In Search of Intergenerational Credit Constraints among Canadian Men: Quantile Versus Mean Regression Tests for Binding Credit Constraints

by

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Abstract

Several recent papers have cited non-linearities in the relationship between incomes of parents and their children as evidence of important intergenerational credit constraints. This paper argues that any pattern in the conditional expectation function can be justified by a properly constructed story with credit constraints. This raises questions about the validity of the approach. Quantile regressions provide an alternative test. Using data from Canadian tax files, this paper finds results contrary to the credit constraints hypothesis; the non-linearities in the regression function are driven by the low-ability (unconstrained) sons rather than high-ability (potentially constrained) sons.

Key Words: intergenerational income mobility, intergenerational credit constraints, quantile regression, lifecycle bias

JEL classification: I22, J24, J62

I. Introduction

In the second half of the 20th century, economists and sociologists have devoted considerable effort to measuring the degree of association between the economic achievement of parents and their children. More recently, a second thrust has developed to explain the observed association. Becker and Tomes (1979 and 1986) provide a model that suggests that intergenerational credit constraints are a potentially important determinant of the degree of income persistence. However, until recently there has been little empirical evidence to support the importance of credit constraints.

The intuition behind the prediction in Becker and Tomes is straightforward. Consider a father-son pair. Why might their incomes be positively related? First, just as height, weight, eye colour, and foot speed are passed from one generation to the next, market ability is also transmitted from father to son. In fact, some observable characteristics are elements of market skill.The correlation between the father's and son's skills indirectly links incomes.This connection is common to all father-son pairs. But the association between the father's and son's success might also be directly related due to credit constraints. Low-income fathers may be unable to borrow enough to efficiently fund the child's education. And so even if two sons were of equal ability, differences in financial resources among fathers cause variance in the educational and economic outcomes of the children. As a result, income regresses to the mean more slowly among credit-constrained families.

Tests of this theory have developed along several different paths. Some studies examine differences in the estimated degree of income persistence across the lifecycle. Using U.S. data, Behrman and Taubman (1990) shows that income persistence is higher when fathers' incomes are observed in the year their sons are 14 years old than when fathers are measured later in the lifecycle. While Behrman and Taubman suggest that this result is evidence of binding credit constraints that limit the educational choices of the family, Grawe (2000) points out that lifecycle income variance patterns can also explain the observation. A second approach divides the constrained from the unconstrained on the basis of bequest information. (The model of Becker and Tomes suggests that constrained fathers will leave little to nothing in the form of bequests.) Mulligan (1997) estimates income persistence separately in the two groups and finds little or no evidence to suggest important credit constraint effects.

This paper examines a third approach that looks for non-linearities in the degree of income persistence over fathers' incomes. The non-linearity in the regression is supposed to capture the fact that the constraint is less binding as fathers' incomes increase. Behrman and Taubman (1990) and Solon (1992) estimate non-linear regressions but do not find the hypothesized pattern. One explanation of their failure to detect the effects of credit constraints is that the sample size in the Panel Study of Income Dynamics (which both studies use) is too small to allow for accurate estimation of flexible non-linear functions. As a result, the authors are forced to use simple quadratic forms that may be mis-specifications. Corak and Heisz (1999) avoid this problem with the use of a vast data set extracted from Canadian tax files. They find non-linearities in mean income persistence that could be consistent with a credit constraints model.

This paper argues that non-linearities in income persistence do not provide the basis for a valid test of the credit constraints model. Section II will formally present the model of Becker and Tomes both with and without credit constraints. The next section shows that the positive relationship between father's income and son's ability makes it possible to explain any non-linear pattern with credit constraints. Section IV shows how quantile regressions provide an alternative to mean regression that does allow for rejection of the model. The reported results show that the Canadian data are not consistent with binding credit constraints. The final section concludes.

II. The Model

In order to theoretically understand the role of credit constraints in income persistence, I present a model originating in Becker and Tomes (1986) but more similar in form to the presentation in Mulligan (1997). Each family is composed of one father and one son. The father is endowed with ability a_f and schooling level h_f. These inputs produce wage income $w(a_f,h_f)$. In addition, the father may have access to financial assets x_f.

The problem of the father is to choose investments in the son so as to maximize $U(c_f,c_s)$ where c_f and c_s represent the consumption of the father and the son respectively. The investments can take one of two forms. First, the father can invest in the child's education h_s. Second, the father may invest in physical assets x_s which earn interest at rate r and are given to the child at adulthood. The father solves

$$\max_{c_f, c_s, h_s, x_s} U(c_s, c_f)$$
$$s.t. \quad c_f + h_s + x_s \leq w(a_f, h_f) + x_f$$
$$c_s \leq w(a_s, h_s) + (1 + r)x_s \tag{1}$$

Assuming there are diminishing returns to the investment in education ($w_{hh}< 0$), the parent's choice between asset bequests and the level of child's education is made by equating the marginal returns to the two forms of investment

$$w_h(a_s, h_s) = 1 + r. \tag{2}$$

If ability increases the productivity of the educational investment ($w_{ha} < 0$) and the interest rate earned on assets is common to all parents, high-ability children will be given more education than low-ability children. This can be seen in Chart 1, which depicts the wage as a function of educational investment for three children with different levels of ability. The optimal level of the child's education is found where the wage function has a slope $1 + r$. The important point to note from the parental decision is that all children of the same ability will receive the same level of education (and earn the same wage) *regardless of the parent's ability or income.*[1]

[1] In the absence of credit constraints, the educational choice problem is an example of the Rotten Kid Theorem in Becker (1974). If the schooling choice were made by the son, he would make exactly the same choice as the father. So long as the child's consumption is a normal good in the father's utility function, they will agree to make the efficient educational choice and maximize the family "pie." When credit constraints are present, the Rotten Kid Theorem fails since the child's choice of education changes the prices that the father faces. That is, devoting one more dollar to education tightens the constraint by one dollar. This failure of the Rotten Kid Theorem follows Bergstrom (1989).

The joint distribution of parent and child wages is derived from parents' investment decisions in combination with the transmission of ability from parent to child. Suppose that ability of the child is given by

$$a_s = y a_f + e_a \tag{3}$$

where e_a is a mean-zero, random term with variance s_a^2. Regression to the mean in observable characteristics suggests regression in unobservables like ability as well $(y < 1)$. In the absence of credit constraints, regression in log income results from regression in ability.

When credit constraints are added to Problem (1) of the form $x_s > k$ for some constant k (often zero), the first order condition becomes

$$\begin{gathered} w_h(a_s, h_s) = (1+r) + \frac{l}{U_2} \\ l > 0 \text{ iff the constraint binds} \end{gathered} \tag{4}$$

where l is the marginal value of relaxing the constraint by one dollar. (I will refer to this version of Becker and Tomes as the "credit constraint model.") The impacts of a binding constraint are easily derived. Since the marginal cost (in terms of utility) of investing in the child's education is higher for constrained families, educational expenditures are lower. So the earned income of constrained children will be lower than that of unconstrained children with like ability and income will regress more slowly to the mean. This market incompleteness would disappear if fathers seeking financial investments could invest in the constrained child's education--after all, the return to a dollar so invested exceeds the market interest rate. The usual assumption made to justify the persistence of the incomplete market is that laws against slavery and indentured servitude are crafted in such a manner that the investor could never reclaim his investment.

