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COMMUNICATING VARIANCES FOR INTERPRETATION OF CHANGES AND TURNING POINTS IN REPEATED SURVEYS

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ABSTRACT

The estimates obtained from repeated surveys are used to monitor and analyse changes in variables of interest over time. Major surveys are often conducted on a monthly or quarterly basis and enable identification of important changes in the level or rate of change of variables, including turning points. The samples used by these surveys are often large and involve overalp of the sample over time through use of a sample rotation procedure. The sample size and the correlation structure in the estimates induced by the rotation pattern affect the sampling errors of the estimates and the analysis of changes in them. Users will analyse and interpret the time series of estimates in various ways often involving estimates for several time periods. Despite the large sample sizes and degree of overlap between the sample for some periods the sampling errors can still substantially affect the estimates of movements and functions of them used to interpret the series of estimates. We consider how to account for sampling errors in the interpretation of the estimates from repeated surveys and how to inform the users and analysts of their possible impact.

KEY WORDS: Movement Estimates; Rotation Patterns; Trend; Turning Points.

1. INTRODUCTION

1.1 Repeated Surveys

Estimates obtained from repeated surveys are used to monitor and analyse changes in variables of interest over time. Major social and economic surveys are often conducted on a monthly or quarterly basis and aim to identify important changes in the level or rate of change of variables, including turning points. These surveys often use large samples and designs that involve overlap of the sample over time through use of a sample rotation procedure. The sample overlap induces a correlation structure in the time series of estimates, which affects the sampling errors of the estimates and the analysis of changes in them.

Users will analyse and interpret the time series of estimates in various ways involving estimates for several time points. Typically the correlation of the variable at the unit level for any two time points will vary according to the time lag involved. It will usually decay as we consider larger lags, although seasonality may have an effect. The correlation between the estimates for two time points will also depend on the sample overlap as well as the unit level correlation. No sample overlap between two time periods will result in zero correlation of the estimates. Even if there is sample overlap the unit level correlation may be low leading to low correlation between the estimates. If there is high overlap and unit correlation then the estimates will have high correlation. These differences in correlation between estimates complicate the analysis and interpretation of the estimate arising from a repeated survey.

Despite the large sample sizes and degree of overlap between the samples for some points, the sampling errors can substantially affect the estimates of movements and functions of them used to interpret the series of estimates. We need to consider how to account for sampling errors in the interpretation of the estimates from repeated surveys and how to inform the users and analysts of their possible impact.

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Examples of major repeated surveys include; Labour force surveys, Retail trade surveys and various other Economic indicator surveys.

2. REPEATED SURVEYS DESIGN AND ANALYSIS

2.1 Rotation Patterns

For repeated surveys the sample is often designed so that there is substantial sample overlap between successive surveys. This reduces the sampling variances of estimates between the two surveys and reduces costs compared with an independent sample each period. Overlapping samples are produced using a rotation pattern that can also be designed to spread the reporting load. A reasonably general class rotation patterns involves selecting a unit for *a* consecutive months, removing it for *b* months and then including it for a further *a* months. This process is repeated until units are included for a total of *m* months. This class can be denoted by a-b-a(m). Setting b=0 gives an in-for-m rotation pattern. See Rao and Graham (1964).

Examples

The Australian Monthly Labour Force Survey (MLFS) uses an in-for-8 rotation pattern. This pattern leads to a theoretical sample overlap at lag s, v_s = 1-s/8 for s=1,..,7. The actual overlap is a little less because of movement of people and households between surveys. With this design there is no sample overlap at lag s=12. This pattern gives high sample overlap at short lags, which helps reduce sampling variances on estimates of short term changes of, say, 1, 2 or 3 months.

The Canadian MLFS uses an in-for-6 rotation pattern. This pattern leads to a theoretical sample overlap at lag s, v_s = 1-s/6 for s=1,..,5. With this design there is again no sample overlap at lag s=12. This pattern also gives high sample overlap at short lags which helps reduce sampling variances on estimates of changes of 1, 2 or 3 months.

The United States Current Population Survey uses a 4-8-4(8) rotation pattern. The resulting sample overlap v_s = 1-s/4 for s=1,..,3, 0 for s=4,...,8 and v_s = 4/8-|s-12|/8 for s=9,…,15. The sample overlap for s=12 is v_{12} = 4/8. The sample overlap reduces quite quickly as the lag being examined increases although it reappears after 9 months.

