollar.

As shown in the chart, the non-parametric measure of capacity utilization is more volatile than the parametric one. The parametric measure is the ratio of actual to capacity output and hence its movement depends on the movements of capacity utilization of capital and the optimal labour input. Since the parametric approach reveals production technology globally based on regression analysis over the whole sample period the period-to-period movements of optimal labour input are smoothed and less volatile. As a result, the ratio of actual to capacity output becomes less volatile than the capacity utilization of capital.

After 2000, the manufacturing sector experienced a sharp decline in capacity utilization. Survey measures show capacity utilization declined in 16 out of the 20 manufacturing industries in that period. Some of the excess capacity that developed post-2000 coincided with the general economic slowdown in North America, the significant appreciation of the Canadian dollar, and increased global competition, which resulted in large declines in manufacturing exports in the period. The emergence of excess capacity in several industries after 2000 was also related to major long-term structural adjustments (Baldwin et al. 2011). For example, the electronic product manufacturing sector underwent readjustment after the collapse of the dot-com bubble in the early 2000s. The non-parametric approach captures such a phenomenon and shows a decline in capacity utilization, but the parametric approach completely misses this well-documented phenomenon.[17]

The post-2000 difference between the non-parametric and the parametric measures of capacity utilization arises from differences in their capability of capturing short-run changes in an economy. The Canadian economy, especially the manufacturing industry, experienced large increases in employment and labour income share after 2000. These changes have no impact on the non-parametric measure of capacity utilization of capital because the actual labour input is used in the non-parametric approach. On the other hand, the parametric approach uses averaged production technology to predict optimal labour input and capacity output year by year. Therefore, it cannot fully capture the post-2000 up-swing of employment and labour income share and hence it underestimates optimal labour input and capacity output, which leads to an upward bias of the estimated actual to capacity output ratio over this period.

Table 1

Alternative measures of multifactor productivity growth for Canadian manufacturing, 1961 to 2007

	Mean	Standard deviation	Mean post 2000	Correlation with output growth
	percent			
Real value-added measure				
Unadjusted for changes in capacity utilization	1.65	3.52	-0.28	0.85
Adjusted for changes in capacity utilization, non-parametric, Gu-Wang	1.64	2.82	0.94	-0.12
Adjusted for changes in capacity utilization, parametric, Berndt-Hesse	1.76	0.10	1.60	...
Real gross output measure				
Unadjusted for changes in capacity utilization	0.54	1.14	-0.09	0.79
Adjusted for changes in capacity utilization, non-parametric, Gu-Wang	0.53	0.90	0.28	-0.20
Adjusted for changes in capacity utilization, parametric, Berndt-Hesse	0.56	0.05	0.49	...

Source: Statistics Canada, authors' calculations based on KLEMS database.

Table 1 presents the MFPG measures on a real value-added and real gross output basis for manufacturing from 1961 to 2007, adjusted for changes in capacity utilization using the

17. Measures of capacity utilization based on gross output for the parametric approach have also been produced.

parametric and non-parametric approaches. For comparison, MFPG from the standard growth accounting framework is also included, labeled as "Unadjusted for changes in capacity utilization."

The MFPG measure derived from standard growth accounting is pro-cyclical, as it includes the effect of the pro-cyclical variation in capacity utilization. Over the period, the correlation coefficient between the MFPG measure from the growth accounting and output growth was about 0.85 for the measure based on gross domestic product (GDP) and 0.79 for the measure based on gross output.

The non-parametric approach proposed in this paper corrects for the effect of cyclical fluctuation in capacity utilization. The correlation coefficients between the non-parametric estimate of MFPG and output growth were close to zero for both the value-added and gross output-based measures. This suggests that the pro-cyclicality of MFPG in Canadian manufacturing could largely be caused by mismeasurement of capital input, and that the effects of other factors (technology shocks, non-constant returns to scale, and price mark-ups) are small. These results support the findings in Basu (1996) for U.S. manufacturing.

The MFPG estimate derived from the parametric approach varies little over time, reflecting the assumption that the variable cost is a quadratic function with technological change represented as a time trend.

The alternative measures give comparable MFP growth rates over the entire period from 1961 to 2007. This suggests that while the MFPG estimate from the growth accounting approach reflects the short-run effect of capacity utilization; when it is averaged over a long period, the measure removes the short-run effects of cyclical capacity utilization.