III. Credit Constraints and Non-Linearities

Taking the insights of Becker and Tomes as given, authors have looked to identify groups which are arguably more or less constrained in order to test for the importance of credit constraints. Behrman and Taubman (1990) (hereafter BT) hypothesizes that families with low-income fathers are more constrained than families with high-income fathers; income persistence should be negatively related to father's income. In order to test this hypothesis, the study uses Panel Study of Income Dynamics (PSID) data to estimate the regression

$$w_s = a + b_1 w_f + b_2 w_f^2 + e \tag{5}$$

where w_s is son's log earnings, w_f is father's log earnings and e is a mean-zero error term. $b_2 < 0$ supports the credit constraint hypothesis while $b_2 > 0$ is contrary to the hypothesis. Solon (1992), without formally stating the hypothesis, also estimates equation (5) using PSID data. Table 1 shows the results reported in these two studies. Both estimate a convex relationship between fathers' and sons' incomes contrary to the proposed credit constraint hypothesis. BT concludes that "the sign of the estimates suggests greater elasticities for higher parents' income, contradicting the Becker and Tomes' conjecture" (p. 122).

The conclusion that the results in Table 1 contradict Becker and Tomes is incorrect. The coefficient estimates do suggest a convex rather than concave relationship, but this is not in contradiction with the prediction made by Becker and Tomes. For reference, I repeat here the hypothesis presented by Becker and Tomes:[2]

> The direct relation between the earnings of parents and children in Eq. (7S.14) is likely to be concave rather than linear, because obstacles to the self-financing investments in children decline as parents' earnings increase.
>
> (7S.14) $w_s = f(a_s, w_f, k_s) + l_s$ (p. S12-S14)

where a_s is the child's endowed ability, l_s is "market luck" of the child (unknown at the time of the educational choice), and k_s includes government funding of education and a parental altruism parameter. The Becker and Tomes conjecture is that *conditional on the ability of the child*, the relationship between fathers' and sons' incomes will be concave. Studies like BT and Solon (1992) that do not control for the child's ability should not necessarily be expected to produce concave estimates.

This point is made very clearly in Corak and Heisz (1999) (hereafter CH). Using a roughly one-in-ten sample from the more than 300,000 father-son pairs drawn from Canadian tax files, they estimate a non-parametric, nearest-neighbourhood version of regression (5) and find that the conditional expectation function is S-shaped over fathers' incomes. The authors suggest that this finding might be explained by the credit constraints model. Fathers with very low income have sons of very low ability who do not seek education beyond that which is freely provided by the state. As a result, few if any low-income fathers face a binding constraint and those who are constrained are only slightly so. However, as the father's income and ability increase, the ability of the son also increases and more sons are severely constrained. The degree of income persistence rises. Finally, if the father's income is high enough, it is unlikely that a son of any level of ability will be constrained—income persistence falls.

In order to arrive at the hypothesis stated in BT, one must implicitly assume that the increase in fathers' incomes outpaces the increase in sons' abilities so that the constraint is less and less binding as fathers' incomes rise. The fact that two very different patterns (strictly concave and S-shaped) could be cited as evidence for the importance of credit constraints is worrisome. In fact, since the child's ability is correlated with the parent's income, *absolutely any* non-linear pattern could be justified by a properly crafted credit constraint story. This is demonstrated in Chart 2. The first two panels display the patterns suggest by BT and CH. Panel c is the "opposite" of the BT hypothesis. This pattern can be justified by reversing the implicit assumption in BT so that the increase in child's ability outpaces the father's ability to finance education. Panel d is the "opposite" of that observed by CH. Here, at low incomes, parents are unable to efficiently provide education to even the lowest ability children. As incomes rise, the financial gains outpace the increase in child ability and the constraint relaxes. Ultimately, the most able children are so able that even the most affluent parents face a binding constraint. Variations on these stories could obviously be created to explain any and every non-linear pattern.

[2] I have slightly changed notation from the original to match the notation used in this paper.

In order to better understand the intuition of Chart 2, suppose the degree of income persistence is given by

$$\begin{aligned} &b(w_f) = g + k(w_f, \overline{a_s}(w_f)) \\ &k(w_f, \overline{a_s}(w_f)) > 0 \text{ iff the constraint binds} \end{aligned} \tag{6}$$

where $\overline{a_s}(w_f)$ is the expected ability of the son conditional on father's log earnings and k captures the effects of the credit constraint. Then

$$\frac{db}{dw_f} = k_1 + k_2 \frac{\partial \overline{a_s}}{\partial w_f}. \tag{7}$$

Let w_f^* be the level of log income such that the father is able to efficiently self-finance the child's education even if the son has the maximum level of ability. Then for $w_f > w_f^*$ the degree of income persistence does not depend on fathers' incomes; $b = g$. However, below this level of fathers' income $b > g$. The pattern in b depends on how much b and g differ at different levels of fathers' incomes. A higher level of father's income relaxes the constraint ($k_1 > 0$) while higher ability of the child tightens the constraint ($k_2 > 0$). Since the abilities of the sons are positively correlated with the earnings of the fathers ($\frac{\partial \overline{a_s}}{\partial w_f} > 0$), the sign of (7) is indeterminate. By carefully adjusting the relationship between fathers' incomes and sons' abilities, $b(w_f)$ can take on any pattern.

Some economists may argue from introspection or on some other basis that the constraint only binds in some range of fathers' incomes. For example, an economist might propose the hypothesis that credit constraints bind for lower-middle class fathers. Does specifying the range over which the credit constraints bind lead to a rejectable test using patterns in mean regression? Again the answer is negative. This is because there is no reason to believe that the relationship between fathers' and sons' incomes is linear in the absence of credit constraints. Even if ability is linearly related across generations, the transformation from ability to wages may be highly non-linear. If empirical estimation did not produce a higher estimated degree of income persistence in the range that was thought to be constrained, the economist might yet claim that the results are consistent with the hypothesis if non-linearities in the absence of credit constraints are offsetting those caused by the constraints. Once again, any observed pattern can be justified.

IV. An Alternative Test of the Credit Constraints Hypothesis

While patterns in mean regression appear to be a poor test of the credit constraints hypothesis, a re-examination of the story told by CH shows that an alternative test is available using quantile regression. While most economists have focused on the change in the degree to which the credit constraint binds in a single family as parental income rises, CH notes that the fraction of families which are constrained also changes.[3] In families with low-income fathers, only the most able sons are credit constrained. So at low levels of father-income, the quantile regressions should be relatively flat for all but the highest quantiles. As parental income increases, the ability of the son also rises and a larger fraction of the families are constrained. Most of the quantiles should now be steeper with only the lowest quantiles (low-ability sons) showing a flatter, unconstrained pattern. Finally, fathers' incomes become high enough that almost no sons are constrained. All but the very highest quantiles should be flatter again. Note that even if income persistence is non-linear in the absence of credit constraints, we can test for the presence of credit constraints; the quantile regressions will be steeper at high quantiles than at low quantiles. Chart 3 shows the predicted shapes of the quantile regression lines.

An alternative to the picture in Chart 3 that bears consideration is that scholarships won by the best and brightest may relax credit constraints for this group. Since society may be especially concerned with the opportunities available to children of low-income families, these scholarships may be concentrated on the high-ability sons born to low-income fathers. This alters the predictions only slightly. If the scholarships are insufficient to entirely eliminate the credit constraints, then the slopes of the highest quantiles will still be steeper than the slopes of the lowest quantiles. In this case, the general prediction remains the same: high quantiles drive the non-linear pattern in the data. The only difference is that the effects will be most apparent in the middle-to-upper quantiles rather than in the topmost quantiles. If the scholarships are large enough so that the child can afford a level of education equal to or in excess of the efficient level, then the non-linearities will be driven by the middle-to-upper quantiles and the topmost quantiles may even be flatter than lower quantiles. So while scholarships complicate the predictions slightly, the essence of the predictions remains the same.