The UK LFS is designed as a quarterly survey but can also be regarded as a monthly sample with a 1-2-1(5) rotation pattern. The sample overlap v_s = 1-s/(3m) for s=3, 6, 9..3m, and v_s = 0 otherwise. There is a small sample overlap v_{12} = 1/5 for s=12.

The Australian Monthly Retail Trade survey uses a rotation pattern for the smaller businesses in which 1/12th of business are rotated out of the survey every three months and no rotation is undertaken in the other two months of the quarter. This leads to smaller busineesses being in the sample for no more than 36 months.

These examples show that there are a number of different rotation patterns in use. Different rotation patterns induce different degrees of sample overlap. They will induce correlations between estimates for time periods with appreciable sample overlap.

2.2 Key outputs and analyses

Repeated surveys can provide estimates for each time periods, $\hat{y}_t$. Given the expense of running them on a repeated basis, the real value of repeated surveys is in their ability to provide estimates of change, such as

$$\hat{y}_t - \hat{y}_{t-s} = \Delta^{(s)}\hat{y}_t .$$

The focus is often on s=1, but for a survey repeated on a monthly basis changes for s=2, 3, 12 are also commonly examined. Having sample overlap at lag s will usually lead to a positive correlation between the estimates. Since

$$Var(\Delta^{(s)}\hat{y}_t) = Var(\hat{y}_t) + Var(\hat{y}_{t-s}) - 2\sqrt{Var(\hat{y}_t)}\sqrt{Var(\hat{y}_{t-s})}Corr(\hat{y}_t, \hat{y}_{t-s}) \quad (1)$$

this reduces the variance of $\Delta^{(s)}\hat{y}_t$ compared with having no sample overlap.

There may also be interest in changes in the rate of change, such as

$$\Delta^{(s)}\hat{y}_t - \Delta^{(s)}\hat{y}_{t-k} = \hat{y}_t - \hat{y}_{t-s} - (\hat{y}_{t-k} - \hat{y}_{t-k-s}).$$

If s=k this becomes

$$\Delta^{(s)}\hat{y}_t - \Delta^{(s)}\hat{y}_{t-s} = \hat{y}_t - 2\hat{y}_{t-s} + \hat{y}_{t-2s}.$$

When a monthly or quarterly repeated survey has been conducted for several years then a time series can be produced and analysed. Seasonally adjusted estimates are often produced to help interpretation of the time series, giving the series $\hat{x}_t$. Seasonal adjustment is not a linear process, but linear approximations are available. To assess the underlying pattern of change trend estimates can also be produced, which raises the question of what is trend? In some cases it is taken to be $\Delta^{(s)}\hat{y}_t$ or $\Delta^{(s)}\hat{x}_t$. The Australian Bureau of statistics often produced trend estimates using Henderson moving averages applied to the seasonally adjusted series (ABS, 1987). Other options are available.

A common opinion is:" the first time something happens it is a blip; the second is a coincidence; the third makes a trend" (*The Economist*, *1990)*. This statement reflects what many users do, that is examine $\Delta^{(s)}\hat{y}_t$ for s=1, 2, 3.

All these approaches to analysis and interpretation are linear functions of the time series ending at time t, $\hat{\mathbf{y}}_t$, $l^T\hat{\mathbf{y}}_t$, where l is some vector of coefficients. The associated sampling variance is

$$Var(l^T\hat{\mathbf{y}}_t) = l^T Var(\hat{\mathbf{y}}_t) l.$$

So, in general we need an estimate of the covariance matrix $Var(\hat{\mathbf{y}}_t)$, which involves estimation of covariances (or variances of differences of estimates) for a number of estimates. With such variance estimates we can construct confidence intervals and test hypotheses, which will often be of the form

$$l^T\hat{\mathbf{y}}_t > a.$$

The rotation patterns in use tend to emphasise producing high sample overlap at lag s=1 and to a lesser extent s=2, 3, and 12. The rotation pattern1-2-1 (5) induces overlap at lags 3, 6, 9, 12 and may be useful for some approaches to analysis, such as examining three month movements and use of three month averages (see Steel, 1997).