Results from the standard growth accounting approach showed a large decline in MFPG in the Canadian manufacturing sector after 2000. From 2000 to 2007, GDP-based MFPG was -0.28% per year, well below the 1.65% annual growth rate over the entire 1961-to-2007 period. When the measure is adjusted for changes in capacity utilization over time, the GDP-based MFPG rate was about 0.94% per year for the 2000-to-2007 period using both the non-parametric and the parametric approaches. The capacity-utilization-adjusted MFPG rate for the post-2000 period is similar to or slightly below the average MFPG rate over the 1961-to-2007 period.

The results suggest that the post-2000 decline in MFPG in manufacturing is largely a result of the decline in capacity utilization. This is consistent with the findings of Baldwin et al. (2011) who use micro-plant-level data from the Annual Survey of Manufacturers.

While both the non-parametric and parametric approaches adjust the MFPG measure for the effect of capacity utilization, the non-parametric approach has a number of advantages.

First, as noted earlier, the capacity utilization measure from the non-parametric approach is more consistent with historical developments in post-2000 capacity utilization in Canadian manufacturing. Consistent with the development of excess capacity in the manufacturing sector in the post-2002 period, the non-parametric approach shows a decline in capacity utilization, whereas the parametric approach shows an increase.

Second, a parametric approach requires accurate estimates of many parameters in the variable cost functions, which, in turn, require a large sample and a consistent estimation method. Short time series may lead to inaccurate estimates of parameters or no convergence. Also, a parametric approach may give biased results owing to aberrant observations (Macdonald 2007).

Furthermore, the non-parametric approach has additional advantages for empirical analysis. First, it offers more practical adjustment for capacity utilization, because it is easier to implement than a parametric approach and is more consistent with the non-parametric tradition used by statistical agencies and researchers to measure MGPG. Second, the non-parametric approach is not sensitive to the choice of the *ex-ante* user cost of capital. As shown in Table 2, the choice of the real rate of return to calculate the user cost of capital has little effect on the MFPG derived from the non-parametric approach.

Table 2
Sensitivity of non-parametric multifactor productivity growth measure to the choice of the real rate of return on capital (Canadian manufacturing)

Real rate of return on capital net of real rate of capital gains/losses	Mean 1961 to 2007	Standard deviation 1961 to 2007	Mean post 2000	Correlation with output growth
		percent		
5 percent	1.60	2.81	1.26	-0.10
10 percent	1.63	2.81	1.04	-0.11
20 percent	1.66	2.84	0.84	-0.12
40 percent	1.68	2.90	0.69	-0.14

Source: Statistics Canada, authors' calculations based on KLEMS database.

The MFPG estimate that is adjusted for capacity utilization and the one that is derived from the standard growth accounting framework for the industries of the business sector at the NAICS 2-digit level are presented in Table 3.

For the total business sector, pro-cyclicality in the standard measure of MFPG has been reduced considerably after adjustment for the effect of changes in capacity utilization. The correlation coefficient between the measured MFPG and output growth dropped from 0.73 to 0.20 for the GDP-based measures, and from 0.66 to 0.09 for the gross output-based measures. This implies that variable capacity utilization accounts for a large portion of the pro-cyclicality in the standard MFPG measure.

Much of the post-2000 decline in MFPG results from cyclical fluctuations arising from changes in capacity utilization. In the business sector overall, the standard measure of MFPG fell 0.37% per year during the 2000-to-2007 periods; MFPG adjusted for variations in capacity utilization rose 0.02% per year over the same period.

The effect of capacity utilization varies across industries. It is generally substantial in goods-producing industries such as manufacturing,[18] but in services-producing industries (except wholesale trade), the effect is much less.

Adjusting for capacity utilization does not always reduce the volatility of measured MFPG (Table 3). Moreover, the volatility increases considerably for mining. This seems to contradict the notion that the capacity-utilization-adjusted MFPG series should be consistently less volatile than the unadjusted series because output growth and capacity utilization are expected to be positively correlated.