A. The Data

The data used to estimate the quantile regressions comes from the same tax file data set employed by CH. The sons are boys age 16-19 in 1982, matched to male adults in the same household (not necessarily biological fathers). A one-in-ten sample was created and then, from this sample, the oldest available son for each family was selected. (Note, the oldest available son may or may not be the oldest son in the family). This resulted in 56,141 father-son pairs. Removing those pairs with "fathers" born prior to 1908 or after 1952 resulted in a sample of 53,390 observations.[4] This is the data set for my analysis. (See CH for a fuller description of the creation of the Canadian men data set.)

For each father and son in the data set, the income reported on T1 tax forms was collected from 1978 to 1998. Through an examination of the mean and variance of reported incomes, several coding irregularities were found. It appears that a significant number of observations in 1978-1982 were assigned a value of $1 when, in other years, they would have been reported as $0. Similarly, in 1996, a significant number of observations were assigned an income of $2. It was

[3] This is, of course, assuming that credit constraints are present.
[4] A report of an exceptionally old or exceptionally young father may be the result of measurement error in year of birth or the adult male figure may be an older sibling or grandfather.

not possible to determine why the data included these anomalies. When I refer to "positive income reports," I mean incomes greater than $1 in 1978-1982 and greater than $2 in 1996.

Since classical measurement error in either the dependent or the independent variable will bias estimates of quantile regressions, I use a five year average of both sons' and fathers' observations. This method of error correction follows Solon (1992), Zimmerman (1992), Peters (1992), Couch and Dunn (1997), and CH. Fathers and sons are observed in 1978-1982 and 1994-1998 respectively.

B. Reviewing the Non-linearity

CH use a non-parametric method to estimate the degree of income persistence. Unfortunately, I know of no analogous method for the estimation of the quantile regressions. Instead, I will use a simpler non-linear estimation method—spline functions. Since these functions can be estimated with both mean and quantile regressions, this will allow me to see whether the quantile regression lines follow the pattern implied by the credit constraints model. Chart 4 plots both the median (o) and mean (*) regression lines for the sample; Table 2 reports the regression results.[5] Fathers' and sons' log income is defined as the log of the average wage and salary income in the five-year span conditional on reporting positive income in all five years.[6] The spline functions were allowed to have seven kinks at the points -4, -3, -2, -1, 0, 1, and 2 on the x-axis. I focus my study on the roughly 98 percent of observations with age-corrected father's income between -2 and 2.[7] The S-shape reported in CH is also found using the spline method of estimation, although it is more pronounced in the median regression than in the mean regression.

The wide variation in the ages of the fathers raises a possible explanation for the observed non-linearity that is entirely spurious. Grawe (2000) shows that the estimated degree of income persistence is negatively related to age in the PSID data set. The explanation for this relationship is that log income variance increases over the lifecycle. If fathers are observed later in life, a larger variance in fathers' log incomes is used to explain the variance in sons' log incomes–the estimated degree of income persistence will be lower.[8] Since there is reason to believe that observations with high or low values for father's log income are older fathers, the S-shaped pattern may be an artifact of the variation in fathers' ages.

To understand why low and high observations of father's income are likely to be found among older fathers, consider Chart 5. Panel *a* shows the permanent income profiles for three types of individuals (solid lines). Actual observations of income will deviate from the permanent profiles due to transitory shocks or classical measurement error (stars). The income profiles follow the familiar concave, upward-sloping pattern. In panel *b*, the transitory income shocks have been eliminated (perhaps by averaging incomes over several adjacent years) and the upward trend in

[5] Estimation of quantile regressions is developed in Koenker and Basset (1978).
[6] The same S-shape is found when "log income" is defined as the average of the five log-income observations or total market income is used in place of wage and salary income.
[7] It is important to nevertheless include splines outside this region so that outlier's do not influence the estimated slopes for the splines between –2 and 2.
[8] The correlation between fathers' and sons' incomes also falls as the age of the father at observation increases. This magnifies the effect of the lifecycle variance pattern.

incomes has been removed. Since the variance of log incomes rises over the lifecycle, the highest and lowest age-corrected incomes will be reported by older fathers. This is visible in panel *b*.

The lifecycle bias is a possible explanation only if the Canadian data exhibit a strong age dependence in estimated income persistence. Since income variance in Canada did not grow as much as in the United States, the results in Grawe (2000) do not speak directly to the Canadian data. In order to evaluate whether lifecycle bias was an important factor in the Canadian data, I estimate the degree of income persistence for all possible combinations of father and son observation years (fathers: 1978-1992 and sons: 1991-1998). In order to avoid a sample selection bias resulting from the retirement of older cohorts of fathers, I restrict the sample to fathers born between 1932 and 1942. Chart 6 plots the results. Each pairing of father-son observation year is represented by a point. I connect the mean estimate (conditional on the year of father observation) by a solid line. The highest average estimate of income persistence is found when fathers are observed in 1978 while the lowest is found when fathers are observed in 1992. The estimated degree of income persistence falls by nearly 50% as the fathers grow 14 years older.

Chart 6 demonstrates that age-dependence of income persistence estimates is a characteristic of the Canadian data. But it may not be the cause of the observed non-linearity. In order to see if the S-shaped pattern observed in the full data set is driven by the lifecycle bias, I re-estimate the median regression spline function using subsets of the data based on year of father's birth. Chart 7 plots the results; Table 3 reports the regression estimates. Fathers are grouped by the following birth years: 1939-1943, 1936-1938, 1932-1935, and 1928-1931. The S-shape shows up in all four of the subgroups; the lifecycle bias is rejected as the explanation of the observed pattern. Since the pattern is not caused solely by differences in the observation age of fathers, I will proceed using the full data set in the remaining analysis.

C. Is it Credit Constraints?

In order to determine whether the observed non-linearities are driven by credit constraints that bind for the most able children in an income class, I compare slopes across different quantiles. Recall that the theory suggests that the upper quantiles will be steeper than lower quantiles when the credit constraint binds. If the source of the non-linearities is credit constraints, we should see that the upper quantiles become steeper (relative to the lower quantiles) in the middle of the fathers' income distribution.

Chart 8 plots the 95^{th}, 90^{th}, 75^{th}, 50^{th}, 25^{th}, 10^{th}, and 5^{th} quantile regression splines; Table 4 records coefficient estimates. Again, log income is defined as the log of average wage and salary income from 1978-1982 (1994-1998) for fathers (sons) conditional on reporting positive income in all five years. Contrary to the credit constraint model, among low-income fathers the steepest quantile regressions are for families with low-income (and presumably low-ability) sons. The regression slope gets steeper in the middle of the fathers' income distribution not because top quantile lines get steeper, but because low quantile slopes increase. The slope of upper quantiles is steeper than lower quantiles for high-income fathers which may be consistent with credit constraints if wealthy parents have such able children that their exceptional wealth is unable to efficiently provide schooling. This seems very unlikely. In total, the patterns observed in Chart 8 and Table 4 contradict the credit constraints hypothesis.[9]

One concern with the results shown in Chart 8 and Table 4 is that the requirement of positive incomes in each of the five years produces a selection bias.[10] It may seem reasonable to assume that sons who do not file taxes are likely to be low-ability sons who are more likely to be found in families with low-income fathers. By excluding these observations from the data set, the quantile regression slopes will be biased toward zero. The real question is whether higher quantiles are more or less biased by the selection than lower quantiles are. The appendix shows that under several distribution assumptions the bias is more severe for the lower quantiles if low-income fathers are more likely to have non-reporting sons. This would strengthen the conclusion that the quantile regression slopes in Chart 8 and Table 4 are inconsistent with the credit constraint hypothesis.