In looking at the interpretation of the recent changes we can look at quantities such as

$$\text{Prob}(l^T\hat{\mathbf{y}}_t > 0 \mid l^T\mathbf{Y}_t) \text{ and } \text{Prob}(l^T\hat{\mathbf{y}}_t \text{sig} \mid l^T\mathbf{Y}_t).$$

2.3 Assessing Power to Detect Changes

While repeated surveys often involve large sample sizes and degree of sample overlap between consecutive time periods, they are often trying to estimate relatively small changes. For example the standard error on estimates of monthly change in the unemployment rate for the Australian MLFS is about 0.1 percentage points, even with the high monthly sample overlap of 7/8ths and with a sample size of over 30,000 households. One month changes in the unemployment rate are often about this size and hence not statistically significant. The value of the survey is in how quickly it will give evidence of an important change.

In an evaluation of options for producing monthly labour force estimates for Great Britain, Steel (1996, 1997) examined the standard errors and power associated with various options. He considered a survey using a sample of 60,000 households and an in-for-6 or 1-2-1(5) rotation pattern and analysis based on the monthly estimates and also quarterly averages. Table 1 shows the standard errors (SEs) for estimates of changes for different lags. At the time the level of unemployment was around 2.4m and averages monthly changes were about 30,000. The first column of figures shows the impact of the high sample overlap in reducing the standard errors on one month change and how the standard error increases as changes over longer periods are examined due to the reduced sample overlap and the decay in the unit level correlation. The second column of figures shows the effect of the sample overlap only occurring at multiples of 3 months. If the analysis focused on three months changes then this option produces lower standard errors. The last two columns five corresponding results when the analysis is based on the quarterly averages, which some users adopt to crudely handle the volatility of the monthly estimates. Here the 1-2-1(5) option is superior to the in-for-6 option because it involves averaging over uncorrelated estimates, whereas the in-for-6 option involves averaging over positively correlated estimates.

Table 1: SEs (000s)of change over 6 months for monthly estimates and quarterly averages, n=60,000, unemployment

	Monthly Estimates		Quarterly Averages	
s	in-for-6	1-2-1(5)	in-for-6	1-2-1(5)
1	28	42	12	11
2	33	42	20	15
3	36	33	24	19
4	39	42	30	20
5	41	42	32	20
6	42	37	34	21
7	42	42	34	22

Steel (1996) also considered the question of the power that the different options would have. He considered a hypothetical situation in which unemployment was steadily increasing at 30,000 a month until April and then decreased at the same rate. Table 2 gives the probablity of obtaining an estimate of the correct sign and also a statistically significant change, using a one-sided 5% test by looking at different lags at each of the months following April. For example if we examine the change between May and April the in-for-6 rotation pattern has an 86% chance of a change of the correct sign but only a 26% chance of a statistically significant change. By July, that is looking at the 3 month change, the probability increase to 100% and 80% respectively. These results are consistent with the opinion expresses in section 2.2 of analysts only be reasonably confident after three changes. Looking at the results for the 1-2-1(5) rotation pattern we see that the larger standard errors for lags 1 and 2 lead to lower probabilities, but the lower SEs for lag 3 results in higher probabilities from June onwards.

Table 2: Power of analysis of *s* month differences – turning point in April, monthly change of 30,000, unemployment

Rotation	in for 6			1-2-1(5)		
Lag s	1	2	3	1	2	3
Analysis Month	Prob (%) estimated change has correct sign					
May	86	50	21	76	50	18
June		97	80		92	82
July			100			100
Analysis Month	Prob (%) estimated change stat sig (one-sided 5% test)					
May	26	5	1	18	5	1
June		57	21		41	24
July			80			87

Producing tables such as Table 2 for different situations may help analysts in interpreting the ability of the survey to inform them of important changes.

3. GRAPHICAL PRESENTATION OF CONFIDENCE INTERVALS ON IMPORTANT CHANGES

3.1 Inferences about changes

In theory we can calculate an estimate of the variance for any linear function and present them in say a table. However, issues of simultaneous inference arise. It would be useful to use simple graphical methods to communicate how to interpret changes in the series of estimate to users and analysts.