This higher volatility should be expected to occur in some circumstances. The measured capacity utilization rate in this analysis is based on the assumption that, in each period, the economy attained short-run optimization with factor prices fully adjusted to changing demand and supply conditions. However, because of a sticky wage rate and the market price of capital in the short-run, changes in output prices may lead to greater volatility in the measured capacity utilization rate than actually exists. If so, the simultaneous movement of output growth and capacity utilization might be weakened, especially in industries like mining that frequently

18. The results also show that the adjusted MFPG for the mining sector is less cyclical than the standard MFPG measure. This may reflect the shift in the mix of the mining sector towards oil sands and low-grade mines, rather than changes in capacity utilization.

experienced dramatic changes in output prices. In this case, the adjusted MFPG may reflect excess volatility due to sticky factor prices in the short run.[19]

Table 3
Multifactor productivity growth (MFPG) by industry, 1961 to 2007

	Mean 1961 to 2007		Standard deviation 1961 to 2007		Mean post 2000		Correlation with output growth	
	Standard	This paper	Standard	This paper	Standard	This paper	Standard	This paper
			percent				correlation coefficient	
MFPG based on gross domestic product								
Total business sector	0.36	0.41	1.67	1.68	-0.37	0.02	0.73	0.20
Agriculture	2.12	2.13	6.88	8.50	1.55	1.39	0.96	0.46
Mining	-1.73	-1.10	6.04	11.39	-5.73	-2.38	0.79	0.03
Utilities	1.21	1.49	4.12	4.94	1.30	1.54	0.64	0.19
Construction	0.29	0.25	3.94	3.70	-0.68	-1.54	0.47	0.31
Manufacturing	1.65	1.64	3.52	2.82	-0.28	0.94	0.85	-0.12
Wholesale trade	1.96	1.96	3.17	3.30	2.28	1.80	0.80	0.31
Retail trade	1.62	1.68	3.15	3.30	1.50	1.38	0.82	0.58
Transportation	1.34	1.38	3.44	3.14	-0.46	-0.54	0.79	0.62
Information	2.02	1.88	2.52	3.56	2.58	0.82	0.63	0.72
Finance, insurance and real estate	-1.05	-0.80	2.84	3.40	0.32	0.15	0.57	0.21
Professional	-2.16	-2.04	3.35	3.36	-0.13	-0.72	0.42	0.58
Administration	-2.09	-2.05	3.94	3.88	-0.37	-0.43	0.22	0.14
Education	1.14	1.14	11.82	11.64	2.34	2.06	0.62	0.61
Health	-0.37	-0.25	4.24	3.68	-2.62	-2.27	0.72	0.59
Arts, entertainment	-3.03	-2.78	6.04	5.60	-1.20	-1.22	0.60	0.52
Accommodation, food	-2.39	-2.19	4.32	3.97	0.57	0.78	0.26	0.31
Other services	-1.14	-1.08	2.96	3.15	0.47	0.15	0.53	0.43
MFPG based on gross output								
Total business sector	0.18	0.20	0.84	0.84	-0.18	0.01	0.66	0.09
Agriculture	1.02	0.99	3.87	4.57	0.56	0.44	0.76	0.27
Mining	-1.31	-0.98	4.39	7.62	-4.02	-1.78	0.69	-0.09
Utilities	0.97	1.19	3.31	3.94	0.92	1.06	0.55	0.15
Construction	0.12	0.11	1.64	1.55	-0.28	-0.62	-0.01	-0.11
Manufacturing	0.54	0.53	1.14	0.90	-0.09	0.28	0.79	-0.20
Wholesale trade	1.28	1.28	2.12	2.19	1.33	1.05	0.65	0.19
Retail trade	1.07	1.12	2.12	2.23	0.92	0.86	0.65	0.58
Transportation	0.81	0.82	1.98	1.78	-0.26	-0.31	0.60	0.45
Information	1.41	1.30	1.79	2.42	1.47	0.46	0.47	0.65
Finance, insurance and real estate	-0.74	-0.58	1.85	2.20	0.18	0.08	0.24	0.15
Professional	-1.62	-1.52	2.48	2.51	-0.09	-0.44	0.35	0.48
Administration	-1.45	-1.42	2.73	2.66	-0.26	-0.26	0.16	0.03
Education	0.62	0.62	5.70	5.57	1.66	1.46	0.59	0.57
Health	-0.26	-0.18	3.17	2.77	-1.93	-1.68	0.67	0.54
Arts, entertainment	-1.75	-1.62	3.43	3.19	-0.62	-0.63	0.52	0.46
Accommodation, food	-1.32	-1.22	2.33	2.14	0.27	0.38	0.21	0.21
Other services	-0.84	-0.79	2.11	2.25	0.30	0.09	0.47	0.37

Source: Statistics Canada, authors' calculations based on KLEMS database.