The selection bias can be eliminated in the quantile regressions, however. While mean regression estimates are highly sensitive to the treatment of zero-income observations (see Couch and Dunn, 1997), so long as the true value and the value assigned to a zero observation is below the quantile line in question, misreporting the actual value of the observation does not bias the quantile slope estimates. If we assume that sons with zero income or those who do not file at all would fall somewhere below the 25th quantile regression line, then including these observations in the sample and assigning them a value of $1 will not bias the estimates for quantiles higher than the 25th quantile.

Chart 9 plots the quantile regression lines when non-reporting or zero-income sons are recoded as earning $1 in the year in question; Table 5 records the regression spline coefficient estimates. The pattern of the quantile regression slopes is not meaningfully altered by this recoding. Again the non-linear pattern is driven by the shape of lower quantiles, contradicting the credit constraints hypothesis.

V. Discussion of Results

This paper questions the common practice of "testing" for the importance of credit constraints by searching for non-linearities in the relationship between sons' and fathers' log incomes. It is shown that any and every non-linear pattern can be represented as the result of a carefully crafted model with credit constraints. Unlike mean regression analysis, quantile regressions provide the basis for a rejectable test of credit constraints even if the relationship is non-linear in the absence of credit constraints. Since the constraint binds most for families with more-able children, high quantiles should drive the observed non-linearities.

[9] Qualitatively similar results are found when log income is defined as the average log income in the five observation years, when the measure of income is total market income, or when sample inclusion requires an income greater than $500 (1992 dollars) in each year.

[10] The reasons a son may not file include 1) income is so low that taxes are not due, 2) the son is dead, 3) the son works in the underground economy, or 4) the son has left the country. In the first case, it is no doubt safe to assume that the son's income is lower than the 25th quantile regression line. If the son is dead or in the underground economy, we might say that the utility to the father is very low, much like a father whose son earns a low income. I assume that few of the sons left the country or that the decision to leave is not correlated enough with parental income so that the results of the quantile regressions are biased.

Using Canadian tax file data, I find that while non-linearities are present in the data, they are not consistent with the credit constraint model. The pattern in income persistence across fathers' incomes is due to the behaviour of families with low-income sons which are least likely to be constrained. More importantly, the regression slopes are often steeper at low quantiles than at high quantiles which is inconsistent with the credit constraint hypothesis. These results are robust to the inclusion of sons who report no income or fail to file taxes altogether.

Upon reflection, this rejection of the credit constraint model is not surprising. The college and university system is financially supported by the public sector. Individuals whose families are unable to pay the tuition fees can apply for government-subsidized loans. Whether this is common to other Western nations remains to be examined. Eide and Showalter (1999) estimate linear quantile regressions using the PSID data and find that the regression slope falls as the quantile moves from lower to higher quantiles. On the surface, this would suggest that the US data are also inconsistent with the credit constraints model.

My results suggest more effort should be made to allow researchers access to administrative data sets. Corak and Heisz (1999) represents an important contribution in this direction. For the first time a data set large enough to precisely identify non-linearities in multiple quantiles has been provided. As data become available for a variety of countries, it may be possible to determine to what extent credit constraints that limit educational choice remain an important issue for public policy in developed nations.

VI. Appendix: Selection Bias in Quantile Regression

In order to motivate this section, consider a young man who can choose to work and be paid his potential (log) income w_s or to remain unemployed and receive welfare benefits B. The problem of the man is (8)

$$\max_{L=0,1}\{U(w_s, L=0), U(B, L=1)\}$$

where L represents leisure time (and, perhaps, the stigma of welfare benefits). The solution to the problem can be characterized by a critical value for potential income w_s^* such that observed income for the man is (9)

$$w_s = h(w_f) + e \text{ if } w_s > w_s^*$$
$$w_s = 0 \text{ otherwise}$$

where w_f is the log income of the father, and e is a mean-zero error term, independent of w_f. $h(w_f)$ is any positive monotonic function of w_f. The error term may be heteroskedastic, but the variance is assumed non-increasing in w_f. The positive correlation between w_s and w_f ensures that the fraction of truncated observations is a decreasing function of w_f.

Consider how truncation at w_s^* alters the value of the p^{th} percentile son observed within any given father-income class. For any given level of w_f, the lowest $F(w_s^* - h(w_f))$ percent of sons will not work where F is the cdf of e given w_f.

Let (10)

$$p(M, w_s^*) = prob(w_s \leq M \mid w_s^*)$$
$$= \frac{\int_{w_s^*}^{M} f(w_s)dw_s}{\int_{w_s^*}^{\infty} f(w_s)dw_s}$$

where $f(w_s)$ is the conditional pdf of w_s. Since the sample is truncated at w_s^*, the value associated with the p^{th} percentile positive observation (M) overestimates the true value (including those who do not work). Applying the implicit function theorem

(11)

$$p_M = \frac{f(M)}{\int_{w_s^*}^{\infty} f(w_s)dw_s}$$

(12)

$$p_{w_s^*} = \frac{-f(w_s^*)\int_{M}^{\infty} f(w_s)dw_s}{\left(\int_{w_s^*}^{\infty} f(w_s)dw_s\right)^2}$$

(13)

$$\frac{dM}{dw_s^*} = -\frac{p_{w_s^*}}{p_M}$$
$$= \frac{f(w_s^*)\int_{M}^{\infty} f(w_s)dw_s}{f(M)\int_{w_s^*}^{\infty} f(w_s)dw_s}$$
$$> 0$$

But is the bias larger or smaller for high quantiles? If the truncation point w_s^* is held fixed, the change in p affects $\frac{dM}{dw_s^*}$ through its effect on M: (14)

$$\frac{\partial\left(\frac{dM}{dw_s^*}\right)}{\partial p} = \frac{\partial\left(\frac{dM}{dw_s^*}\right)}{\partial M}\frac{\partial M}{\partial p}.$$

Since $\frac{\partial M}{\partial p} > 0$, we can focus on the sign of $\frac{\partial\left(\frac{dM}{dw_s^*}\right)}{\partial M}$. (15)

$$\frac{\partial\left(\frac{dM}{dw_s^*}\right)}{\partial M} = f(w_s^*)\int_{w_s^*}^{\infty} f(w_s)dw_s \frac{-f(M)^2 - f'(M)\int_M^{\infty} f(w_s)dw_s}{\left(f(M)\int_{w_s^*}^{\infty} f(w_s)dw_s\right)^2}$$

It is clear in expression (15) that if *f'(M)>0*, the truncation biases higher percentiles less than lower percentiles. For example, if the sons' potential incomes are distributed uniformly (conditional on w_f), truncating the distribution shifts upward the position of the 90th percentile observation less than the position of the 20th percentile observation.

Assuming instead that sons' potential log incomes are distributed normally with conditional mean *m* and conditional variance s^2, I substitute the particular expressions for the normal distribution for *f* and *f'*. Since the numerator in expression (15) determines the sign, I will only pursue this portion of the expression:

$$-\frac{1}{s^2 2p} e^{-\left(\frac{M-m}{s}\right)^2} + \left(\frac{M-m}{s^2}\right)\frac{1}{\sqrt{s^2 2p}} e^{-.5\left(\frac{M-m}{s}\right)^2} \int_M^{\infty} f(w_S)dw_S \tag{16}$$

Rearranging (16) shows that it has the same sign as

$$-1+\left(\frac{M-m}{s}\right)\sqrt{2p}\, e^{.5\left(\frac{M-m}{s}\right)^2}\left[1-\Phi\left(\frac{M-m}{s}\right)\right] \tag{17}$$

Where Φ is the standard normal c.d.f.. Chart 10 plots expression (17); it is always negative. Just as in the case of the uniform distribution, a one unit increase in the truncation point shifts upward the position of the high percentile observations less than the position of the low percentile observations.