Suppose the main focus is on $\Delta^{(s)}\hat{y}_t$. An approximate 95% confidence interval for $\hat{y}_t$ can be constructed as $\hat{y}_t - 2\sqrt{\hat{V}ar(\hat{y}_t)}$ to $\hat{y}_t + 2\sqrt{\hat{V}ar(\hat{y}_t)}$, where $\hat{V}ar(\hat{y}_t)$ is an estimate of the variance of accounting for the sample design. Similar confidence interval can be constructed for $\hat{y}_{t-s}$.

Plotting these intervals for each time point is valid for indicating the CI for each estimate. However, if users examine whether or not these confidence intervals overlap to assess the statistical significance of the change then they will be in error, as they imply a confidence interval of

$$\hat{y}_t - \hat{y}_{t-s} - 2\left(\sqrt{\hat{V}ar(\hat{y}_t)} + \sqrt{\hat{V}ar(\hat{y}_{t-s})}\right) \text{ to } \hat{y}_t - \hat{y}_{t-s} + 2\left(\sqrt{\hat{V}ar(\hat{y}_t)} + \sqrt{\hat{V}ar(\hat{y}_{t-s})}\right)$$

instead of an appropriate interval of

$$\hat{y}_t - \hat{y}_{t-s} - 2\left(\sqrt{\hat{V}ar(\Delta^{(s)}\hat{y}_t)}\right) \text{ to } \hat{y}_t - \hat{y}_{t-s} + 2\left(\sqrt{\hat{V}ar(\Delta^{(s)}\hat{y}_t)}\right),$$

where $Var(\Delta^{(s)}\hat{y}_t)$ is given by (1).

Assume $Var(\hat{y}_t) \approx Var(\hat{y}_{t-s})$. If we scale the original confidence intervals on the estimates of levels by a factor of $\sqrt{\frac{1-Corr(\hat{y}_t,\hat{y}_{t-s})}{2}}$, that is use

$$\hat{y}_t - \sqrt{2(1-Corr(\hat{y}_t,\hat{y}_{t-s}))}\sqrt{\hat{V}ar(\hat{y}_t)} \text{ to } \hat{y}_t + \sqrt{2(1-Corr(\hat{y}_t,\hat{y}_{t-s}))}\sqrt{\hat{V}ar(\hat{y}_t)},$$

then the confidence interval for $\hat{y}_t - \hat{y}_{t-s}$ implied by looking at whether the intervals overlap is appropriate.

This factor is given in Table 3 are based on correlation models developed by Steel (1996) for the variables of unemployment and employment. The factor of 0.71 corresponds to the case that the estimates are uncorrelated. The factors are all less than 1 and show that is necessary to make the adjustment. For each variable the factor depends on the lag being examined. It is straightforward to present adjusted intervals for one key lag, say s=1. Simple methods to simultaneously present these adjusted intervals for several lags need to be developed to aid users appropriately interpret the estimates.

Table 3: Factor to adjust level confidence intervals for analysis of movements at different lags, original series

	Unemployment		Employment	
s	In-for-6	1-2-1(5)	In-for-6	1-2-1(5)
1	0.47	0.71	0.38	0.71
2	0.56	0.71	0.47	0.71
3	0.61	0.54	0.54	0.42
4	0.66	0.71	0.60	0.71
5	0.69	0.71	0.66	0.71
6	0.71	0.62	0.71	0.52
7	0.71	0.71	0.71	0.71

REFERENCES

Australian Bureau of Statistics (1993,) "*A Guide to Interpreting Time Series - Monitoring "Trends", An Overview*", catalogue no 1348.0 , Canberra, Australia: Australian Bureau of Statistics.

McLaren, C. H. and Steel, D. G. (2000), "The Impact of Different Rotation Patterns on the Sampling Variance of Seasonally adjusted and Trend Estimates", *Survey Methodology*, 26, pp. 163-172.

Rao, J. N. K. and Graham, J. E. (1964), "Rotation Designs for Sampling on Repeated Occasions", *Journal of the American Statistical Association*, 69, pp. 492-509.

Steel, D. (1997), "Producing Monthly Estimated of Unemployment and Employment According to the International Labour Office Definition", *Journal of the Royal Statistical Society*, 160, pp. 5-33.

Steel, D. G. (1996) "Options for Producing Monthly Estimates of Unemployment According to the ILO Definition", unpublished report, London, UK: Central Statistics Office.

The Economist (1990), "Spotting a Trend", *The Economist*, March 17 edition, London, UK