19. We are grateful for Jianmin Tang for pointing this out.

6 Conclusion

Several studies have attempted to adjust for the impact of capacity utilization in the measurement of MFPG.

This paper describes a non-parametric approach that distinguishes between in-use and in-place quasi-fixed inputs that evaluates all variables and elasticities in a consistent manner. In this framework, capital utilization is derived from data on the *ex-ante* user cost of capital and is based on the economic theory of production.

This framework is used to estimate MFPG for manufacturing and other industries and the results are compared to those yielded by the Jorgenson non-parametric approach and the parametric approach developed by Berndt and Hesse (1986). The results suggest that variable capacity utilization is important in explaining the pro-cyclicality in the standard non-parametric measure of MFPG in Canadian goods-producing industries, especially manufacturing.

The approach proposed in this study indicates a large post-2000 decline in capacity utilization in Canadian manufacturing, which is consistent with other empirical evidence. Consistent with the evidence on the effect of capacity utilization on MFPG using micro-data from the Annual Survey of Manufacturers in Baldwin et al. (2011), the results show that the post-2000 decline in MFPG in the manufacturing sector was largely a result of the decline in capacity utilization.

7 Appendix

7.1 Short-run fixity and shadow value of capital

This appendix discusses a number of conceptual issues associated with the Berndt and Fuss (1985) non-parametric approach for measuring MFPG with variable capacity utilization. It first shows that the shadow value of capital at the actual level of capital used is zero when capital is underutilized. It then argues that the Berndt and Fuss approach compares output with a bundle of inputs that does not correspond to that output.

Consider a profit-maximization problem in the short run when capital is fixed. The short-run fixity of capital imposes a constraint on production. The shadow value approach is often used to value such a constraint. In theory, the term "shadow value of capital" refers to the value of relaxing the constraint on capital. Given output and input prices, firms choose labour input and capital to maximize their profits subject to the constraint that capital used is less than capital-in-place, that is

$$Max_{K^u,L} \quad PY(K^u, L) - P_L L \qquad s.t. \quad K^u \le K^s. \tag{31}$$

The associated Lagrange function is

$$\Lambda(L, K^u, \lambda) = PY(L, K^u) - P_L L - \lambda(K^u - K^s). \tag{32}$$

The Lagrange multiplier (λ) refers to the shadow value of capital. The first-order conditions for profit maximization are:

$$\begin{aligned} \frac{\partial \Lambda(L, K^u, \lambda)}{\partial L} &= P\frac{\partial Y(L, K^u)}{\partial L} - P_L = 0 \\ \frac{\partial \Lambda(L, K^u, \lambda)}{\partial K^u} &= P\frac{\partial Y(L, K^u)}{\partial K^u} - \lambda = 0. \end{aligned} \tag{33}$$

The optimal level of labour input and capital input $\{L, K^u\}$ can be solved from (33), with each bundle of labour and capital-in-use corresponding to a given level of output.

The Kuhn-Tucker conditions associated with (32) can be written as

$$K^u \le K^s \quad \text{and} \quad \lambda(K^u - K^s) = 0, \tag{34}$$

which implies that

$$\lambda = \begin{cases} P\dfrac{\partial Y(L, K^u)}{\partial K} > 0, \ \text{if} \ K^u = K^s \\ \quad 0, \ \text{if} \ \ K^u < K^s. \end{cases} \tag{35}$$