In the model of potential earnings and unemployment above, the fraction of truncation is negatively correlated with income. So the quantile slopes are biased toward zero. And the magnitude of this bias is larger for lower quantiles since $\frac{\partial\left(\frac{dM}{dw_s^*}\right)}{\partial p}<0$.

Table 1. Estimates of income persistence using quadratic functions. Absolute t-values given in parentheses.

	Behrman and Taubman (1990)	Solon (1992)
b_1	-0.46 (2.7)	-0.108 (0.14)
b_2	0.039 (4.3)	0.0258 (0.63)

Table 2. Median and mean spline function regression estimates. Absolute t-statistic in parentheses.

	Median	Mean
[-2,-1]	0.045 (0.92)	0.085 (1.78)
[-1,0]	0.269 (17.05)	0.260 (15.70)
[0,1]	0.269 (15.70)	0.306 (18.32)
[1,2]	0.142 (1.86)	0.265 (3.54)

Table 3. Median spline function regression slope estimates for four father-birth-year groups. Absolute t-statistics in parentheses.

	Father Birth Years			
	1939-1943	1936-1938	1932-1935	1928-1931
[-2,-1]	-0.061 (0.56)	0.190 (1.40)	0.016 (0.20)	0.046 (0.37)
[-1,0]	0.323 (9.41)	0.344 (8.07)	0.289 (9.69)	0.223 (5.14)
[0,1]	0.322 (8.87)	0.257 (6.06)	0.255 (8.63)	0.233 (5.49)
[1,2]	0.202 (1.00)	0.201 (0.86)	0.192 (1.59)	0.039 (0.234)

Table 4. Father-son log income quantile regression slope estimates. Absolute t-statistics in parentheses.

	Quantiles						
	95	90	75	50	25	10	05
[-2,-1]	-0.059 (0.85)	-0.129 (5.85)	-0.093 (2.27)	0.045 (0.92)	0.080 (1.13)	0.291 (2.34)	0.344 (2.27)
[-1,0]	0.098 (3.61)	0.143 (7.48)	0.211 (14.32)	0.290 (17.05)	0.368 (15.18)	0.383 (8.82)	0.379 (6.95)
[0,1]	0.423 (14.90)	0.313 (16.15)	0.294 (20.26)	0.269 (15.70)	0.269 (10.91)	0.270 (6.14)	0.240 (4.31)
[1,2]	0.625 (6.06)	0.505 (5.85)	0.232 (3.43)	0.142 (1.86)	0.149 (1.40)	0.288 (1.60)	0.103 (0.47)

Table 5. Father-son log income quantile regression slope estimates; zero-income and non-reporting sons included. Absolute t-statistics in parentheses.

	Quantiles				
	95	90	75	50	25
[-2,-1]	-0.087 (1.74)	-0.081 (1.72)	-0.104 (2.42)	0.002 (0.03)	0.623 (3.49)
[-1,0]	0.075 (4.44)	0.143 (9.03)	0.241 (16.89)	0.420 (18.52)	0.777 (13.37)
[0,1]	0.374 (21.6)	0.296 (18.56)	0.268 (18.55)	0.219 (9.34)	0.004 (0.06)
[1,2]	0.369 (6.82)	0.197 (3.86)	0.023 (0.51)	-0.032 (0.44)	-0.262 (1.37)

Chart 1. Choosing a child's education given ability.

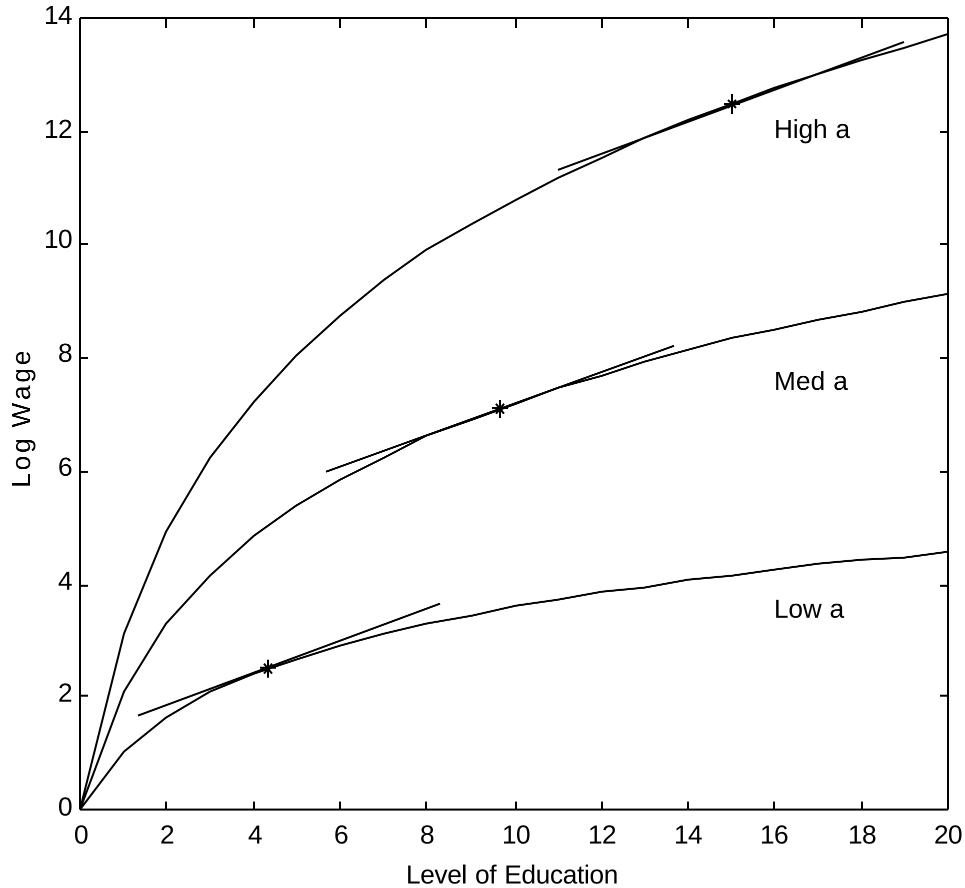


Chart 2. Four patterns that can be justified by a version of the credit constraints model.