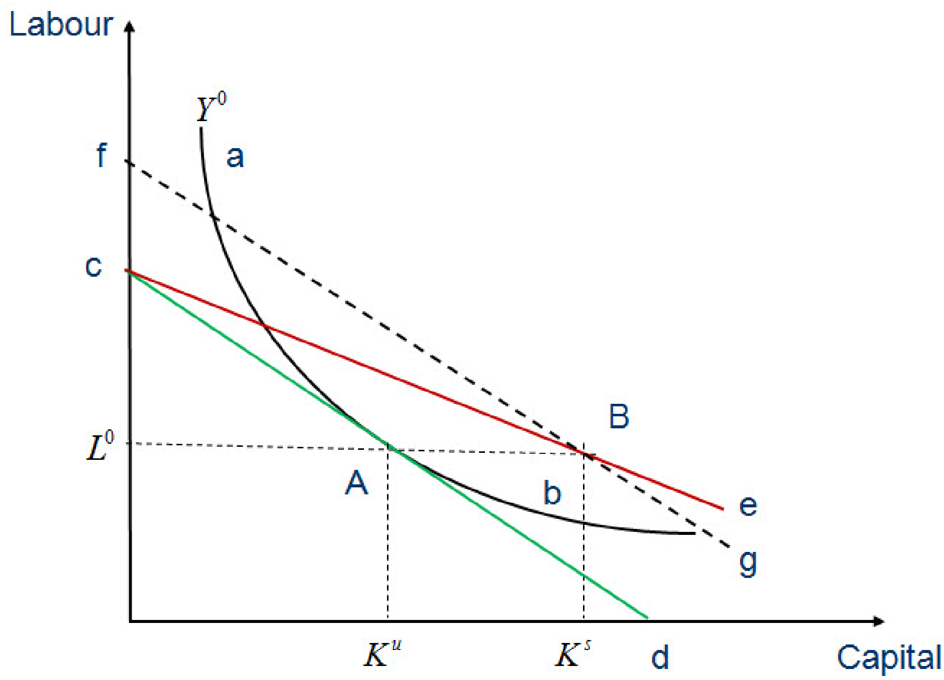
Equation (35) suggests that the shadow value of capital is equal to zero when capital is underutilized. This is intuitive, because adding extra capital adds nothing to profits when existing capital has not been fully used. The other implication of (35) is that the shadow value of capital is equal to the value of the marginal product of capital when capital in use is equal to the actual level of capital, that is, when the constraint on capital stock is binding.

Figure 1 sets out the MFPG measurement problem faced by the analyst. Curve ab represents the isoquant corresponding to output Y^0. The dashed line fg is the isocost line that represents $P_L L^0 + P_K K^s = C$. In the short run, $C > PY^0$. Our solution is to replace K^s by K^u such that $P_L L^0 + P_K K^u = PY^0$ (isocost line cd). The Berndt-Fuss solution is to find Z_K such that $P_L L^0 + Z_K K^s = PY^0$ (isocost line ce). Point A is the short-run optimization point at which $Y^0 = Af(L^0, K^u)$, $P\,\partial Y(L^0, K^u)/\partial L = P_L$, and $P\,\partial Y(L^0, K^u)/\partial K = P_K$. As a result, income shares evaluated at point A are a valid approximation of output elasticities that are used for measuring MFPG. With output and input prices being unchanged, point B is not an optimization point, because

$$\frac{\partial^2 f(L,K)}{\partial L \partial K} \geq 0 \quad \Rightarrow \quad P\frac{\partial Y(L^0, K^s)}{\partial L} > P\frac{\partial Y(L^0, K^u)}{\partial L} = P_L. \qquad (36)$$

Therefore, applying marginal analysis at point B is not valid. In addition, Berndt and Fuss (1986) compare output at point A with inputs at point B to estimate MFPG. Consequently, Berndt and Fuss (1986) implicitly assume full utilization of capital.

Figure 1



Notes: Figure 1 explains the MFPG measurement problem.

The horizontal axis represents capital stock, and the vertical axis represents labour.

Curve *ab* represents the isoquant corresponding to the observable output that is denoted as Y^0.

The dashed line *fg* is the isocost line that represents observable labour and capital costs in production. The observable labour cost equals wage rate times labour input and observable capital cost equals market user cost of capital times capital in place.

Point B represents the combination of the observed labour input (L^0) and the observed capital in place (K^s).

In the short run with capital being under-utilized, total cost exceeds total revenue because capital is fully paid but not fully utilized. The solid line *cd* is the isocost line that is tangent to the isoquant *ab* at point A. So the isocost *cd* represents minimal cost for producing the observed output (Y^0), and point A represents the optimal combination of labour input (L^0) and capital in use (K^u). In this situation, total cost and total revenue are the same. So isocost line *cd* is parallel with but below the isocost line *fg*.

The Berndt-Fuss approach is to keep capital in place (K^s) unchanged, but value it with the internal user cost of capital (Z^k) instead of the market user cost of capital, such that total cost and total revenue are the same. As a result, their isocost line (*ce*) becomes flatter and crosses isocost line *cd* at point c and isocost line *fg* at point B.

Point A is the short-run optimization point at which the observable output (Y^0) is produced by using the observed labour input (L^0) and capital in use (K^u), and the values of marginal products of capital and labour are equal to their corresponding market prices. Therefore, income shares evaluated at point A are a valid approximation of output elasticities that are used for measuring MFPG.

With output and input prices being unchanged, point B is not an optimization point, because the value of marginal products of labour at point B surpasses the market price of labour (wage rate). Therefore, the marginal analysis at point B is not valid. Actually, the output that can be produced at point B will be greater than at point A. Berndt and Fuss (1986) inconsistently compare output at point A with inputs at point B to estimate MFPG. Consequently, Berndt and Fuss (1986) implicitly assume full capital utilization.

7.2 Parametric approach for measuring multifactor productivity growth with variable capital utilization

This section presents a parametric approach for estimating MFPG with changing capacity utilization. The approach is developed by Berndt and Fuss (1986), Berndt and Hesse (1986), and Morrison (1985). It starts by estimating a short-run variable cost function. It then estimates the capacity output, which corresponds to the minimum point on the average total cost curve if constant returns to scale prevailed in the long run. MFPG is calculated as the log change in the average total cost at the capacity output. Or equivalently, it can be calculated as the difference between capacity output growth and combined input growth. Because MFPG is calculated at the capacity output, it does not include the effect of changes in capacity utilization. By contrast, the MFPG measure from the traditional growth accounting method is calculated at the actual level of output. It includes the effect of changes in capacity utilization as the growth in actual output and may partially reflect increases in the utilization of existing inputs rather than from changes in technology.

We follow Berndt and Hesse (1986) in this section. A translog variable cost function is employed. Let the variable inputs be labour (L), energy (E), materials (M), and services (S), and denote their respective prices as P_I for $I=\{L,E,M,S\}$. Let K be capital stock, and t be the effect of technical progress. Capital stock is assumed to be fixed in the short run. The variable inputs (labour, energy, materials, and services) and fixed capital input are used to produce output Y. The translog short-run or variable cost function is written as

$$\begin{aligned}\ln(VC)=\alpha_0+\sum\nolimits_I \alpha_i \ln(I)+\sum\nolimits_I \frac{\beta_{ii}}{2}\left[\ln(I)\right]^2+\sum\nolimits_{I\neq J}\sum\nolimits_J \beta_{ij}\ln(I)\ln(J)\\ \text{for } I,J=\left\{Y,K,P_L,P_E,P_M,P_S,e^t\right\},\quad \text{and } i,j=\left\{Y,K,L,E,M,S,t\right\}\ ,\end{aligned} \tag{37}$$

where $\beta_{ij}=\beta_{ji}$. There are 36 parameters in equation (37). Several restrictions must be imposed for the cost function to be well defined. The condition of homogeneity of degree one in the prices of variable inputs implies the following constraints on the parameters:

$$\begin{aligned}&\sum\nolimits_{i=\{L,E,M,S\}}\alpha_i=1\\ &\sum\nolimits_{i=\{L,E,M,S\}}\beta_{ij}=0,\ \text{for } j=\left\{Y,K,L,E,M,S,t\right\}.\end{aligned} \tag{38}$$

The assumption of constant returns to scale implies additional constraints:

$$\begin{aligned}&\sum\nolimits_{i=\{Y,K\}}\alpha_i=1\\ &\sum\nolimits_{i=\{Y,K\}}\beta_{ij}=0,\ \text{for } j=\left\{Y,K,L,E,M,S,t\right\}.\end{aligned} \tag{39}$$

There are 16 restrictions on parameters given by equations (38) and (39). However, one of them is redundant because the summation of equations over $j=\{Y,K\}$ from (38) is identical to that for $j=\{L,E,M,S\}$ from (39). Therefore, there are 15 independent restrictions, and a total of 21 independent parameters must be estimated. Substituting restrictions (38) and (39) in the variable cost function (37) yields a variable function that is homogenous of degree one in $\{Y,K\}$ and in $\{L,E,M,S\}$. After some algebra, we have

$$\ln(VC)-\ln(K)-\ln(P_S)$$

$$=\alpha_0+\sum_I \alpha_i \ln(I)+\sum_I \frac{\beta_{ii}}{2}\left[\ln(I)\right]^2+\sum_{I\neq J}\sum_J \beta_{ij}\ln(I)\ln(J) \quad (40)$$

$$\text{for } I,J=\left\{\frac{Y}{K},\frac{P_L}{P_S},\frac{P_E}{P_S},\frac{P_M}{P_S},e^t\right\}, \quad \text{and } i,j=\{Y,L,E,M,t\}\ .$$