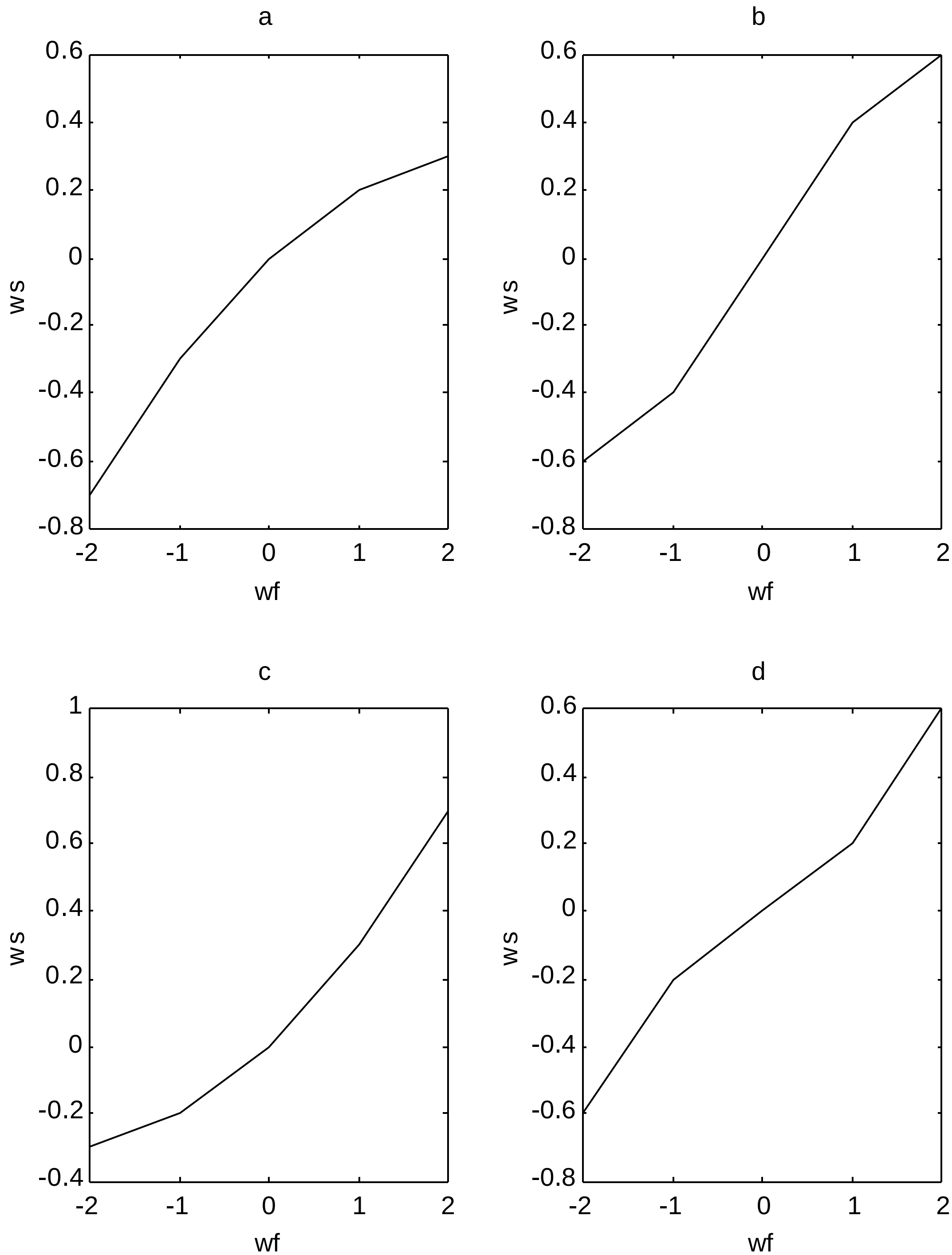


Chart 3. Predicted quantile regression line pattern.

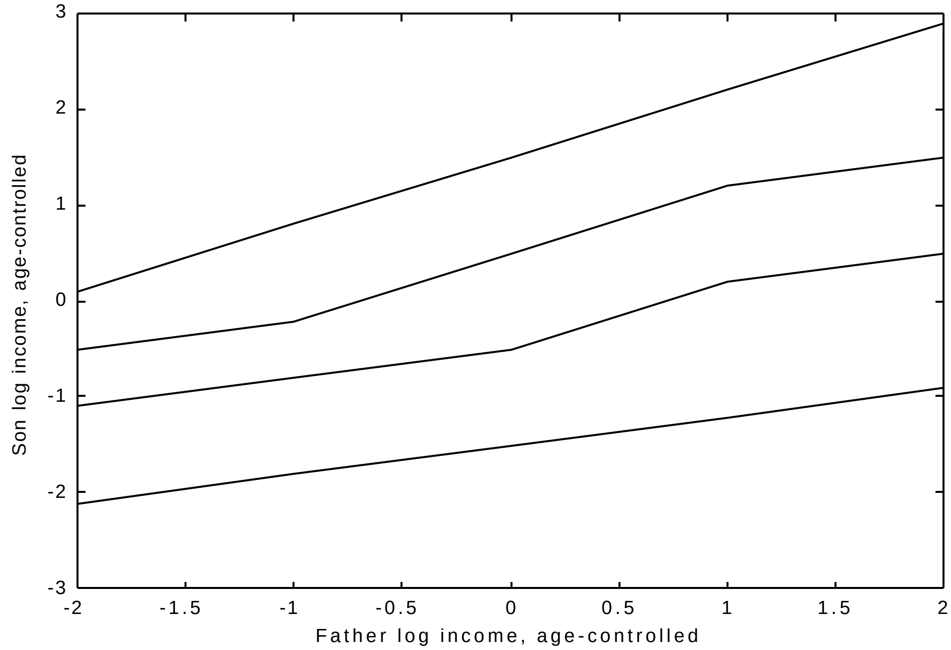


Chart 4. Median (o) and mean (*) spline regression using the full date set.

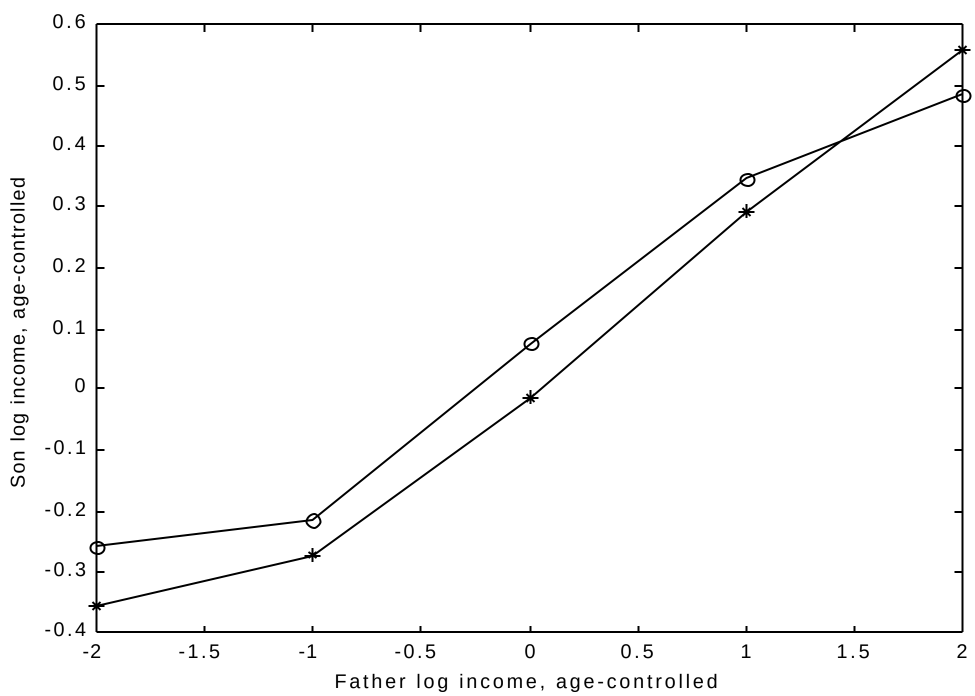


Chart 5. Age-income profiles before and after detrending and measurement error correction.
a, Raw age-income profiles. *b*,detrended error-purged age income profiles.

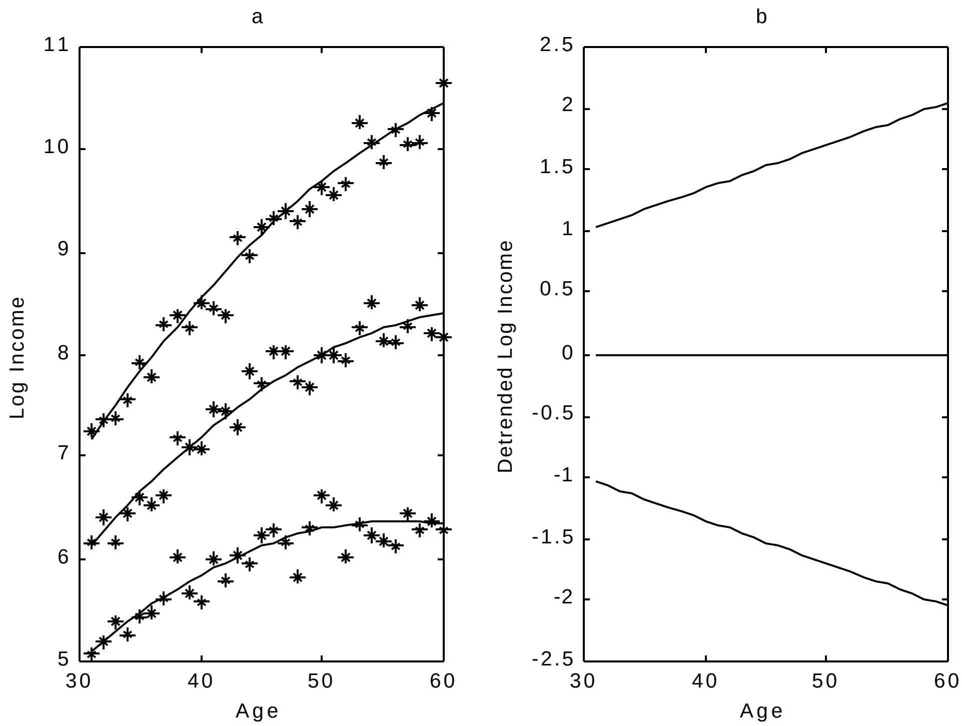


Chart 6. Estimates of income persistence across years of father observation.

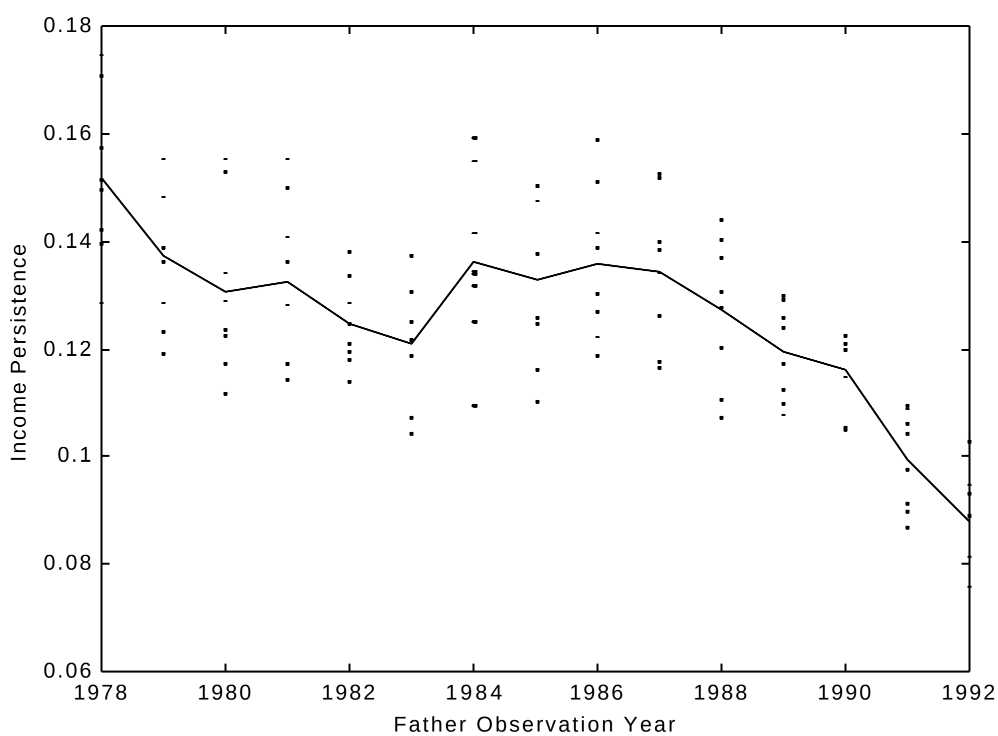


Chart 7. Estimated spline functions for subsets based on father's year of birth; *a*, 1939-1943; *b*, 1936-1938; *c*, 1932-1935; *d*, 1928-1931.

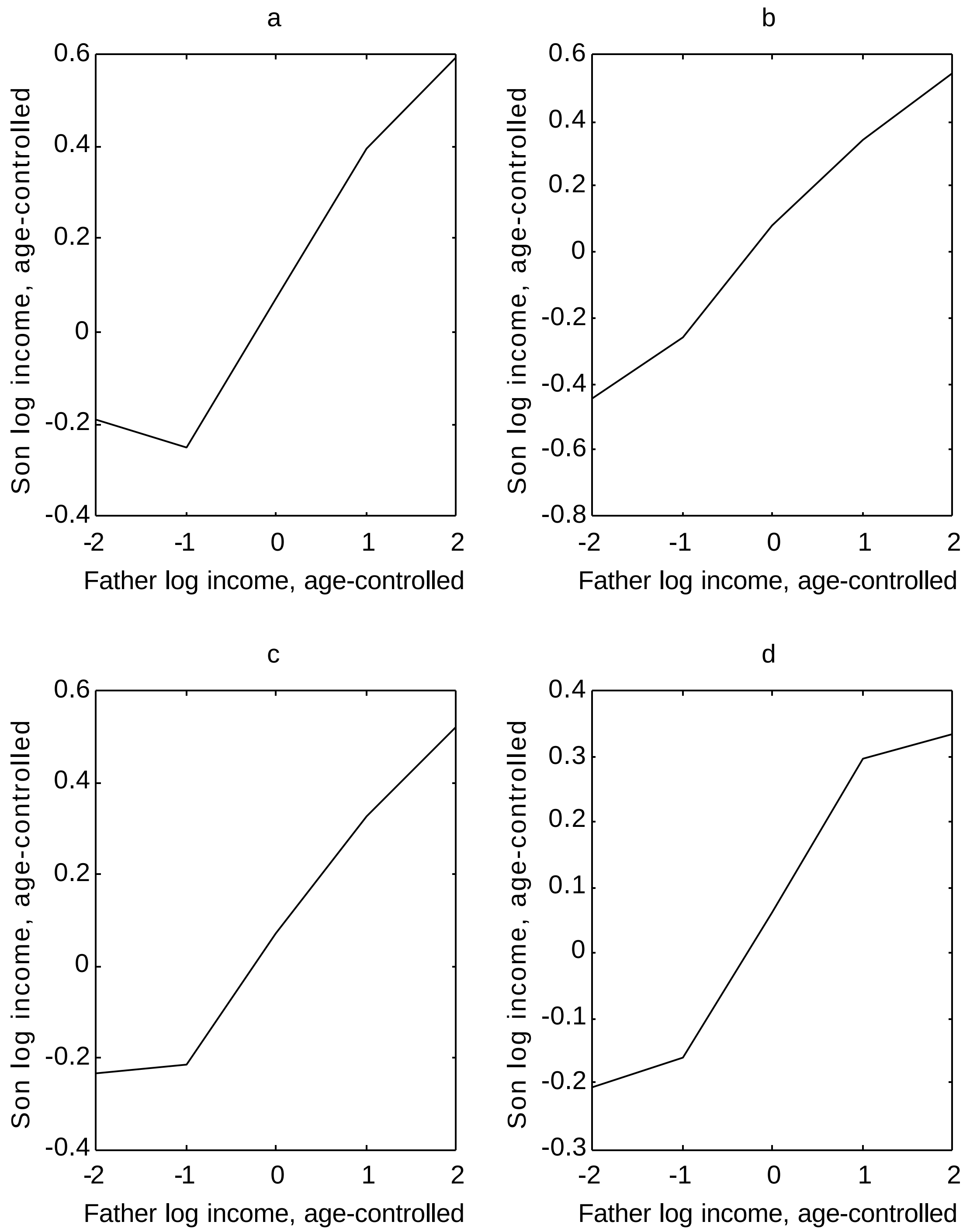


Chart 8. Non-linear quantile regression lines for Canadian men.