The variable inputs are chosen to minimize short-run variable cost. Hotelling's lemma implies

$$\frac{\partial VC}{\partial P_I}=I, \quad \text{for } I=\{L,E,M,S\}. \quad (41)$$

For profit maximization, the price of output is set equal to marginal cost, that is, $\partial VC/\partial Y=P$. The shares of labour, energy and materials in variable costs and the ratio of nominal output to variable cost can be written as

$$S_I\equiv\frac{P_I I}{VC}=\frac{\partial VC}{\partial P_I}\frac{P_I}{VC}=\frac{\partial\ln(VC)}{\partial\ln(P_I)} \quad \text{for } I=\{L,E,M\}, \text{ and } S_Y\equiv\frac{PY}{VC}=\frac{\partial\ln(VC)}{\partial\ln(Y)}. \quad (42)$$

Logarithmically differentiating (40) with respect $I=\{Y,P_L,P_E,P_M\}$ gives

$$S_i=\alpha_i+\sum_J \beta_{ij}\ln(J)$$
$$\text{for } J=\left\{\frac{Y}{K},\frac{P_L}{P_S},\frac{P_E}{P_S},\frac{P_M}{P_S},e^t\right\},\ j=\{Y,L,E,M,t\},\ \text{and } i=\{Y,L,E,M\}. \quad (43)$$

In the empirical analysis, the variable cost equation (40) and four "share" equations in (43) are estimated using a full information maximum likelihood estimator (FIML).

MFPG is calculated as capacity output growth minus the growth of combined inputs that are used to produce capacity output. Capacity output (Y^*) minimizes total cost (ATC)

$$Min_Y(ATC)=Min_Y\left(\frac{VC}{Y}+\frac{P_K K}{Y}\right) \quad (44)$$

where P_K is the expected user cost of capital.

The first-order condition for the minimization of total cost is

$$\frac{1}{Y^*}\frac{\partial VC}{\partial Y}-\frac{VC^*}{Y^{*2}}-\frac{P_K K}{Y^{*2}}=\frac{1}{Y^*}\frac{VC^*}{Y^*}\frac{\partial\ln(VC)}{\partial\ln(Y)}-\frac{VC^*}{Y^{*2}}-\frac{P_K K}{Y^{*2}}=\frac{VC^*}{Y^{*2}}S_Y^*-\frac{VC^*}{Y^{*2}}-\frac{P_K K}{Y^{*2}}=0$$
$$\Rightarrow \quad VC^*(S_Y^*-1)=P_K K, \quad (45)$$

where VC^* is a function of Y^* and Y^{*2}, and S_Y^* is a function of $\ln(Y^*)$. The first-order condition is used to solve for capacity output. Because the equation is non-linear in Y^*, there is no closed form solution for Y^*. A numerical procedure is employed to solve for Y^*.

Once capacity Y^* is known, it is possible to calculate the minimal variable cost VC^*, the quantity of variable inputs used to produce capacity output $\{L^*, E^*, M^*, S^*\}$ and the share of variable inputs in variable cost S_L^*, S_E^*, S_M^*, S_S^* and S_Y^* from equations (40) and (43). MFPG can then be calculated as

$$\Delta \ln(MFP) = \Delta \ln(Y^*) - \overline{s}_L \Delta \ln(L^*) - \overline{s}_E \Delta \ln(E^*) - \overline{s}_M \Delta \ln(M^*) - \overline{s}_S \Delta \ln(S^*) - \overline{s}_K \Delta \ln(K), \tag{46}$$

where

$$I^* = \frac{S_I^* VC^*}{P_I} \quad \text{for } I = \{L, E, M, S\} \text{ , and}$$
$$s_I = \frac{S_I^*}{S_Y^*} \quad \text{for } I = \{L, E, M, S\} \quad \text{and} \quad s_K = 1 - (s_L + s_E + s_M + s_S). \tag{47}$$