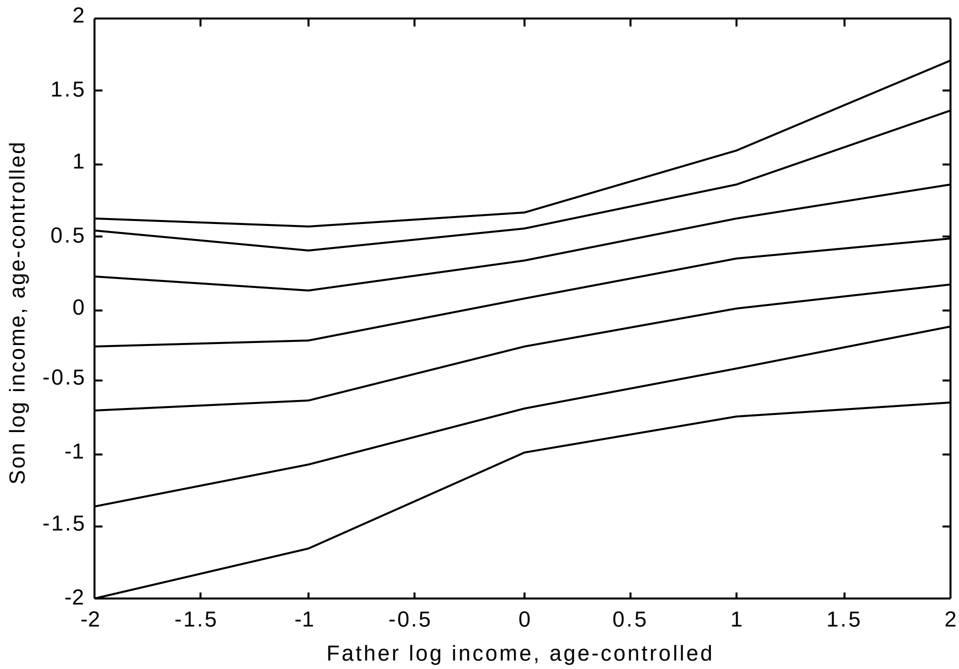


Chart 9. Non-linear quantile regression lines for Canadian men; zero-income and non-reporting sons included.

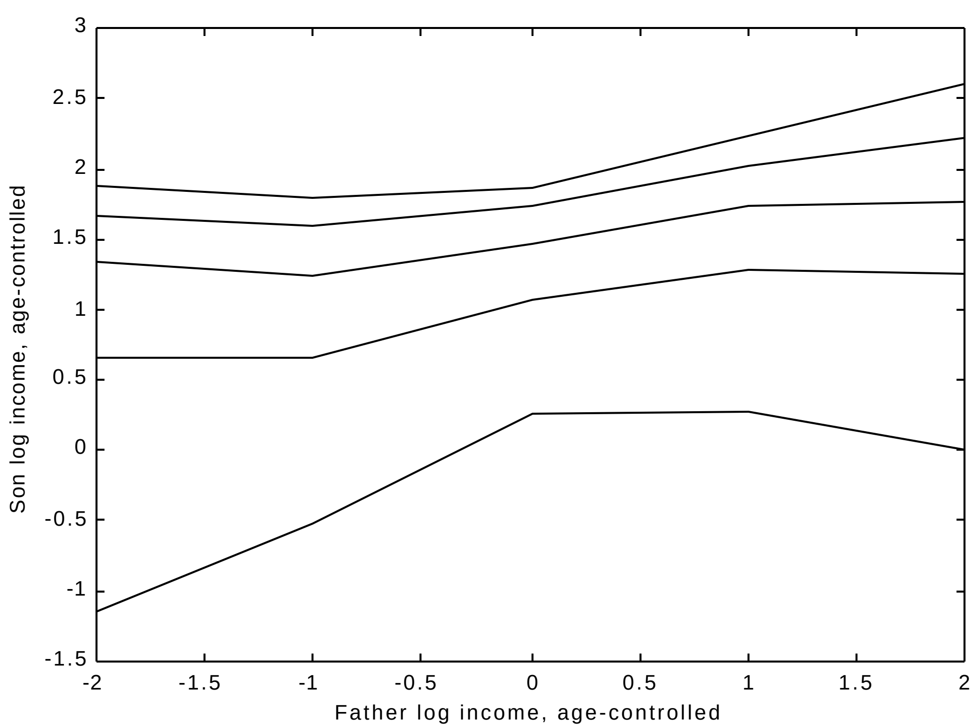
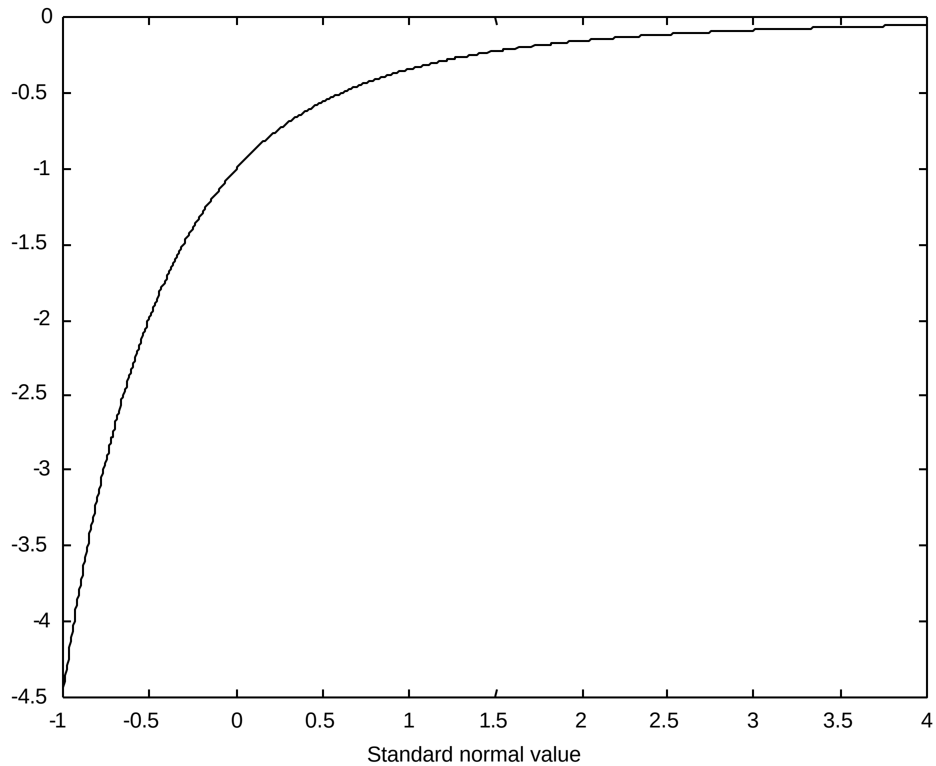


Chart 10. Value of expression (17).



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