The long-run output price, prime capacity utilization, and dual (cost) capacity utilization can be calculated as

$$P^* = \frac{S_Y^* VC^*}{Y^*}, \quad CU_P = \frac{Y}{Y^*}, \quad CU_C = \frac{VC + P_K K}{VC^* + P_K K}. \tag{48}$$

7.3 Price mark-up and capacity utilization

The price mark-up (μ) is often defined as a ratio of output price (P) to marginal cost of output ($MC_Y \equiv \partial C / \partial Y$), i.e.

$$\mu \equiv \frac{P}{MC_Y}. \tag{49}$$

It is related to the rate of return to scale (γ) and the price elasticity of output (ε_{PY}). The rate of return to scale is referred to as the output elasticity with respect to cost, which is

$$\gamma = \frac{\partial \ln Y}{\partial \ln C} = \frac{\partial Y}{\partial C}\frac{C}{Y} = \frac{P}{MC_Y}\frac{PY-(PY-C)}{PY} \equiv \mu(1-\zeta), \tag{50}$$

where the economic profit rate (ζ) is the ratio of economic profits to total revenue. Also, firms are modeled as choosing the amount of output produced such that marginal revenue is equal to the marginal cost of output, which yields

$$\begin{aligned} &\frac{\partial (PY)}{\partial Y} = P + Y\frac{\partial P}{\partial Y} = P(1+\varepsilon_{PY}) = MC_Y \\ &\Rightarrow \quad \mu = \frac{P}{MC_Y} = \frac{1}{1+\varepsilon_{PY}}. \end{aligned} \tag{51}$$

The marginal cost (MC_Y) in (49) can be replaced with the marginal variable cost (MVC_Y) because they are the same in the short run. Because price levels are not observable, we can only calculate the index for price mark-up by using corresponding price indices. Let the superscript I denote index, we can approximate the price mark-up index as

$$\mu^I = \frac{P^I}{\sum_J \varpi_J P_J^I}, \quad \text{with} \quad \varpi_J \equiv \frac{P_J J}{\sum_J P_J J}, \quad \text{and} \quad J \in \left(\text{variable inputs}\right). \tag{52}$$

Using data from the Statistics Canada KLEMS database, the price mark-up index can be calculated based on (52).

To investigate the influence of the assumptions of perfect competition and constant return to scale, the correlation coefficient between the price mark-up and the capacity utilization estimated from (20) is calculated. Let ρ be the correlation coefficient and N be the sample size; the t-statistics can then be calculated as

$$t = \rho\sqrt{\frac{N-1}{1-\rho^2}}. \tag{53}$$

The results of the correlation coefficient for the total business sector and 2-digit NAICS industries are presented in Table 4. The correlation coefficient is small and statistically insignificant in the total business sector and in all industries except agriculture, mining and education.

Table 4
Correlation coefficient between price mark-up and capacity utilization

	Based on gross domestic product		Based on gross output	
	coefficient	t-statistic	coefficient	t-statistic
Total business sector	0.0779	0.5183	0.0710	0.4721
Agriculture	0.2754	1.8999	0.4439	3.2860
Mining	0.5389	4.2431	0.4852	3.6803
Utilities	0.2079	1.4097	0.2504	1.7155
Construction	-0.0052	-0.0344	0.0308	0.2042
Manufacturing	0.0889	0.5918	0.0901	0.5999
Wholesale trade	0.0533	0.3543	0.0736	0.4899
Retail trade	0.0956	0.6368	0.1064	0.7098
Transportation	0.0650	0.4323	0.0692	0.4602
Information	-0.0535	-0.3556	-0.0381	-0.2529
Finance, insurance, real estate	0.1712	1.1529	0.1493	1.0019
Professional	0.2551	1.7500	0.1846	1.2459
Administration	0.0409	0.2713	-0.1961	-1.3265
Education	-0.3936	-2.8397	-0.3988	-2.8844
Health	-0.1603	-1.0770	-0.2018	-1.3666
Arts, entertainment	0.0532	0.3531	-0.0349	-0.2318
Accommodation, food	-0.2442	-1.6702	-0.2895	-2.0066
Other services	0.1569	1.0538	0.1345	0.9004

Note: The critical value of t-distribution at 95% significant level with 44 degrees of freedom (sample size mines two) is 2.02.
Source: Statistics Canada, authors' calculations based on KLEMS database.

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